International Journal of Engineering Innovation & Research
Volume 6, Issue 1, ISSN: 2277 – 5668

Partial Wave Analysis of Resonances Amplitude Based on Monte Carlo Methods

Hao Yang[a,b], Lingfang Yin[a] and Xiangyang Xu[b*]
[a]Jiangsu University of Science and Technology, Zhenjiang City, Jiangsu Province, P. R. China.
[b]Geodetic Institute, Faculty of Civil Engineering and Geodetic Science, Leibniz University Hanover, Hanover, Germany
*Corresponding author email id: xu@gih.uni-hannover.de

Date of publication (dd/mm/yyyy): 07/02/2017

***Abstract* – In this paper an innovative method which is titled as partial wave analysis is applied to engineering practice based on Monte Carlo Methods. Though analyses of the point likely data, significant contributions can be attributed to amplitude distributions which are observed in surface approximation. The data here is same as the point cloud which including the information of three dimensional space and intensity value of every point. The computational techniques of the performed partial wave analysis, namely a independent extended maximum likelihood fit followed by a surface approximation.**

***Keywords* – Monte Carlo Methods, Partial Wave Analyses, Resonances, Intensity, Amplitude.**

I. INTRODUCTION

In the 90s of the last century high statistics experiments have leaded to a better insight in the spectrum of hadrons [1]. In particular the finding of crypto exotic and J^{PC} exotic states tremendously improved the experimental situation in meson spectroscopy. All this was possible only with sophisticated analysis methods like the decomposition of measured phase-space distribution into partial waves and to express the partial waves in terms of complicated dynamical functions [1].

The basic task is to find all resonances, with their static properties like mass, width, spin and parities. This is a very demanding task, since a lot of resonances overlap. In addition complicated production processes or scattering with many waves in the intermediate state complicate the situation [2].

Generally PWA begins with some physics assumptions concerning the reaction process. First of all we summarize them for the analysis presented in this work and also point out their implications. Furthermore, a PWA is always based on the spin formalism, which employs certain reference frames and decides on the concrete representation of spin states and angular distributions.

Therefore I will briefly introduce a very common approach. The computational techniques of the performed PWA [5], namely a mass-independent extended maximum likelihood fit followed by a mass-dependent fit.

II. EXPERIMENT

The COMPASS collaboration has extracted large data sets, covering an unprecedented range of invariant masses, and hopes to clarify the situation. In this experiment [3] took data with a 190 GeV π^- beam impinging on a liquid hydrogen target, aiming at collecting large samples of data for spectroscopy.

COMPASS is one of the fixed target experiments at CERN. The COMPASS spectrometer was assembled in 1999-2000 and commissioned during a technical run in 2001. The first data taking period started in summer 2002 and ended in fall 2004. The experiment had its first shut down in 2005 for repairs, refits and new installations. The years 2006 and 2007 were dedicated to the muon run, 2008 and 2009 to the hadron programme. In 2010 a polarized proton was used again to study structure functions. The construction of the spectrometer is shown in Figure 1 [4].

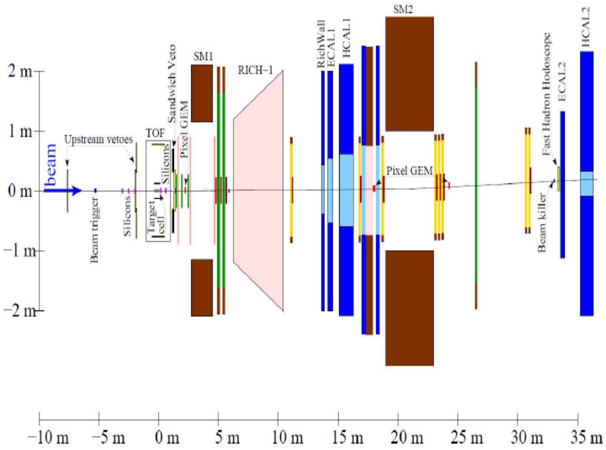

Fig. 1. Schematic design of the spectrometer

The Fig.1 provides an overview of the 2-stage COMPASS spectrometer indicating important parts. The target was surrounded by a Recoil Proton Detector (RPD) measuring the signature of diffractive processes, the recoil proton. The final states, neutral and charged, of decaying resonances excited in the target were identified and measured by the spectrometer behind the target. The principle of a spectrometer is bending of charged particle tracks by dipole magnets in order to measure their momentum with high accuracy and acceptance over a wide momentum range [4].

III. EVENT SELECTION

The reactions $\pi^- p \rightarrow Xp$, $X \rightarrow \pi^- f_1$, $f_1 \rightarrow \pi^-\pi^+\eta$, $\eta \rightarrow \pi^-\pi^+\pi^0$, $\pi^0 \rightarrow \gamma\gamma$ with an incoming π^-beam of 190 GeV/c were described. The event selection was explained in detail with the goal to select favored events and minimize background events by applying cuts. The mass distributions of $\pi^-\pi^+\pi^0$ and $\pi^-\pi^+\eta$ are shown in Fig. 2 [5].

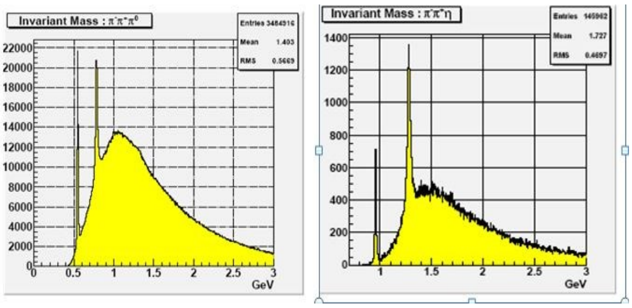

Fig. 2. Distribution of the events as a function of the invariant mass in the π^-p reaction after the selection $\pi^0\rightarrow\gamma\gamma$. The left one is invariant mass of $\pi^-\pi^+\pi^0$ with the cut window: $|m_{\gamma\gamma} - m_{\pi 0}| < 20\text{MeV/c}^2$; the right one is invariant mass of $\pi^-\pi^+\eta$ with the cut window: $|m_{\pi-\pi+\pi 0} - m_\eta| < 20\text{MeV/c}^2$.

IV. DATA ANALYSIS

4.1 Amplitude Analysis

In this paper we focus the reactions on two pseudo scalars decays. The angular momentum l of a system of two pseudo scalars is given by their orbital angular momentum. The amplitude for this system with angular momentum *l* and angular momentum projection at z-axis given by m is by elementary quantum mechanics given by the spherical harmonic Y_l^m with the usual spherical coordinates. For a fixed mass of the two pseudo scalars decays system, its phase space is completely described by the angular variables. The spherical harmonics are not eigen functions of parity, while the strong interaction conserves parity[6]. Therefore, the requirement of parity conservation is most usefully implemented by defining states as eigen functions with respect to reflections on the production plane which equals the parity operation followed by a rotation which takes the particle momenta back to their original values [7]. The quantum number corresponding to this operation is called reflectivity and takes value -1 and +1. Then full set of quantum numbers are ε ,l, m where ε = ±1, $0 \leq m \leq l$ and m = 0 only for ε = -1. The corresponding basis for the two - pseudo scalar states is formed by the function 1 [8]:

$$Y_m^{\in\ell}(\theta,\phi) = c_m\left(Y_m^{\ell}(\theta,\phi) - \in(-1)^m Y_{-m}^{\ell}(\theta,\phi)\right)$$

Where c_m is $(1/2)^{1/2}$ for the case that m is not 0 (reflecting the orthogonality of the Y_l^m) and 1/2 for the case that m is 0 where the two spherical harmonics are equal. It is worth noting that ε is -1 times the eigen value of the reflection operator when applied to the state.

The natural frame of reference in which to define the angles going into the spherical harmonics of function 1 is the Gottfried-Jackson frame, also called t-channel helicity frame. This frame is defined as the right-handed rest-frame of the produced state whose axes are chosen such that the x-axis lies in the production plane, the y-axis is orthogonal to it, and the z-axis is along the line of the exchanged momentum, i.e. also fixing the sign, the beam direction as seen in the rest frame.

Traditionally, analysis of the two pseudo scalar decay system includes all amplitudes of spin up to a total of two and m≤1. Following the convention in the literature, we shall label the waves using a letter from spectroscopic notation S, P, D . together with a subscript +, − or 0, which indicates naturality and spin sub state at the same time, where + indicates m = 1 natural exchange waves, − indicates m = 1 unnatural exchange wave, and 0 indicates m = 0 waves, which can only correspond to unnatural exchange [10]. We include two additional waves in the analysis: the m = 2 natural exchange spin-2 wave, labeled D_{++}, and the m = 1 natural exchange spin-4 wave, labeled G_+. Note that one can't add the D_{--} wave at the same time as the D_{++} wave as this would lead to an under determination of the angular distributions [9], but this wave is expected to be strongly suppressed.

We show in Figure 3 exemplary angular distributions derived from these amplitudes. For the main waves of interest, which are all natural-parity exchange waves with m = 1, the distribution in ϕ_{GJ} is independent of the particular wave, and therefore not depicted. We can thus show the relevant information in two dimensions, plotting the relative phase shift between the waves on the horizontal axis and $\cos\vartheta_{GJ}$ along the vertical axis. In the Figures we see that changes in relative phase between the waves alter the distributions in $\cos\vartheta_{GJ}$ significantly. This is what the fit utilizes to extract phase information [10].

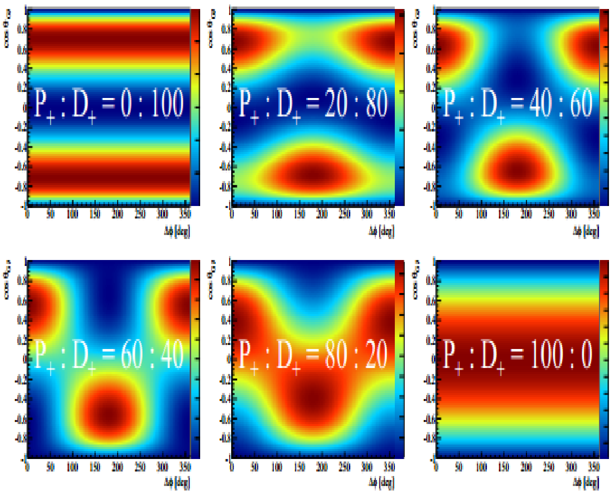

Fig. 3. Interference of P_+ and D_+ waves for different relative strength[10]. Every plots shows for the relative contribution to the amplitude indicated on the plot the evolution of the angular distribution in terms of $\cos\vartheta_{GJ}$ as a function of phase-shift. In real data, the phase-shift will be a function of mass which will be presented in Fig. 4.After calculated, the angular distribution can be written in a general form, using spherical harmonic function 2 [5].

$$I(\theta,\phi) = \left|\sum_{0}^{l}\sum_{m=-l}^{l} n_l^m Y_l^m(\theta,\phi)\right|^2$$

Here the parameters n_l^m is independent. And for every parameter n_l^m, it should include the real part and imaginary part. With the computer programming we can get the Fig. 4 (more detail see [5]).

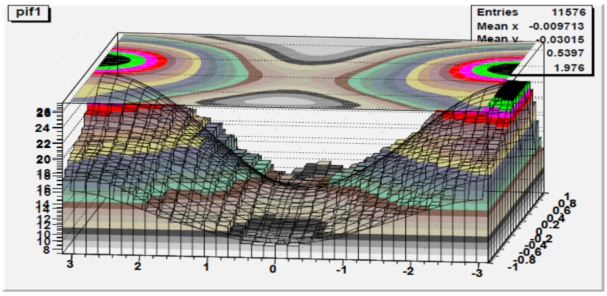

Fig. 4. The $cos\vartheta_{GJ}$ vs $\phi_{GJ}$of f_1 in $\pi^- f_1$ system [5]. The coloured Figure is the data for $\pi^- f_1$ decay, the lines correspond to the fit function, the plots are the projection of the fit. A special function as indicated in the Figure with x = $cos\vartheta_{GJ}$ and y = ϕ_{GJ}. This angular distribution is in the G-J frame. The mass range of $\pi^- f_1$ is from 1.6GeV to 1.8GeV. The mass range of f_1 is from 1.25GeV to 1.31GeV.

4.2 Partial Wave Analysis

The partial Wave Analysis (PWA) is a technique used in hadrons spectroscopy to extract information about the spin-parity and decay properties of resonances produced in hadronic interactions [11]. A partial wave analysis (PWA) of $\pi^-\pi^-\pi^+\eta$ final state events from diffractive pion dissociation at COMPASS has been carried out.

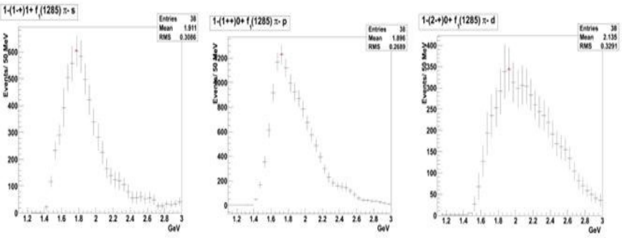

Fig. 5. Intensity of S, P, D waves

The left one is the intensity of S wave; the middle one is the intensity of P wave and the right one is the intensity of D wave. The particular final state $\pi^- f_1$ is a subset of the various 2-body intermediate states leading to $\pi^-\pi^-\pi^+\eta$. This subset has been described by several partial waves. The intensities have been performed for 3 waves with the $I^G J^{PC} M^{\varepsilon} L$ as $1^-1^{-+}1^+S$, $1^-1^{++}0^+P$ and $1^-2^{-+}1^+D$ in Fig. 3 [5].

V. RESULT

We have performed partial-wave analyses of the $\pi^- f_1$systems. According to the intensity in Figure 3 we observed the resonances a_1(1640), π_1(1600) and π_2 (1880). With the result of phase motion in Figure 4, the phase shift between P and D wave is 180 degree which checks the exit of the resonance.

The particular final state $\pi^- f_1$ is a subset of the various 2-body intermediate state leading to $\pi^-\pi^-\pi^+\eta$. This subset has been described by 7 partial waves. A subsequent mass-dependent fit has been performed for 3 waves. The particles $a_1(1640)$, π_1 (1600) and $\pi_2(1670)/(1880)$ are resolved with good quality, confirming the Particle Data Group (PDG) average values for mass and width. In addition, the $a_1(1930)$, π_2 (2005), $a_4(2040)$ mesons are included in the data. The relative branching ratios have been estimated for the decays of the exotic meson $\pi_1(1600)$. A first attempt has been made to extract the branching ratio $1^{-+}1^{+}(\pi^- f_1)$ relative to that of $1^{-+}1^{+}(\pi^-\eta')$. $BR(1^{-+}1^{+}(\pi^- f_1)/BR(1^{-+}1^{+}(\pi^-\eta'))$is about 2 with error 0.5. This is lower than the value 3.8 with error 0.8 in the 2010 PDG [5]. Moreover, the branching ratio of $1^{-+}1^{+}\pi^-\rho(770)$ relative to that of $1^{-+}1^{+}(\pi^- f_1)$ has been estimated, $BR(1^{-+}1^{+}\pi^-\rho(770))/BR(1^{-+}1^{+}(\pi^- f_1))$is about 1.5 with error 0.3. Thus it has been found that $BR(1^{-+}1^{+}\pi^-\rho(770)):BR(1^{-+}1^{+}(\pi^-\eta')):BR(1^{-+}1^{+}(\pi^- f_1))= 3:1:2$.

An emphasis of the analysis certainly is the spin-exotic $1^{-+}1^{+}$ wave obtained with the mass-dependent fit. Exotic quantum number mesons which cannot be accommodated by quark- antiquark states have been a long sought-for prediction of QCD. Recent reviews of the field, which also give references, are Refs. [12-14]. Due to its exotic quantum numbers, this state cannot be conventional quark-antiquark meson and is a hot candidate for hybrid [5]. Several theory models predict a 1^{-+} (J^{PC}) hybrid in the light-quark sector with a mass between 1.5 and 2.0 GeV/c^2 [15].

VI. CONCLUSIONS

In the scope of this paper, a partial wave analysis (PWA) of final state events from diffractive pion dissociation at COMPASS has been carried out. In the regime of high momentum transfer ($|t| > 0.1$ GeV^2) more than 4 million events have been studied, employing a set of 35 partial waves in a mass-independent PWA. The relative branching ratios is $BR(1^{-+}1^{+}\pi^-\rho(770))$: $BR(1^{-+}1^{+}(\pi^-\eta')):BR(1^{-+}1^{+}(\pi^- f_1))= 3:1:2$.

An emphasis of the analysis certainly is the spin-exotic $1^{-+}1^{+}(\pi^- f_1)$ wave obtained with the mass-dependent fit. Due to its exotic quantum numbers, this state cannot be a conventional quark- antiquark meson and will be considerably a hot candidate for a quark- antiquark- gluon hybrid.

ACKNOWLEDGMENTS

The authors of this paper are supported by Natural Science Foundation of Jiangsu Province (BK20160558) and Deutsche For schungsgemeinschaft.

REFERENCE

[1] Peters, K. A. Primer on partial wave analysis. Phys. Lett. B352:463-467, 1995.

[2] Keters, J. Partial wave analysis. Int. J. Mod. Phys. A21: 618-624, 2006.

[3] Abbon, P. et al. (COMPASS collaboration). The COMPASS Experiment at CERN. Nucl. Inst. Meth, A577:455–518, 2007.

[4] Ferrero, A. Light mesons spectroscopy and search for non-quark-antiquark mesons at COMPASS, International Workshop Freiburg, 19 March 2007.

[5] Yang, H. Diffractive production of $\pi^- f_1$ in pion-proton interactions at COMPASS, PhD thesis, Ludwig Maximilians Universität, 35-88, 2012.

[6] Chung, S.U. and Trueman, T.L. Positivity conditions on the spin density matrix: A simple parameterization. Phys. Rev., 633–646, 1975.

[7] Chung, S.U. Techniques of amplitude analysis for two pseudoscalar systems. Phys. Rev., 7299–7316, 1997.

[8] Chung, S. Formulas for Partial-Wave analysis. Technical report, BNL, 2010.

[9] Lednev, A. Study of the reaction $\pi^- p \rightarrow \eta \pi^0 n$ at 32, 38 and 100 GeV/c. Hadron Spectroscopy – Seventh International Conference, number 432 in AIP Conference Proceedings, 253–263, 1997.

[10] Schlüter, T. The $\pi^- \eta$ and $\pi^- \eta^0$ Systems in Exclusive 190 GeV $\pi^- p$ Reactions at COMPASS (CERN), 65-89, 2012.

[11] P. Cummings, et al. The New BNL Partial Wave Analysis Programs. Phys. Rev. D11: 633-646, 1975.

[12] Klempt, E. and Zaitsev, A. Glueballs, Hybrids, Multiquarks. Experimental facts versus QCD inspired concepts. Phys. Rept., 454:1–202, 2007.

[13] Martin, A. A Study of Isospin 1 Meson States Using 10GeV/c $K^- K^0$ Production Data. Phys. Lett., B74:417, 1978.

[14] Meyer, C. and Van Haarlem, Y. Status of exotic-quantum-number mesons. Phys. Rev., C82:025208, 2010.

[15] Yang, H.; Xu, X.; Neumann, I. The Benefit of 3D Laser Scanning Technology in the Generation and Calibration of FEM Models for Health Assessment of Concrete Structures. Sensors 14, 21889-21904, 2014.

[16] Q. Weitzel. Precision Meson Spectroscopy: Diffractive Production at COMPASS and Development of a GEM-based TPC for PANDA, PHD thesis, Technic Universite Munich, 2008.

AUTHORS' PROFILES

First Author:
Hao Yang's research topic are dominantly the benefit of 3-D laser scanning technology in the generation and calibration of finite-element method models, multisensor integration information system for structural health monitoring, and Monte Carlo Method.

Second Author:
Xiangyang Xu's research topic mainly falls in combination of geodetic measurements with numerical models and processing of point clouds data based on surface approximation, and Monte Carlo Method.