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Local Geoid Modelling by the Combination of GPS Data and Geopotentialmodel

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***Abstract* – An important advantage of the Global Positioning System GPS is providing a three dimensional position of a point in a Geodetic system. Since GPS derived heightsare ellipsoidal, the heights required for surveying, and engineering applications are Orthometric Geoidheights, transference from geodetic height Orthometric height must occur. Achieving using GPS derived Orthometricheight is one of the most important goals of Geodesy and has been the purpose of many studies. This approach will eliminate the costly and time-consuming levelling observations. One of the methods to achieve GPS derived Orthometric height is to have an accurate local Geoid model.**

Publishing the new global GOCEGeopotential with a higher accuracy than previous models can contribute to acquiring a reliable model by combining this model with GPS data andtesting and evaluating it in Iran. This paper discusses the GOCEGeopotential model first and then combines this model with GPS data using appropriate basis functions to achieve an optimum outcome. The results of testing Geoid in southern Iran suggests an accuracy of 52 cm for the GOCEGeopotential model, while using a combination of GPS data with the GOCEGeopotential model suggests an accuracy of 33 cm for this model.

***Keywords* – Gauss, Geoid, Geopotential Model.**

I. Introduction

Geoid is one of the equipotential surfaces of the earth that effectively approximates the mean sea level (MSL).Geoid as the height datum is of high importance in geodesy. Today due to increased use of gravitational satellites, positioning, altimetry, new various data such as the global Geopotentialmodel, Digital Elevation Models and sea gravitational data a major part of geodesians' mission is the accurate determination of Geoid.

Many global models of Geoidhave been proposed so far with one of their famous being the global model EMG96 and recently the GOCEGeopotential model.Geoidis expanded to the spherical harmonic series and its coefficients have been measured and reported. Although coefficients with medium-to long-wavelength Geoids are usually accurately recognizable by observing the satellite orbit, determining the medium to short wavelength still depends on the surface gravity anomaly information or gravimetricdata. The desirableGeoidto determine the Orthometric height or the geometric height difference between two points is the Geoidwithhighly visible long to medium wavelength.

Due to the lack of surface gravimetric information and inaccuracy of defined Geoids in terms of short wavelength in Iran, a method is required to use a combination of the leveling information, GPSdata and the Geopotentialmodel to determine the local Geoid. Using more GPS data and information, the optimal Geopotential model, the trend analysis and selecting the correct basis functions, determining the unknown coefficients matrix and choosing a procedure with an appropriate degree as the correction surface, are items considered for improving the accuracy of the model in this research [1]

Local data are combined (or shared) as the observed Geoid height with Geoid information of an accurate global Geopotential model like GOCEusing GPS in the form of a mathematical local model at heights in the national height network in an areanear the coast in order to enhance the Geoid accuracy compared to the global Geoid model in that area.However the model accuracy depends on the spatial density of local observations carried out by GPS.

II. Global Models of the Earth's Gravitational Potential

The lack of coverage and low accuracy of some of the gravity data on one side and abundance and availability of accuratesatellite dataon the other side has led to the idea of using satellite information in order to acquire a global Geoid model. Determining the Geoid by gravimetric satellites at low altitudes, have dramatically increased the accuracy of the earth's gravity field surveying. The Geopotential models are the earth's potential expansion coefficients to spherical harmonics.

The coefficients of the long wavelength of the earth's potential expansion obtained by satellite methods have high accuracy in computations;however, the acquired Geoid lacks the gravity field details due to the large distance from the orbiter to the earth and will have a very smooth surface. Moreover the Geoid obtained from the earth information, has the long wavelengths bias despite enough details due to non-uniform distribution of earth observations throughout the earth planet. Therefore, an optimal outcome can be achieved by appropriately combining the coefficients with earth observations.

Recent Satellite gravimetrymethods such as lowering the satellite height, measuring the satellite orbit by satellites at higher heights such as GPS, measuring the effects of nonabsorbent forces on the satellite and measuring the acceleration of gravity at the satelliteheight has led to an increased accuracy and spatial resolution of the potential field.

Assuming that solving the Laplace equation for the gravitational potential of the earth ($W(r,\bar{\theta},\lambda)$) is harmonic, it can determine the harmonic coefficients[2]. Mathematical models that relate observable quantities along the orbit to measurable quantities at the sea or earth levels are as follows:

The general form of the gravitational potential expansion to spherical harmonic series:

$$W_g(r,\theta,\lambda)=\sum_{n=0}^{\infty}(\frac{a}{r})^{n+1}\sum_{m=0}^{n}\overline{J}_{nm}Y_{nm}^{c}(\theta,\lambda)+\overline{K}_{nm}Y_{nm}^{s}(\theta,\lambda) \quad (1)$$

R is the geocentric distanceat the earth's level, ($\lambda$and φ)are the geographicallongitude and latitude, J_{nm} and K_{nm} are normalized harmonic spherical coefficients of the gravitational potential expansion, is half the diameter of the reference elliptical length and $Y_{nm}^{s}(\theta,\lambda)$ and $Y_{nm}^{c}(\theta,\lambda)$ are surface spherical harmonics.

$$Y_{nm}^{c}(\theta,\lambda)=P_{nm}(\cos\theta)\cos m\lambda \quad (2)$$

$$Y_{nm}^{s}(\theta,\lambda)=P_{nm}(\cos\theta)\sin m\lambda \quad (3)$$

$P_{nm}(t)$ Is a Legendre function of degree 2 and order m.

The model, which is used in Geodesy satellite to express the gravitational potential, is as follows:

$$W_g(r,\theta,\lambda)=\frac{GM}{r}[1-\sum_{n=0}^{\infty}(\frac{a}{r})^{n}\sum_{m=0}^{n}\overline{J}_{nm}Y_{nm}^{c}(\theta,\lambda)+\overline{K}_{nm}Y_{nm}^{s}(\theta,\lambda)] \quad (4)$$

Normal earth gravity field $(U(r,\overline{\theta}))$ based on the Zonal Spherical Surface Harmonics is calculated by equation (5):

$$U_g(r,\theta)=\frac{GM'}{r}[1-\sum_{n=2,4,6,8}^{n}(\frac{a'}{r})^{n}J_{n0}^{N}P_{n0}(\cos\theta)] \quad (5)$$

M': Normal elliptical mass

$U_g(r,\theta)$: The normal potential

$U_g(r,\theta)$ Can be created and calculated at every point using the GRS80elliptical characteristics.

Only Zonal coefficients and of even degree exist in the normal gravity field.

The actual potential difference and the normal potential is called potential anomaly and is represented by T. The potential anomaly $(T(r,\overline{\theta},\lambda))$ can be expanded to the spherical harmonics series as well.

$$T(\vec{r})=W(\vec{r})-U(\vec{r}) \quad (6)$$

$$T(r,\theta,\lambda)=\frac{G(M-M')}{r}-\frac{GM}{r}\sum_{n=1}^{\infty}(\frac{a}{r})^{n}\sum_{m=0}^{n}\left[\overline{J}_{nm}-\frac{M'}{M}\overline{J}_{n0}^{N}\right]Y_{nm}^{c}(\theta,\lambda)+\overline{K}_{nm}Y_{nm}^{s}(\theta,\lambda) \quad (7)$$

Once the potential anomaly is determined at every point, other anomaly quantities such as the Geoid height, the gravity anomaly, the gravity sway are measureable as they are potential anomaly functions.

$$N(r,\varphi,\lambda)=\frac{T(r,\varphi,\lambda)}{\gamma(\varphi)} \quad (8)$$

The Geopotential models qualities are evaluated in two different methods:

1. Examining the variance of the models error degree
2. Comparing the height obtained from various models with GPS derived Geoid height[3].

The first method measures the precision of the models and the second method measures the accuracy of the models.

The Geopotential models express the errors of this spectral band of Geoid in addition to proving information about the long wavelength of Geoid. Such errors that arise from insufficient satellite data, lack of earth's gravity data and systematic errors in the satellite altimetry are usually categorized into two main groups of errors:

1. Commission errors: these errors are caused by the noise in the Geopotential coefficients and are calculated as follows:

$$\delta N_C^2=\frac{R^2}{2\gamma^2}\sum_{n=2}^{N_{max}}\sum_{m=0}^{n}[(\delta_{nm}^{c})^2+(\delta_{nm}^{s})^2] \quad (9)$$

In this equationN_{max} is the maximum degree used in the spherical harmonic expansion. δ_{nm}^{c} and δ_{nm}^{s} are the estimated values of spherical harmonic expansion that are usually declared along with coefficients of the spherical harmonic values. Equation (9) is a global moderate estimation of the Geoid height or in other words is an internal criterion of cumulative error for the Geiod height. This criterion can be used in comparing different Geopotential models and choosing the optimum model. Figure 1 indicates the commission error values for the model GOCE up to degree 240.

1. The OMISSION errors: these errors are associated with coefficients that do not exist in the model and are caused by the spherical harmonics series truncation. The truncation removes the terms higher than the maximum degree of spherical harmonics.

Degree of gravity anomaly variance is calculated by the following equation [4]:

$$c_n=\gamma^2(n-1)^2\sum_{m}(\overline{C}_{nm}^2+\overline{S}_{nm}^2) \quad (10)$$

The law of Kaulacan be used in order to determine the precision of the spherical harmonic coefficients and calculate the error degree variance for the coefficients not existing in the model. Figure 2 indicates the degree of omission error values for the model GOCE up to degree 1200.Fig.2.

Estimation of these two types of errors for the GOCEmodel is indicated in table1.

III. Local Geoid Modelling

Fitting an algebraic procedure based on the selected reference pointsis one of the modelling techniques. This technique derives from the Geiod height as an analytical level based on mathematical formulas. A sheet or a procedure with a combination of polynomial algebraicwith the appropriate degree is used to determine the local Geoid model.

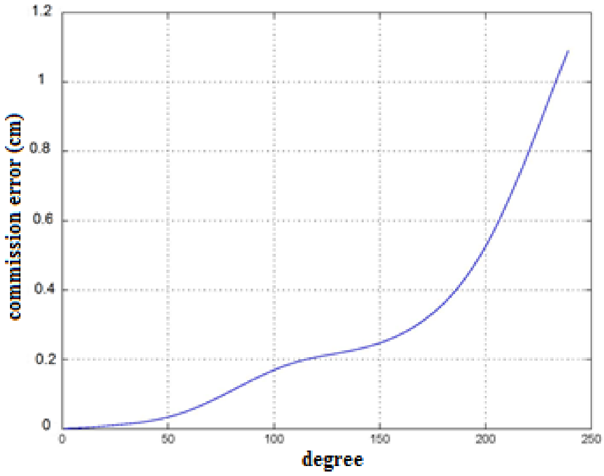

Fig.1.Commission error values of the GOCE model

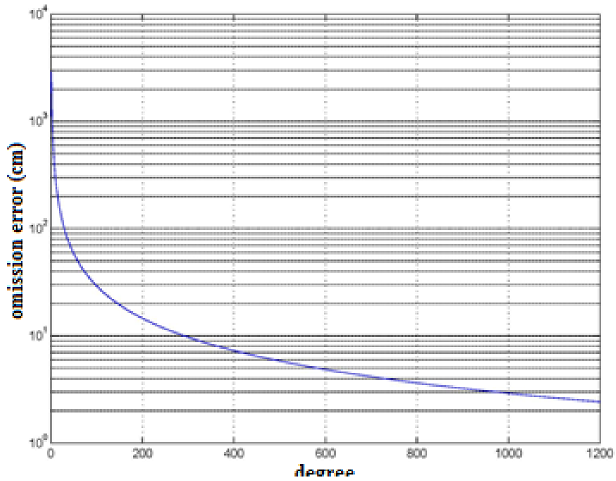

Fig.2. The commission error values of the GOCE model up to degree 1200

Table 1: Commission and omission errors estimation in GOCE satellite

Spherical harmonic degree	Commission error (cm)	Ommission error (cm)
100	0.17	29.11
150	0.25	19.41
200	0.55	14.55
240	1.09	12.13
300	-----	9.70
400	-----	7.28
500	-----	5.82
700	-----	4.16
1000	-----	2.91
1200	-----	2.42

Several factors affecting the accuracy in determining the Geoid model are the number of reference stations (GPS) and their distribution, the accuracy of the Geoid height that is by the Orthometric height accuracy obtained from levelling observations and the extent of the area where the GPS points are distributed [5]

The region of interest in southern Iran is located in the coordinate range of (32.5830, 47.5000) and (25.5830, 61.4500) and covers area of approximately 600.000 square kilometers.

Various resources provided the GPS data database and the data were evaluated using the 3σtest and those data values outside the range of three times the standard deviation of the total data were considered as wrong data and removed. 704GPS points that 165of them were from the national cartographic center and 539points were from consulting companies workingactivated in different surveying projects across the countryand approved by the cartographic center, were used in this stage. The points' position, density and distribution are demonstrated in figure 3.

The first step is to determine the GPSGeoid height at height network points. In order to do the, GPS measurement is performed at points with visible Orthometric height (H) and the geodetic height (h) is determined and calculated for the points by the following equation:

$$N_{GPS} = h - H \tag{11}$$

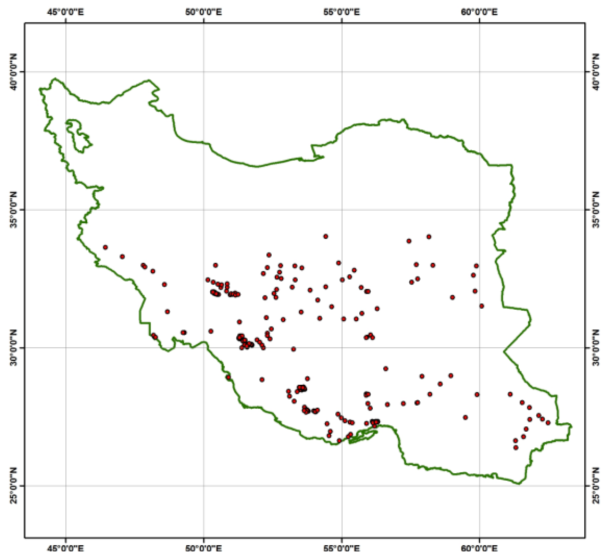

Fig.3. Position and distribution of GPS points

The important point about this information is the failure to mention the accuracy of GPS observations and the accuracy of levelling that can cause challenges in the future calculations. The variance and covariance matrix of the model observations can usually be derived from the proration results of the levelling and GPS networks. The average height error of GPS points in Iran's network is about 12.5 cm. (Experts in the cartographic center-personal contact). However, Iran is leveling network estimates an average precision of about 7 cm .

Geiodis calculated up to degree and order 240 in the next stage using the Geopotential model for points with visible GPS Geoid height. The harmonic coefficients and their precision are first obtained from the website ICGEM.

Then a program is written by matlabwith its input as the geographical longitude and latitude and its output as the Geoid height based on the GOCE potential model. Afterwards, the purpose is to achieve an optimal estimation of the Geoid height signal. The main idea of

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this work is to apply the correction algebraic procedures. A correction procedure is used in this stage to restore the Geoid obtained from other observations such as the Geoid height obtained from the satellites.

Small size of the study area on one side and the inappropriate points' distribution on the other side, are effective factors in selecting the procedure level 2 for this area, as Geoid is a relatively smooth surface in a small area [1].

$$N(X,Y)=\sum_{i=0}^{n}\sum_{j=0}^{n-i}a_{ij}y^{i}x^{j} \quad (12)$$

$$N(X,Y)=a_{00}+a_{01}x+a_{10}y+a_{02}x^{2}+a_{11}xy+a_{20}y^{2} \quad (13)$$

$$X=R*(\varphi-\varphi_{0}) \qquad Y=R*\cos\varphi*(\lambda-\lambda_{0}) \quad (14)$$

R is selected as the average radius of the earth in the region in this equation and (φ_0,λ_0) is considered as the geodetic coordinate at the center of the region of interest in this study. Height of the Geoid N_P is approximated at the points using a series of basis functions. N_P is obtained by modulating the GeoidGOCE using algebraic procedures. If the vandermond matrix composed of selected basis functions is demonstrated with ϕ, a procedure with an appropriate degree is fitted to that and thus the approximated height of Geoid $\underline{N}_G$ is obtained by the following equation:

$$\underline{N}_{P}=\underline{\phi}^{T}\underline{\lambda}^{(o)} \quad (15)$$

In the equation above, $\underline{\lambda}^{(o)}$ are the coefficients of the model GOCE or in other words the unknown coefficients that are estimated using the least squares regression as follows:

$$\underline{G}_{G}\hat{\underline{\lambda}}^{(o)}=\underline{U}_{G} \quad (16)$$

$$\underline{G}_{G}=\underline{\phi}\underline{C}_{G}^{-1}\underline{\phi}^{T} \quad (17)$$

$$\underline{U}_{G}=\underline{\phi}\underline{C}_{G}^{-1}\underline{N}_{G} \quad (18)$$

In this equation, $\underline{N}_G$ is the Geoid height obtained by the GOCE satellite and $\underline{C}_G$ is the covariance matrix of Geoid $\underline{N}_G$ and $\underline{G}_G$ is the Gram matrix and yet reverse of the covariance matrix $\hat{\underline{\lambda}}^{(o)}$ and is obtained by equation (19).

$$\hat{\underline{\lambda}}^{(o)}=\underline{C}_{G}^{-1}\underline{U}_{G}=\underline{G}_{G}^{-1}\underline{C}_{G}^{-1}\underline{\phi}\underline{N}_{G} \quad (19)$$

$$\underline{C}_{\hat{\lambda}^{(o)}}=\underline{G}_{G}^{-1} \quad (20)$$

The covariance matrix is $\underline{N}_P$ which is calculated by equation 19.

$$\underline{C}_{\hat{P}}=\underline{\phi}^{T}\underline{C}_{G}^{-1}\underline{\phi} \quad (21)$$

$$N(x,y)=\phi^{T}(x,y)(\hat{\underline{\lambda}}^{(o)}+\underline{\delta\lambda}) \quad (22)$$

The Geoid height $\underline{N}_D$ at the point's r will be added to equation (13) as the new observation in the next step. By doing so, the vector of $\hat{\underline{\lambda}}^{(o)}$ coefficients changes as much as $\underline{\delta\hat{\lambda}}$ and $N(x,y)$ will fit using values $\underline{N}_G$ and $\underline{N}_D$ and the equation will be as follows:

Therefore, the calculated or in other words the analytical Geoid will be provided in the region. Also incorporating the weight matrix will result in the following equation:

$$\underline{\phi}(\underline{P}_{G}+\underline{P}_{GD}+\underline{P}_{DG}+\underline{P}_{D})\underline{\phi}^{T}(\underline{\lambda}^{(o)}+\underline{\delta\hat{\lambda}}) =\underline{\phi}[(\underline{P}_{G}+\underline{P}_{DG})\underline{N}_{G}+(\underline{P}_{GD}+\underline{P}_{D})\underline{N}_{D}] \quad (23)$$

Here P is the weight vector matrix of N and $\underline{C}_D$ and $\underline{C}_G$ is respectively the covariance matrices between $\underline{N}_G$ and $\underline{N}_D$ and $\underline{C}_{GD}$ is the cross covariance matrix between $\underline{N}_G$ and $\underline{N}_D$ and I is the unit matrix in these equations.

The following equation is obtained by subtracting equation 16 from 23:

$$\underline{\phi}(\sum\underline{P})\underline{\phi}^{T}\underline{\delta\hat{\lambda}}=\underline{\phi}[\underline{P}_{DG}\Delta\underline{N}_{G}+(\underline{P}_{D}+\underline{P}_{GD})\Delta\underline{N}_{D}] \quad (24)$$

In which:

$$\sum\underline{P}=(\underline{P}_{G}+\underline{P}_{GD}+\underline{P}_{DG}+\underline{P}_{D}) \quad (25)$$

$$\Delta\underline{N}_{G}=\underline{N}_{G}-\underline{N}_{P} \quad (26)$$

$$\Delta\underline{N}_{D}=\underline{N}_{D}-\underline{N}_{P} \quad (27)$$

It should be noted if there is no dependency between the data, in other words with null hypothesis ofand $\underline{C}_{GD}$, equation 24 can be written as simplified in equation 28:

$$\underline{P}=\begin{bmatrix}\underline{P}_{G} & 0\\ 0 & \underline{P}_{D}\end{bmatrix}=\begin{bmatrix}\underline{C}_{G} & 0\\ 0 & \underline{C}_{D}\end{bmatrix}^{-1} \quad (28)$$

$$\underline{\phi}(\underline{C}_{G}^{-1}+\underline{C}_{D}^{-1})\underline{\phi}^{T}\underline{\delta\hat{\lambda}}=\underline{\phi}\underline{C}_{D}^{-1}\underline{N}_{D}-\underline{\phi}\underline{C}_{D}^{-1}\underline{\phi}^{T}\hat{\underline{\lambda}}^{(0)} =\underline{\phi}\underline{C}_{D}^{-1}\underline{N}_{D}-\underline{\phi}\underline{C}_{D}^{-1}\underline{N}_{P}=\underline{\phi}\underline{C}_{D}^{-1}(\underline{N}_{D}-\underline{N}_{P}) \quad (29)$$

$$\underline{\delta\hat{\lambda}}=[\underline{\phi}(\underline{C}_{G}^{-1}+\underline{C}_{D}^{-1})\underline{\phi}^{T}]^{-1}\underline{\phi}\underline{C}_{D}^{-1}(\underline{N}_{D}-\underline{N}_{P}) \quad (30)$$

A correction procedure can be finally defined as follows:

$$\underline{\delta N}(X)=\underline{\phi}^{T}(X)\underline{\delta\hat{\lambda}} \quad (31)$$

Then we will integrate the Geoid obtained from the GOCE potential model which is measured with and Geoid which measured by evaluation derived from GPS observation have been shown in certain points with

The correction procedure of the equation 31, which is defined by adding to the basic information at the point r, is directly added to at every point of the region (not only point's r). Therefore:

$$\underline{N}(X)=\underline{N}_{G}(X)+\underline{\phi}^{T}(X)\underline{\delta\hat{\lambda}} \quad (32)$$

$$\underline{N}(X)=\underline{N}_{G}(X)+\underline{\phi}^{T}(X)\underline{C}_{\delta\hat{\lambda}}^{-1}\underline{\phi}(r)[\underline{P}_{DG}\Delta\underline{N}_{G}(r)+(\underline{P}_{D}+\underline{P}_{GD})\Delta\underline{N}_{D}(r)] \quad (33)$$

Considering the data independence, Equation 30 can be written as follows:

$$\underline{N}(X)=\underline{N}_{G}(X)+\varphi^{T}(X)[\underline{\phi}(r)(\underline{C}_{G}^{-1}+\underline{C}_{D}^{-1})\underline{\phi}^{T}(r)]^{-1}\underline{\phi}(r)\underline{C}_{D}^{-1}(\underline{N}_{D}-\underline{N}_{P}) \quad (34)$$

In this equation x represents the points that require corrected Geoid calculation. R represents the points where the GPSGeiod is visible as well. P_D Represents the weight matrix of the Geoid obtained from GPS points in the region and calculated using equation 33:

$$\underline{P}_D = \underline{P}_{N_{GPS}} = \underline{C}^{-1}_{N_{GPS}} = \underline{C}^{-1}_{h-H} = (\underline{C}_h + \underline{C}_H)^{-1} \quad (35)$$

As per the equation 28, the weight matrices of quantities $\underline{N}_G$ and $\underline{N}_D$ quantities play an important role in the result. Therefore, different weight estimates of Geoidmust be introduced properly to the right filter. If all observations have a definite accuracy in the process of calculating each of the quantities $\underline{N}_G$ and $\underline{N}_D$, the accuracy of $\underline{N}_G$ and $\underline{N}_D$ is also determined using the law of propagation of errors and is directly used in the combination process.

The equation 34 shows a two-dimensional linear filter in which the linear operators are functions of the covariance matrices and $\underline{C}_G$ basic functions that form. This filter, which is compatible with basic functions, operates in a certain frequency band on and combines them based on their accuracy. Therefore, choosing the basic functions and determining the relative weight of the observations in this filter requires considerable attention.

Since the Geoid height is a quantity with an infinite dimension, it is not possible to correct all the coefficients of the Geoid frequencies using this filter and a limited number of point's r in a region, but only moderate Geoid frequencies can be restored in the region. The notable thing is that this filter corrects only moderate frequency bands and does not correct neither of high nor of low frequencies. This filter is not applicable worldwide but only in southern Iran with a certain planned procedure [6].

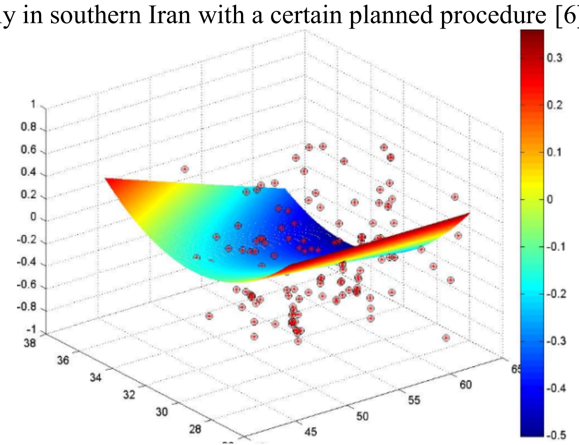

Fig.4. Developed procedure in addition to GPS points

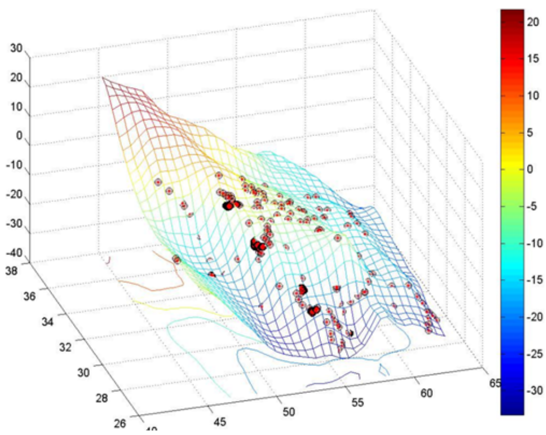

Fig.5. Developed Geoid in addition to GPS points

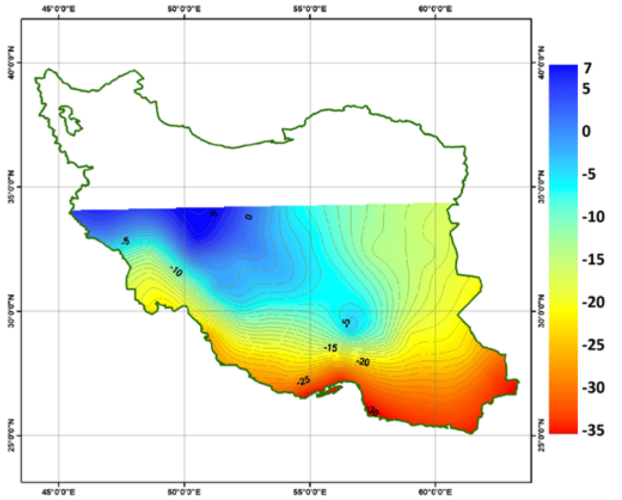

Fig.6. Geoid obtained from the combined local model for the southern iran

The procedure developed in this study is indicated in figure 4 in addition to the GPS points' position in figure 4. Figure 5 represents the geiodprocedures using the model GOCE. The Geoid map of the combined model of local Geoidspecific to the region is indicated in figure 6. The GOCEGEOPOTENTIALGeoid in the southern Iran compared with Geoid obtained from GPS points in this region and the differences are indicated in figure 7. The local combined model Geoidis compared with Geoid obtained from GPS points in this region. Geoid obtained from GPS points in this region and the differences are indicated in figure 8.

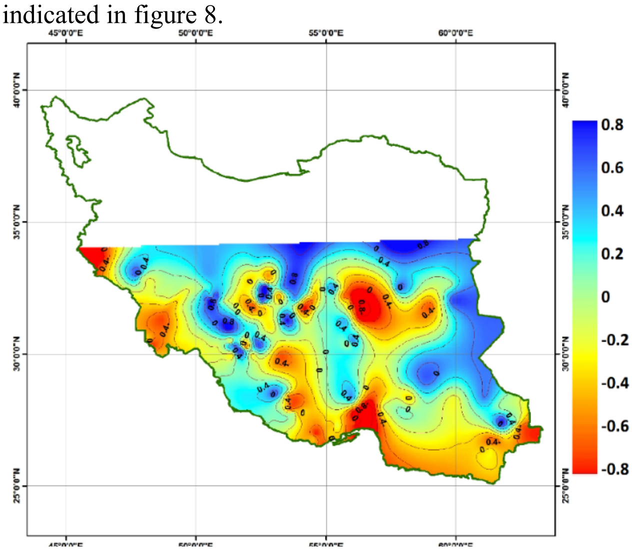

Fig.7. The difference between the Geoid measured by model GOCE and GPS derived Geoid

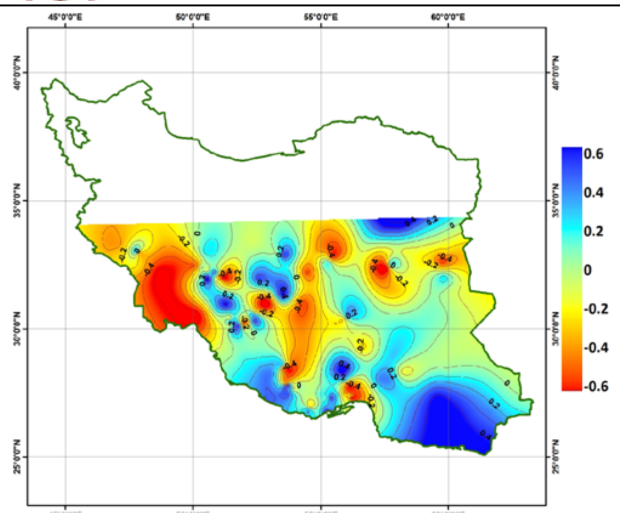

Fig.8. The difference between the measured Geoid by the combined local model and Geoid obtained from GPS points in this region

The root mean square of errors, the Geoid model and the GPS Geoid heights can be measured and then compared together in order to assess the accuracy of the local Geoid height obtained from the Geopotential model.

Geopotential model	**EGM96**	**GOCE**	**Local Geoid**
RMSE (cm)	0.542011	0.519391	0.330705

The local Geiod error obtained in this study is statically analyzed for 704 test points in this stage. This error is indicated with the symbol δ that is calculated using the equation (34).

$$\delta = N_{GPS} - N_{Local} \quad (34)$$

The sample mean and standard deviation were respectively estimated at -0.0066 and 0.331 in the mean deviation of zero test. Therefore, the zero testing is rejected. The goodness of fit for normal distribution was conducted in order to test the normality of the quantity δ distribution and the results are indicated in table 2.

Table 2: The normality test results of δ distribution

N	Statistic	704
Range	Statistic	3.69
Minimum	Statistic	-2.373
Maximum	Statistic	1.316
Mean	Statistic	-0.0066
Mean	Std. Error	0.0135
Std. Deviation	Statistic	0.331
Variance	Statistic	0.112
Skewers	Statistic	0.461
Skewers	Std. Error	0.094
Kurtosis	Statistic	3.936
Kurtosis	Std. Error	0.188

The distribution curve kurtosis δ is 0.461, being zero indicates a complete asymmetry of the distribution curve, and thesteepness of the distribution curve is 3.936 suggesting that the slope of the distribution curve is not very close to the normal curve. The Kolmogorov-Smirnov test statistic was used to test the normality of the data. The Asymp. Sig value is less than 0.05 in this table;therefore, the normality of the data is rejected. The residuals distribution histogram created in the local Geoid, is indicated for the test points of the model in figure 9. If the data histogram is close to the normal data histogram, it can be concluded that the data are graphically normal. The normal curve that is symmetric as one of itsfeatures is a bell-shaped curve. The data are not close to the normal curve and symmetric in figure 9.

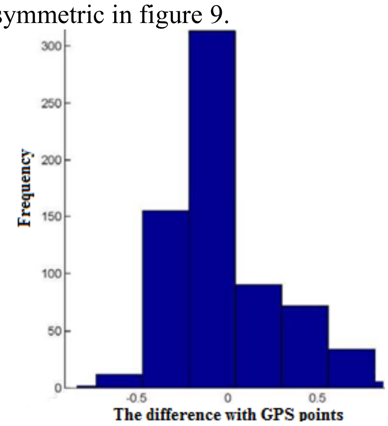

Fig.9. Histogram of the model error in GPS points

The results indicate that the quantity δ is not normally distributed and the goodness of fit test, which indicates the effect of systematic error resources,is rejected. Systematic errors can be found in bias resources such as inconsistency at national network heights, the effects of sea surface topography in the zero station of the height network, no normal correction to leveling observations, refraction effect in leveling observations, effective errors in GPS observations and so forth.

IV. Conclusion and Results

The Geoid accuracy obtained from the GOCE potential model was estimated at about 52 cm for the region of interest and the local Geoid accuracy obtained by combining GPS data and the GOCE potential model in which the long wavelength of Geoid was restored using polynomial basis functions was estimated at about 32 cm. The Geopotential models GOCE and EGM96 are almost about the same accuracy and precision in the southern Iran.

Using a combination of models can improve the Geopotentialmodels. The reasons of quality improvement in new models can be using more GPS data and removing the incorrect data, using newer and more accurate Geopotential models as well as an optimal combination of GPS data and the Geopotential model using a correction procedure. The Geoid modelling accuracy using this method is considerably dependent on the two factors of sufficient GPS points and their appropriate distribution. Lack of these factors results in the use of low degree polynomials as basic functions, which is the main cause of the failure to achieve a high accuracy in this study compared to studies in other countries.

Other factors that may improve the model accuracy are taking into account the effect of the topography in computing, using the least square collocation method, identifying the systematic errors such as device or environmental error,and making necessary corrections, determining the accuracy of all leveling and GPS observations. It is hoped that a database of GPSpoints is created by the help of relevant authorities such as the cartographic center, the geographical organization, the oil company, and the surveying consulting companies that have been working in different parts of the country for years, so that the Geoid with high accuracy will be achieved in addition to other applications.

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