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Macroscopic Traffic Flow Model in Line with Kerner Theory with Consideration of Driver Aggressiveness

Mwarania Amendeo*
Mt. Kenya University, Nkubu Campus, P.O.Box 511, Meru, Kenya. Email : amedgikunda@yahoo.com
* Correspondence author

Kirimi Jacob
Chuka University, P.O Box 109-60400
Chuka, Kenya
Email : h.kirimi@yahoo.com

Abstract **- In this paper, we outline the three phase traffic theory. A macroscopic traffic flow model which factors the driver aggressiveness is developed. The features of this model are discussed in detail and compared against those of existing models like the Aw- Rascle model. The model is written in conservation form and solved using hybrid numerical scheme based on the Godunov method. When simulated results are compared with those of Aw- Rascle model, our model is found to be more realistic since it is able to predict the behavior of a vehicle that approaches a slow moving vehicle when the traffic density ahead is not changing.**

Keywords **- Conservation laws, 3-phase traffic flow, Phase transitions.**

I. Introduction

Congestion of vehicle traffic within urban areas is a problem experienced worldwide. It has adverse effect on quality of life due to delays, accidents and environmental pollution. One way of eliminating the problem is to increase the capacity of existing roadways by addition of lanes. However this is not always practical due to limited financial resources, space, environmental effects etc. The only option remaining is utilization of existing infrastructure by employing prudent traffic management and operation strategies.

The understanding of traffic congestion is the key to development of many fields of transportation science and engineering. For this reason, a huge numbers of publications are devoted to empirical studies of traffic congestion and associated traffic flow theories (Prigogine and Herman 1971, Whitham 1974, Helbing 2001, Hoogendoorn 2001, Kimathi 2012).

Traffic congestion in a road network is a consequence of traffic breakdown in initially free flowing traffic. Traffic breakdown is the abrupt decline of velocity from high values in free flow to lower values in congested traffic, and normally happens at highway bottlenecks such as on- and off-ramps. Lane drops, accident area, area with poor weather conditions etc.

A traffic flow theory aims at describing in a precise mathematical way the vehicle to vehicle interactions and interactions between vehicles and infrastructure.

The models developed from the traffic flow theories can be categorized into Microscopic, Macroscopic and Mesoscopic (Gas kinetic) traffic flow models.

Definition 1.1: *Macroscopic Models*

Macroscopic Models contain two independent variables which are location (x), time (t) and three state variables namely flow rate (q), vehicle density occupancy (ρ) and average vehicle speed (v) . It describes traffic at a high level of aggregation. Macroscopic traffic models are governed by the continuity equation which expresses the relation between the rates of change of density w.r.t. time and the flow w.r.t. location and the velocity dynamics equation. To describe time varying and spatially varying average velocities V(x, t) such as those that occur in traffic jams or stop and go traffic, we need a dynamic velocity equation. (Necoara, Schutter and Hellendoorn 2004)

Definition 1.2: *Microscopic Models*

Microscopic models describe the characteristics that reflect the behavior of individual vehicles in traffic flow and interactions with other vehicles. The flow variables are single vehicle space coordinates and their time dependence, time headway and a space gap between two vehicles following each other. (Hoogendoorrn and Bovy 2001)

Definition 1.3: *Mesoscopic (Gas Kinetic) Models*

Mesoscopic (Gas Kinetic) models describe traffic in less aggregate manner than macroscopic models and in probabilistic terms. Traffic is represented by (small) groups of traffic entities, the activities and interactions of which are described at a low detail level. (Nicoara, Schutter and Hellendoorn 2004). One of the macroscopic models that have been developed is a macroscopic 3-phase traffic theory (Kimathi2012). The model is based on 3-phase traffic theory (Kerner2002). This theory was developed by Kerner Boris between 1996 and 2002. It entails the following three phases of traffic:

Definition 1.4: *Free Traffic Flow (F)*

Usually observed when the vehicle density in traffic is small enough. Here, the vehicles have an opportunity to move with their desired maximum speeds (if not restricted by road conditions or traffic conditions. If density in free flow increases, the flow rate increases too. This leads to a decrease in average vehicle speeds due to vehicle interactions.

Definition 1.5: *A wide Moving Jam (J)*

This is a localized traffic pattern that maintains the mean velocity of the downstream front of the jam as the jam propagates. The density is very high. The moving jam is spatially restricted by the downstream jam front and upstream jam. Within the downstream jam, vehicles accelerate from low speed states within the jam to higher speeds in traffic flow downstream of the moving jam. Within the upstream jam, front vehicles must slow down to the speed within the jam. Both jam fronts move upstream. Within the jam fronts, the vehicle speed, flow rate and density vary abruptly.

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Definition 1.6: *Synchronized Flow (S)*

This is any congested traffic that does not exhibit the above distinctive features of wide moving jams. In particular, the downstream jam front is often fixed at the bottleneck. In the case, that synchronized flow happens to move, the mean velocity of the downstream front changes in a wide range during the pattern propagation. Another notable feature of synchronized flow is the formation of diverse spatiotemporal traffic patterns upstream of the bottleneck.

This leads to the following types of synchronized flow patterns.

Definition 1.7: *Localized Synchronized Pattern (LSP)*

The downstream front of the synchronized pattern is fixed at the bottleneck and upstream front propagates upstream in the course of time only to get localized at the same distance upstream of the bottleneck.

Definition 1.8: *Widening Synchronized Pattern (WSP)*

The downstream front is fixed at the bottleneck but the upstream front continuously propagates upstream in the course of time.

Definition 1.9: *Moving Synchronized Pattern (MSP)*

This is realized when both upstream and downstream fronts of a synchronized pattern propagate upstream he road for as long as it does not encounter another bottleneck upstream where it is likely to induce traffic breakdown.

The term spatiotemporal pattern implies the distribution of traffic flow varies in space and time.

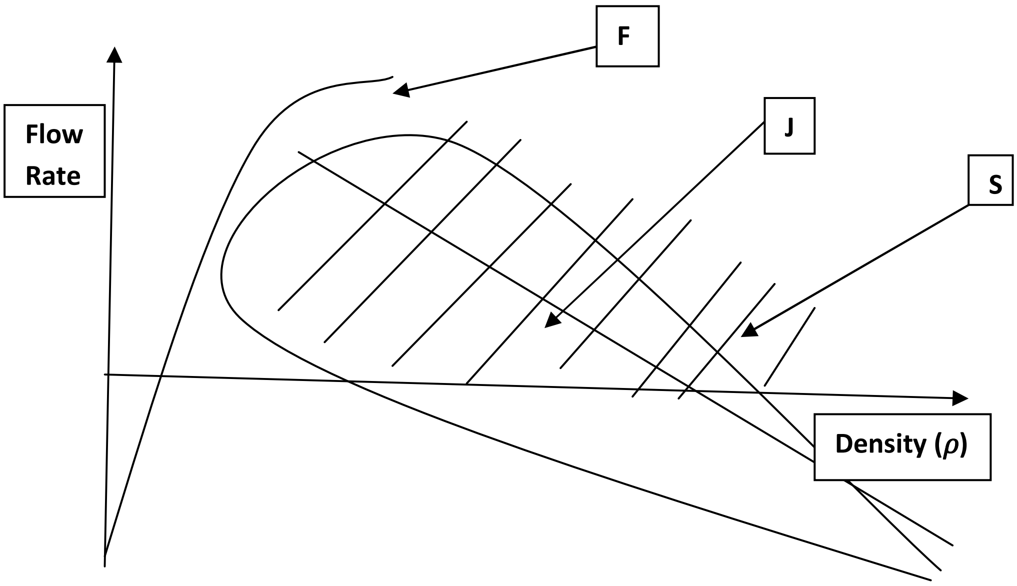

Fig.1. The three phases of traffic

The synchronized region is divided into stable and metastable regions by the wide moving Jam region. Metastable state is the state of synchronized flow which is stable with respect to small enough local disturbances in flow. If great enough local disturbances in this state appears the disturbance grows leading to the formation of a wide moving jam.

The main aim of this study is develop a macroscopic model for traffic flow based on Kerner traffic flow theory with considerations of driver aggressiveness in the context of Kenyan roads.

II. Equation of Continuity

In traffic flow, the number of vehicles is conserved hence we use the equation of continuity.

$$\partial_t \rho + \partial_x \rho u = 0 \qquad (2.1)$$

It expresses the relation between the rates of change of the density with respect to time and flow with respect to space.

Dynamic velocity equation

This is used to describe time-varying and spatially varying average
velocities $u(x,t)$ such as those that occur in traffic jams or stop-and go-jams

$$\partial_t(\rho u) + \partial_x(\rho u^2) - \alpha\left(\frac{\rho}{1-\rho}\right)\partial_x u = 0 \qquad (2.2)$$

The variable α is a measure of driver aggressiveness and in this study we shall approximate it as a function of velocity as follows:

$$\alpha = Ae^{-u} \qquad (2.3)$$

where A is a constant. The range for α will be $0.3 < \alpha < 1$. As α tends towards 0, the model produces the negative travel velocities. If α = 0.3, the model becomes the 3-phase traffic flow (2012) and if α = 1, the model becomes Aw Rascle.

Substituting (2.3) into (2.2) gives

$$\partial_t(\rho u) + \partial_x(\rho u^2) - A\left(\frac{e^{-u}\rho}{1-\rho}\right)\partial_x u = 0 \qquad (2.4)$$

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Features of the new model

Depending on how we recast (2.4), we can rewrite the model equations in two forms. That is;

Non conservative: This is in terms of the primitive variables ρ and u.

$$\partial_t \rho + \partial_x(\rho u) = 0 \quad (2.5)$$

$$\partial_t u + \left(u - \frac{Ae^{-u}\rho}{\rho(1-\rho)}\right)\partial_x u = 0 \quad (2.6)$$

Using the expression (2.17) while substituting C for A on (.6) yields

$$\partial_t u + \left(u - e^{-u}\rho p'(\rho)\right)\partial_x u = 0 \quad (2.7)$$

Therefore the Non conservative form of our model is

$$\partial_t \rho + \partial_x(\rho u) = 0$$

$$\partial_t u + \left(u - e^{-u}\rho p'(\rho)\right)\partial_x u = 0$$

Comparing with the Aw-Rascle model, its Non conservative form is

$$\partial_t \rho + \partial_x(\rho u) = 0 \quad (2.8)$$

$$\partial_t u + \left(u - \rho p'(\rho)\right)\partial_x u = 0 \quad (2.9)$$

In order to express the equations in Conservative Form we multiply (2.5) with p' to obtain $p'\rho\partial_x u = -p'\partial_t\rho - p'u\partial_x\rho$. Substituting the result in (2.7) and multiplying by e^u, yields

$$e^u\partial_t u + ue^u\partial_x u + p'\partial_t\rho + p'u\partial_x\rho = 0$$

Grouping this expression gives

$$\partial_t\left(e^u + p\right) + u\partial_x\left(e^u + p\right) = 0 \quad (2.10)$$

Multiplying (1.2.22) by $\left(e^u + p\right)$

$$\left(e^u + p\right)\partial_t\rho + \left(e^u + p\right)\partial_x\rho u = 0 \quad (2.11)$$

Multiplying (2.10) by ρ and adding the result to (2.11) achieves the conservative form after regrouping the terms:

$$\partial_t\rho + \partial_x\rho u = 0 \quad (2.12)$$

$$\partial_t\left[\rho\left(e^u + p\right)\right] + \partial_x\left[\rho u\left(e^u + p\right)\right] = 0 \quad (2.13)$$

Where the conservative variables are ρ and

$$y = \rho e^u + \rho p$$

To obtain the Conserved form of Aw-Rascle model, we multiply (2.8) by ρ and $\partial_t(u + p(\rho)) + u\partial_x(u + p(\rho))$ by $(u + p(\rho))$ and the add up the two equations to obtain

$$\partial_t\rho + \partial_x\rho u = 0 \quad (2.14)$$

$$\partial_t(\rho(u + p(\rho))) + \partial_x(\rho u(u + p(\rho))) = 0 \quad (2.15)$$

Therefore the conservative variables of the model are ρ and $y = \rho u + \rho p(\rho)$

The properties of the system are largely dictated by the Eigen values of the Jacobian Matrix

$A(U)$, are determined by the characteristic polynomial det $(A - \lambda I) = 0$

Expressing the new model (2.5) and (2.7) in the form

$U_t + A(U)U_x = 0$

(2.16)

We have $U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \rho \\ u \end{bmatrix}$ and

$$A(U) = \begin{bmatrix} u & \rho \\ 0 & u - e^{-u}\rho p'(\rho) \end{bmatrix} = 0$$

So, $\begin{bmatrix} u - \lambda & \rho \\ 0 & \left(u - e^{-u}\rho p(\rho)\right) - \lambda \end{bmatrix} = 0$

This yield $(u - \lambda)\left(-\lambda + \left(u - e^{-u}\rho p'(\rho)\right)\right) = 0$

Expanding the equation gives;

$$\Rightarrow \lambda^2 - \lambda\left(2u - e^{-u}\rho p'(\rho)\right) + u^2 - ue^{-u}\rho p'(\rho) = 0$$

Applying the quadratic formula gives

$$\lambda = \frac{\left(2u - e^{-u}\rho p'(\rho)\right) \pm e^{-u}\rho p'(\rho)}{2}$$

leading to $\lambda_1(u) = u - e^{-u}\rho p'(\rho)$ *and* $\lambda_2 = u$ (2.17)

These Eigen values are the characteristic speeds that govern the propagation of information in the traffic system. The largest Eigen value is equal to the flow velocity.

This now means that no traffic information travels faster than the traffic and so the anisotropic character of the vehicular traffic flow is preserved.

The Eigen values are real and distinct hence the system of equations is purely hyperbolic. Expressing the Aw-Rascle model in the form (2.16) yields $U = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \rho \\ u \end{bmatrix}$

and $A(U) = \begin{bmatrix} u & \rho \\ 0 & u - \rho p'(\rho) \end{bmatrix}$

The Eigen values become;

$$\lambda_1(u) = u - \rho p'(\rho) \text{ and } \lambda_2(u) = u \quad (2.18)$$

This shows that the system is purely hyperbolic and that the anisotropic character of vehicular traffic is preserved.

We now refer to the waves associated with λ_1 as 1-wave and those associated with λ_2 as 2-waves. In order to determine those waves, we now calculate the right eigenvectors $R^{(i)} = \left(r_1^{(i)}, r_2^{(i)}\right)$ of the matrix $A(u)$ corresponding to the Eigen values λ_i

We now evaluate $R^1(u)$.

$$\begin{pmatrix} u & \rho \\ 0 & u - e^{-u}\rho p'(\rho) \end{pmatrix}\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \left(u - e^{-u}\rho p'(\rho)\right)\begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

This means that

$$\begin{bmatrix} uR_1 + \rho R_2 \\ R_2\left(u - e^{-u}\rho p'(\rho)\right) \end{bmatrix} = \begin{bmatrix} \left(u - e^{-u}\rho p'(\rho)\right)R_1 \\ \left(u - e^{-u}\rho p'(\rho)\right)R_2 \end{bmatrix}$$

Leading to $R_2 = -R_1\left(e^{-u}\rho p'(\rho)\right)$ and setting $R_1 = 1$ hence we have

$$R^1(u) = \begin{pmatrix} 1 \\ -e^{-u}p'(\rho) \end{pmatrix}. \quad (2.19)$$

Similarly For $R^2(u)$

$$\begin{pmatrix} u & \rho \\ 0 & u-e^{-u}\rho p'(\rho) \end{pmatrix}\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = u\begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

Yielding to $R_2 = 0$

Therefore $R^2(u) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (2.20)

Similarly we work out the Eigen vectors for the Aw-Rascle model to obtain

$$R^1(u) = \begin{pmatrix} 1 \\ -p'(\rho) \end{pmatrix} \quad (2.21)$$

$$R^2(u) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2.22)$$

We now determine the kind of waves associated to each Eigen value $\lambda_i, i = 1,2$ by checking whether the dot product $\nabla\lambda_i(u)\cdot.R^{(i)}(u)$ vanishes or not.

For $\lambda_1(u)$ we have

$$\begin{pmatrix} \partial_\rho\lambda_1 \\ \partial_u\lambda_1 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ -e^{-u}p'(\rho) \end{pmatrix} = -\partial_\rho\left(e^{-u}\rho p'(\rho)\right) + \left(1+e^{-u}\rho p'(\rho)\right)\left(-e^{-u}p'(\rho)\right)$$

$$= -\partial_\rho\left(e^{-u}\rho p'(\rho)\right) - e^{-u}p'(\rho) - e^{-2u}\rho\left(p'(\rho)\right)^2 \neq 0$$

This implies that the 1st characteristic field is genuinely non-linear.

Evaluating $\lambda_2(u)$.

We have $\begin{pmatrix} \partial_\rho\lambda_2 \\ \partial_\rho\lambda_2 \end{pmatrix}\cdot\begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0$.

Hence the 2nd characteristic field is linearly degenerate. Comparing this model with the Aw Rascle model, we have

$\nabla\lambda_1(u)\cdot r^1(u) = -\partial_\rho\left(p'(\rho)\right) \neq 0$.

This shows that the 1st characteristic field is genuinely non linear.

$\nabla\lambda_2(u)\cdot r^2(u) = 0$

This shows that the 2nd characteristic field is linearly degenerate.

Now we compute the Riemann Invariants for the new model across each wave.

Across the λ_1-wave, we have $\frac{d\rho}{1} = \frac{du}{-e^{-u}p'(\rho)}$ which on re-arrangement $p'(\rho)d\rho = -e^u du$ and on integration yields $p(\rho) + e^u = C$

Across the λ_2-wave, we have $\frac{d\rho}{1} = \frac{du}{0}$ which leads to $u = C$ on integration.

The left and right Riemann Invariants now become;

$$I_L(\rho, \rho y) = p(\rho) + e^u \quad (2.23)$$

$$I_R(\rho, \rho y) = u \quad (2.24)$$

The Riemann Invariant for the Aw-Rascle model is:

Across the λ_1 − wave, we have $\frac{d\rho}{1} = \frac{du}{-p'(\rho)}$

Hence $I_L(\rho, \rho y) = p(\rho) + u$ (2.25)

And across the λ_2 − wave we have $\frac{\partial\rho}{1} = \frac{\partial u}{0}$ and on integration gives

$$I_R(\rho, \rho y) = u \quad (2.26)$$

Owing to the above facts above, we can now confirm that, in the new model the 1-wave will either be a rarefaction or shock waves and the 2-waves will be contact discontinuities.

For a shock wave the two constant states U_L and U_R are connected through a jump discontinuity in a genuinely non linear field and the following conditions apply:

- The Rankine Hugonoit Conditions

$$F(U_R) - F(U_L) = S_i(U_R - U_L) \quad (2.27)$$

The Entropy Conditions

$$\lambda_i(U_L) > S_i > \lambda_i(U_R) \quad (2.28)$$

For a contact wave, the two data states U_L and U_R are connected through a single jump discontinuity of speed S_i in a linearly degenerate field and the following conditions apply:

- The Rankine Hugonoit Conditions
  $$F(U_R) - F(U_L) = S_i(U_R - U_L) \quad (2.29)$$
- Constancy of the generalized Riemann Invariant across the wave
  $$\frac{du_1}{R_1^i} = \frac{du_2}{R_2^i} \quad (2.30)$$
- The parallel characteristic conditions
  $$\lambda_i(U_L) = S_i = \lambda_i(U_R) \quad (2.31)$$

For a rarefaction wave with two data sates U_L and U_R are connected through a smooth transition I a genuinely non linear field and the following conditions are met:

- Constancy of the generalized Riemann Invariant across the wave
  $$\frac{du_1}{R_1^i} = \frac{du_2}{R_2^i}$$
- Divergence of characteristics
  $$\lambda_i(U_L) < \lambda_i(U_R) \quad (2.32)$$

The Riemann problem

Using the conservative form of the model (2.12) and (2.13) we set up the Riemann Problem with piecewise constant initial data as follows:

$$\partial_t U + \partial_x F(U) = 0$$

$$U(x,0) = \begin{cases} U_L & if\ x < 0 \\ U_R & if\ x > 0 \end{cases} \quad (2.33)$$

Where $u=(\rho,y)^T$, $F(u)=(\rho u, yu)^T$ and $U_L(U_R)$ is the piecewise constant traffic state on the left (right) of the jump located at $x=0$. Since $\lambda_1(\rho,u)<\lambda_2(\rho,u)$ for all u, then 1-waves must exceed 2-waves. Thus the general solution of the Riemann Problem includes a 1-wave connecting the left state u_L to an intermediate state U_M (to be defined) and a 2-wave connecting this intermediate state U_M to the right state U_R.

Since 1-wave is either shock or rarefaction waves, we shall have the following type of solution.

- 1-shock connecting U_L to U_M followed by a 2-contact discontinuity connecting U_M to U_R.
- 1-rarefaction wave connecting U_L to U_M followed by a 2-contact discontinuity connecting U_M to U_R. The right edge of the fan is denoted by S_H (rarefaction head) and the left edge denoted by S_T (rarefaction tail).

In the case that $U_M = U_R (U_M = U_L)$
Then U_L and U_R are connected by only a 1–wave (2-wave).

In order to determine the intermediate state U_M we need to the Riemann Invariant (2.23), (2.24)

For numerical purpose (in the next chapter) we present the solution to the Riemann Problem set $x = 0$

As follows
$$U_G=\begin{cases} U_M \text{ if } S_1<0 \\ U_L \text{ if } S_1>0 \\ \tilde{U} \text{ if } S_T<0<S_H \end{cases} \qquad (2.34)$$

Where the intermediate state $U_M=(\rho_M, y_M)$ is computed from the Lax curves (Riemann Invariants) as below

$$e^{u_M}+p(\rho_M)=e^{u_L}+p(\rho_L) \qquad (2.35)$$

Since $u_M=u_R$ and $y=\rho e^u+\rho p$ we obtain from (2.27) the following expressions;

$$\rho_M=\left(e^{u_L}+p(\rho_L)-e^{u_R}\right)\frac{1}{p}$$

$$y_M=\rho_M\frac{y_L}{\rho_L} \qquad (2.36)$$

Hence obtaining the explicit form of the intermediate state U_M in terms of U_L, U_R.

To obtain the solution $\tilde{U}=\left(\tilde{\rho},\tilde{y}\right)^T$ inside the rarefaction we consider the speed of the characteristic rays inside the fan and also use the Lax curves to have;

$$e^{\tilde{u}}+p\left(\tilde{\rho}\right)=e^{\tilde{u}_L}+p(\rho_L)$$

$$\tilde{u}-e^{-\tilde{u}}\tilde{\rho}\,p\left(\tilde{\rho}\right)=0$$

$$\because\frac{x}{t}=0 \qquad (2.37)$$

Which upon solving simultaneously for $\tilde{\rho}$ and $\tilde{u}$ yields the desired form of the Riemann solution $\tilde{U}$.

To compute the speeds S_1, S_T, S_H we note that U_L and U_R can always be connected by a 1–rarefaction wave provided they lie on the same integral curve (on which rarefaction curves lie) and that the condition below must be satisfied;
$\lambda_1(\rho_L,u_L)<\lambda_1(\rho_M,u_M)$

Otherwise the two states are connected by a 1-shock and so from the Rankine Hugoniot condition we have

$$S_1=\frac{\rho_L u_L-\rho_M u_M}{\rho_L-\rho_M}$$

Moreover, since the right (left) edge of the 1-rarefaction wave is said to carry the value $U_M\ (U_L)$, we write;

$$S_T=u_L-e^{-u_L}\rho_L p'(\rho_L)$$
$$S_H=u_M-e^{-u_M}\rho_M p'(\rho_M) \qquad (2.38)$$

Finally, we illustrate on the $(\rho,\rho u)$ phase plane how the derived Aw-Rascle type traffic model (2.23)-(2.24) handles transitions from a left state U_L to the right state U_R with the aid of the above mentioned i – Lax curves (for $i=1,2$).

III. Method of Solution

The numerical solution to the conservative system (2.14)-(2.15) is achieved by use of a hybrid numerical scheme proposed by Chalons and Goatin 2007, based on Godunov method. To numerically solve the homogeneous system $\partial_t U+\partial_x F(U)=0$ (3.1) the spatial domain is discretisized into M cells.

$C_j=[x_{j-1/2}, x_{j+1/2}]$ For j= 1,2…M of the same size as Δ_x

The cell interface and cell centre are respectively defined as

$x_{j-1/2}$= (j-1) Δ_x , $x_{j+\frac{1}{2}}$ = j Δ_x and x_j = (j – ½) Δ_x

Suppose that at time $t=t^n$, the initial data for (3.1) is given as $U(x,t^n)$. Then the first step of Godunov scheme is the evolution of the solution to a time $t^{n+1}=t^n+\Delta t$. That is achieved through considering the cell averages.

$$U_j^n=\frac{1}{\Delta x}\int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}}U(x,t^n)\,\partial x \qquad (3.2)$$

Which now produces a piecewise constant approximation of the solution U (x,t^n) as
$U(x,t^n)=U_j^n$. For all $x\in C_j, j=0,1,\ldots\ldots m$ (3.3)

The second step is obtaining the solution for the local Riemann problem say $RP(U_j^n, U_{j+1}^n)$ at the cell interface $x_{j+1/2}$ with data U_j^n *and* U_{j+1}^n, respectively on the left side and right side of position $x_{j+1/2}$

The solutions to this Riemann problem are self similar solutions $U_{j+\frac{1}{2}}\left(\tilde{x}/\tilde{t}\right)$, $\tilde{x}=x-x_{j+\frac{1}{2}}$, $\tilde{t}=t-t^n$, $x\in\left[x_j, x_{j+\frac{1}{2}}\right]$, $t\in\left[t^n,t^{n+1}\right]$

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i.e they are functions of Riemann Problem local co-ordinates $\tilde{x}/\tilde{t}$ and are constituted by the 1-and 2-wave. Now for a sufficiently small time step Δt, such that there are no eave interactions, we obtain the global solution $\tilde{U}(x,t)$ in the entire spatial domain for $t \in [0, \Delta t]$ by gluing together the solution of the local Riemann Problem set at each interface of the cell as below.

$$\tilde{U}(x,t) = U_{j+\frac{1}{2}}\left(\tilde{x}/\tilde{t}\right), \text{ for all } (x,t) \in [x_j, x_{j+1}] \times [0, \Delta t] \qquad (3.4)$$

Having obtained the solution $\tilde{U}(x,t)$ the final step of the Godunov scheme entail the evolution of the solution to a time $t^{n+1} = t^n + \Delta t$ by defining a new set $\{U_{j+1}^n\}$ of average values as follows;

$$U_j^{n+1} = \frac{1}{\Delta x} \int_{x_{j-\frac{1}{2}}}^{x_{j+\frac{1}{2}}} \tilde{U}(x, t^{n+1})\, dx \qquad (3.5)$$

Within $C_j = \left[x_{j-\frac{1}{2}}, x_{j+\frac{1}{2}}\right]$.

To guarantee that the interaction of the $i-$waves, $i = 1, 2$ is entirely contained within cell Cj we impose the following CFL condition

$$\Delta t \le \frac{C_{cfl} \Delta x}{\max\{|\lambda_i(U)|, i = 1, 2\}} \qquad (3.6)$$

C_{cfl} is called the Courant number and is usually set to 1. The CFL condition together with the integral form of the conservation law allow us to alternatively express U_j^{n+1} in the following form

$$U_j^{n+1} = U_j^n + \frac{\Delta t}{\Delta x}\left[F_{j-\frac{1}{2}}^n - F_{j+\frac{1}{2}}^n\right] \qquad (3.7)$$

With the intercell numerical flux given by

$F_{j+\frac{1}{2}}^n = F\left(U_{j+\frac{1}{2}}(0^-; U_j^n, U_{j+1}^n)\right)$ where $u_{i+\frac{1}{2}}(0)$ denotes the exact solution $u_{i+\frac{1}{2}}\left(\frac{x}{t}\right)$ of the Riemann Problem evaluated at $\frac{x}{t} = 0$

IV. Numerical Results

Case1

$\rho^L = 0.4$	$\rho^R = 0.4$
$u^L = 0.8$	$u^R = 0.2$

Table 4.1

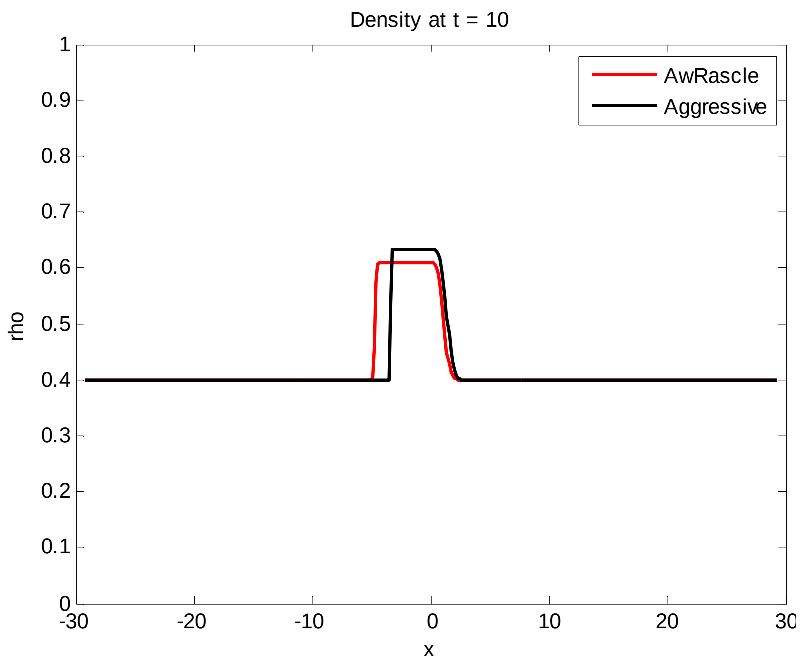

Figure 4.1.

With x = 0, , in this case the solution is given by a 1-shock followed by a 2-contact wave for the Aw-Rascle and a 1-Shock followed by a phase transition for the new model.

Case 2

$\rho^L = 0.3$	$\rho^R = 0.6$
$u^L = 0.7$	$u^R = 0.3$

Table 4.2

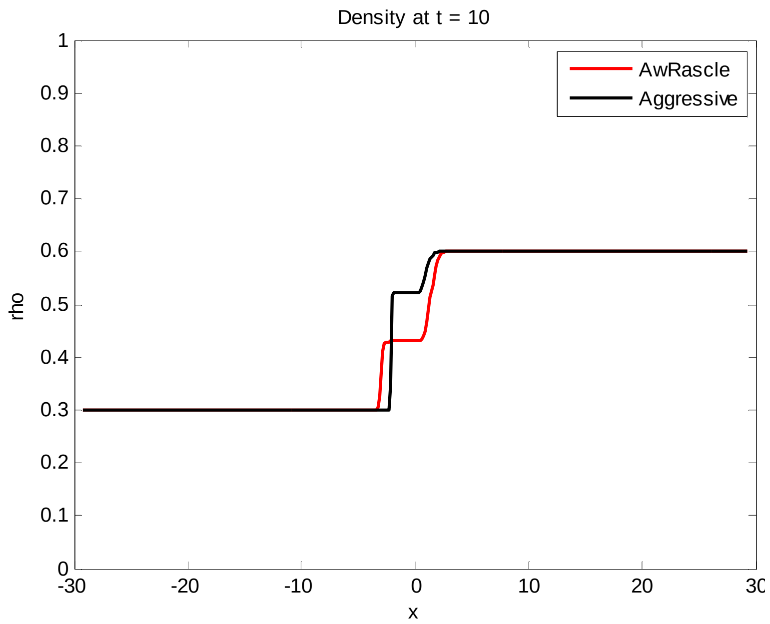

Figure 4.3.

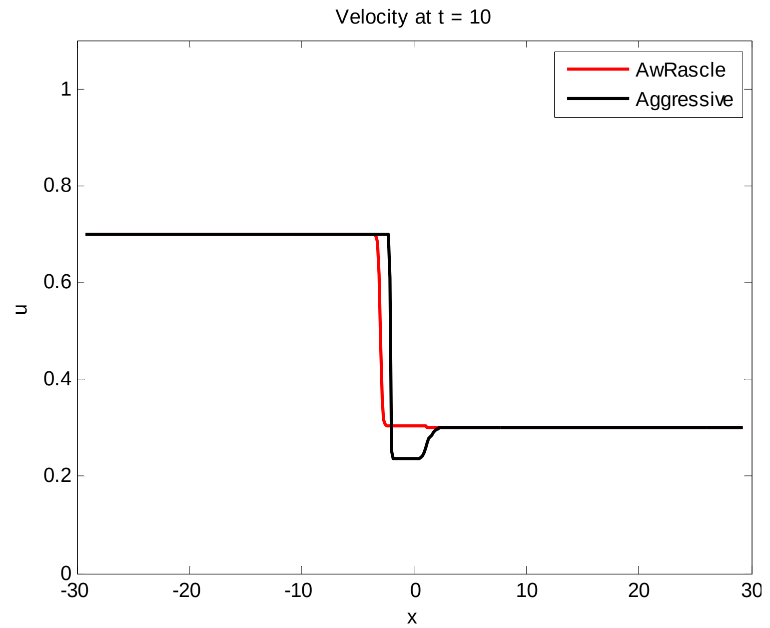

Figure 4.4.

With x = 0 in the case above the solution is a phase transition followed by a 2-contact discontinuity for the Aw-Rascle model and for the new model the solution is a 1-shock followed by a phase transition.

Discussion of Results

In case 1, Aw-Rascle model shows that when an oncoming vehicle comes across slow moving vehicles but the density ahead is not changing, it relaxes to adjust to the speed of the slow vehicles. The new model shows that in a similar situation, the vehicles slow down but maintain a speed higher than that of vehicles ahead. This shows a bigger possibility of passing by changing lanes. The vehicles finally adjust to the speed of vehicles ahead. Before getting to the bottle neck, the vehicle was in free flow region by overtaking, it forces the vehicle behind it in the target lane to over break hence causing local disturbance in the flow region. This is evidenced by the increase in density by the new model as opposed to the

Aw-Rascle model at the intermediate state. This shows a phase transition.

In the case 2, Aw-Rascle model shows that if a fast moving vehicle approaches a slow moving vehicle and the density of vehicles ahead is higher, the vehicle should decelerate to adjust to the velocities ahead. With the new model, vehicles should move at a speed slower than that of the vehicles then accelerate to the speed of the vehicles ahead.

We observe that the average densities increase and average velocities decrease significantly. Whenever we have such a situation, there is a spontaneous emergence of a growing narrow moving jam and this takes place in synchronized flow region. This emergence of a narrow moving jam can be caused by unexpected breaking, lane change and emergence of vehicles from other roads thus causing fluctuations in traffic flow variables. The case depicts a phase transition from F. S. The two cases above clearly show that our new model is able to reproduce the features of a three phase traffic flow.

V. Conclusion

In this study, macroscopic traffic flow model motivated by Kerner theory with considerations of driver aggressiveness in the context of Kenyan roads has been presented.

The hyperbolic nature of the derived macroscopic model is studied. The model has been shown to be hyperbolic through its features. By construction of the solution to the Riemann Problem, set up using the conservative form of the model, the model features have well been explored further.

The numerical method for solving the macroscopic model in conservation form is discussed and tests are carried out to show the effectiveness of the numerical method used.

Using this numerical method we go ahead to simulate traffic flow on road ways. Through these simulations we assess the ability of the derived macroscopic traffic flow model.

The model was compared with the Aw-Rascle and the features of the model have been shown to reproduce the features of a three traffic flow which the Aw-Rascle model cannot reproduce. It has equally been shown that the model respects the frontal aspects of traffic and it does not produce negative travel.

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