Abstract – In this study, the controllers are improved by way of the sliding mode control and passive control methods for continuous time nonlinear Shimizu-Morioka chaotic system. The control structure of Shimizu-Morioka chaotic system for both methods were calculated as analytically, and then applied a numerical example. Shimizu-Morioka chaotic system like other chaotic systems show nonlinear behavior and this causes the systems to be unstable. The Shimizu-Morioka non-linear chaotic system for the analysis of stability was linearized continuous time via Taylor Series. In addition, the state variables were analyzed whether they reach the equilibrium point or not, while controllers at different times were activated. Both the sliding mode and passive methods reached to an equilibrium point, but the results of the simulations sliding mode control were better performance than passive control technique based on the performance criteria.

Keywords – Chaos, Equilibrium Point, Passive Control, Shimizu-Morioka, Sliding Mode Control (SMC).

I. INTRODUCTION

Chaotic systems that have limited predictability are exhibited of non-linear behavior depending on the initial conditions. In most of the events weather events and changes in temperature show the non-linear behavior. Therefore, chaotic systems can be used in the analysis of natural phenomena. Lorenz simulating weather models are the first application of this event [1, 2]. Chua, Rössler, Rikitake, Rucklidge, Chen, Lü, and Shimizu-Morioka are other important examples chaotic systems. These systems were applied to many linear and nonlinear control methods, because of output regulation [3], stabilization and synchronization [4,5], arbitrary output [6], and ending chaos [7]. One of the most important aims of the control methods is to stabilize the equilibrium point [8].

The Shimizu-Morioka chaotic system is a classical three dimensional chaotic system studied by Shimize and Morioaka [3]. Shimizu-Morioka chaotic system was applied to many control methods, such as feedback control [3], adaptive control [4], exact linearization [6], and sliding mode control [9]. On the other hand, the Shimizu-Morioka chaotic system is very important in fields like fluid dynamics and laser physics [10].

The chaotic systems have non-linear complex structures. So, the control of chaotic systems can be nonlinear control methods like sliding mode control method or like Taylor Series expansion process in linear control methods [6, 9]. In the last two decades, against disturbances, instability, uncertainties etc. [4, 8, 16] sliding mode control method has been applied to many areas, such as electric power systems [11], satellite systems [12], biomedical systems [13], car control systems [14], aerospace systems [15], mathematics etc. [5].

Chaotic systems is difficult to control because of having nonlinear and complex structures. For this reason sliding mode control method has been selected as a powerful control method. Sliding mode method studies are conventional sliding mode controllers and observer design, second-order sliding mode control and differentiators, higher-order sliding mode controllers and differentiators and higher-order sliding mode observer design etc. [17-18].

In order to control a nonlinear system with linear control systems, nonlinear systems have to be linearized. There are a lot of linearization applications in various area in the literature. The non-linear chaotic system can be linearized by means of Taylor Series expansion process. If time-dependent changes of system function are linearized constantly, the eigenvalues can be calculated. Therefore, stability can be easily analyzed using time dependent changes of eigenvalues [19, 20].

In recent years, passive controller is another important method used to control the chaotic systems. Passive control techniques have been developed for Lorenz [21], Lü [22], Rabinovic [23] ve Rucklidge Chaotic Systems [24]. Passive nonlinear systems use the nonlinear analog of the minimum phase property. This property in nonlinear systems is the stability of zero dynamics. Moreover, the important issue in the analysis is to determine when a finite dimensional nonlinear system can be rendered passive via feedback. In other terms the design issue is to identify nonlinear systems that are feedback equivalent to passive control [25].

In this study, stability at equilibrium point, output regulation and synchronization of Shimizu-Morioka chaotic system is studied with sliding mode control and passive control methods.

This paper is organized as follows. A literature review is given in the introduction. Brief definition of a Shimizu-Morioka chaotic system is given in part 2, then, the design of the sliding mode controller is given in section 3, afterwards, numerical simulation of chaos control by way of sliding mode control and passive control methods and finally, the conclusion is given in Section 5.
II. SHIMIZU-MORIOKA CHAOTIC SYSTEMS

The Shimizu-Morioka chaotic system is defined by the following nonlinear equations.

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= x - xz - ay \\
\dot{z} &= x^2 - bz
\end{align*}
\]

(1)

The Shimizu-Morioka systems is described by,

\[
\begin{pmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix} =
\begin{bmatrix}
0 & 1 & 0 \\
1 - z & -a & 0 \\
x & 0 & -b
\end{bmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
\]

(2)

Where x, y and z are state variables, and a, and b are positive constant parameters when they choose parameter values for a the Shimizu-Morioka System is \(a = 0.75\) and \(b = 0.54\).

Using a MATLAB/Simulink model, the time series of the Shimizu-Morioka chaotic system of the \(x, y, z\) and \(xy, xz, yz\) and \(xyz\) phase portraits were obtained as shown in Figure 1 and Figure 2 When \(x_0 = 0, y_0 = 0.01\) and \(z_0 = 0\).
III. DEVELOPMENT OF SLIDING MODE CONTROLLER FOR SHIMIZU-MORIOKA CHAOTIC SYSTEM

Sliding mode control of Shimizu-Morioka chaotic system was developed using only one controller. The system can be presented with equation (3).

\[
\begin{align*}
\dot{x} &= y + u \\
\dot{y} &= x - xz - ay \\
\dot{z} &= x^2 - bz
\end{align*}
\]

(3)

Where \( u \) is the control input.

Then, selection of a sliding surface from Equation (4), following equations can be written:

\[
\begin{align*}
s &= \dot{e} + \lambda e \\
\dot{s} &= \ddot{e} + \lambda \ddot{e}
\end{align*}
\]

(4)

(5)

The trajectory error state could be chosen as \( e = x_r - x \), where \( x_r \) is constant, so \( \dot{x}_r = \ddot{x}_r = 0 \).

\( \dot{x}_r \) and \( \ddot{x}_r \) are obtained as, \( \dot{e} = \dot{x}_r - \dot{x} = o - \dot{x} = -\ddot{x} \) and \( \ddot{e} = \ddot{x}_r - \ddot{x} = o - \ddot{x} = -\ddot{x} \).

\[
\text{sign}(s) = \begin{cases} 
1, & s > 0 \\
0, & s = 0 \\
-1, & s < 0 
\end{cases}
\]

(6)

\[
\dot{s} = -\rho\text{sign}(s) - k.s
\]

(7)

A proportional reachability rule like Eq. (7) is chosen, then following equations can be written.

\[
\begin{align*}
\dot{s} &= \dot{e} + \lambda e = \ddot{x}_r - \ddot{x} + \lambda (\ddot{x}_r - \ddot{x}) = -\rho \text{sign}(s)k.s \\
\ddot{s} &= \ddot{e} + \lambda \ddot{e} - \ddot{x} - \dddot{x} + \lambda \dddot{x} = -\rho \text{sign}(s)k.s
\end{align*}
\]

(8)

The input control signal is obtained like equation (9).

\[
u = -\frac{\ddot{x} + k_s + \rho \text{sign}(s)}{\lambda}
\]

(9)

The stability analysis of control procedures is very important to evaluate the design of nonlinear controller. Therefore, stability analysis is evaluation of eigenvalues of the system linearization matrix at the equilibrium point \( E_0 \) (0, 0, 0) [9, 26].

IV. PASSIVE CONTROL METHOD

In the modern control theory, passive control technique has been widely used. The nonlinear system was illustrated by equations of characteristic.

\[
\begin{align*}
u(t) &\text{ input and } y(t) \text{ output vectors, } f(x) \text{ and } g(x) \text{ are smooth vector fields, } h(x) \text{ is also a smooth mapping. In addition, nonlinear system can be described as } [23]: \\
\dot{x} &= f(x) + g(x)\cdot u(t) \\
y &= h(x) \tag{10}
\end{align*}
\]

Equation (10) modifies the system equation to;

\[
\begin{align*}
\dot{z} &= f_0(z) + p(z, y) \cdot y(t) \\
y &= b(z, y) + a(z, y) \cdot u(t)
\end{align*}
\]

(11)

Where \( a(z, y) \) is nonsingular for any \( (z, y) \) [23]. Zero dynamics describe the dynamics of the system (11) where \( y = 0 \). A system whose zeroes dynamics is asymptotically stable is called minimum phase system [25]. Let a system state function \( V(x) \) called storage function for system (11) and \( W(x) \) called Lyapunov function. If the system (11) \( x = 0 \) is an equilibrium point, has relative degree \( \{1,1,1,1,...,1\} \) and is weakly minimum phase, then can be locally feedback equivalent to a passive system with proper storage function \( V(x) \) [25]. Therefore, passive controller is as follows [23]:

\[
u(t) = a(z, y)^{-1}\left[-b^T(z, y) - \frac{\partial W(z)}{\partial z} - p(z, y)\right] - ay + v
\]

(12)

V. NUMERICAL SIMULATION

The control input signal is obtained by using equation (9). Where \( x = y, \dot{x} = 0 \), and sliding mode controller gains have been selected \( \lambda = 5, \rho = 0.198 \) and \( k = 9.9 \) with the initial conditions \( x_0 = 0 \), \( y_0 = 0.01 \) and \( z_0 = 0 \). The controllers are activated \( t = 40 \) second in all simulations.

\[
u = \frac{a}{\lambda} - y + \frac{k_s + \rho \text{sign}(s)}{\lambda} = \frac{0.75}{5} - y + \frac{9.9(y - 5x)\text{sign}(s)}{5} \]

(13)

\[
\begin{align*}
f_1 &= \dot{x} = y \\
f_2 &= \dot{y} = x - xz - ay \\
f_3 &= \dot{z} = x^2 - bz
\end{align*}
\]

(14)

The system was linearized by means of Taylor’s series expansion and then the Jacobian Matrix was determined by using the first terms of linear elements. A function is expressed in a great ratio by the first terms of the expansion while higher degree terms are ignored.

\[
f(x) = f(\bar{x}) = \frac{\partial f}{\partial x} \bigg|_{x=\bar{x}} (x - \bar{x}) + \text{hig. deg. ter.} \]

(15)

Taylor’s series expansion based linearization state space model can be written as follows.

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

(16)
As shown in this equation:

\[
\begin{bmatrix}
\frac{\partial y}{\partial x} \\
\frac{\partial (x - x - ay)}{\partial y} \\
\frac{\partial (x^2 - b)}{\partial y} \\
\frac{\partial (x^2 - b)}{\partial z} \\
\frac{\partial (x^2 - b)}{\partial z}
\end{bmatrix}
\]

\[
J = \begin{pmatrix}
0 & 1 & 0 \\
1 - z & -a & -x \\
2 - x & 0 & -b
\end{pmatrix}
\]

The eigenvalues are calculated by means of solving the characteristic equation,

\[
A = [\lambda I - J]
\]

\[
\begin{vmatrix}
\lambda - 1 & 1 & 0 \\
1 - z & \lambda - 0.75 & -x \\
2 - x & 0 & \lambda - 0.54
\end{vmatrix} = 0
\]

Initial eigenvalues for \( x_0 = 0, y_0 = 0.001 \) and \( z_0 = 0 \).

Which has the eigenvalues \( \lambda_1 = -1.4430, \lambda_2 = 0.6930, \lambda_3 = -0.5400 \) and \( \lambda_3 = 0.6930 \), the \( \lambda_3 \) is an unstable eigenvalue of \( A \) matrix, so Shimizu-Morioka chaotic system is unstable at the equilibrium point \( E_0=(0, 0, 0) \) from Lyapunov stability theory [9, 26]. Furthermore, The result of theoretical eigenvalues at the equilibrium point \( E_0 = (0, 0, 0) \) were also confirmed in Figure 3 in result of simulation, \( \lambda_1 = -1.4430, \lambda_2 = 0.5400 \) and \( \lambda_3 = 0.6930 \), respectively.

The designed and used sliding mode control model for Shimizu-Morioka chaotic system

On the other hand, the control of Shimizu-Morioka chaotic system was developed using passive control theory. The control model given by,

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= x - xz - ay + u \\
\dot{z} &= x^2 - bz
\end{align*}
\]

Where \( u \) is controller to be designed. Suppose that \( z_1 = x, z_2 = z \) and \( y \) is to be output of the system. The system can be expressed by,

\[
\begin{align*}
\dot{z}_1 &= y \\
\dot{z}_2 &= z^2 - bz_2 \\
\dot{y} &= z_1 - z_1z_2 - ay + u
\end{align*}
\]

Equation (20) modified form direction of the equation (11)

\[
\begin{align*}
\dot{z}_1 &= 0 \quad 0 \quad 1 \quad 0 \\
\dot{z}_2 &= -b \\
\dot{y} &= z_1 - z_1z_2 - ay + u
\end{align*}
\]

Choose the following storage function,

\[
v(z, y) = W(z) + \frac{1}{2} y^2
\]

Where,

\[
W(z) = \frac{1}{2} (z_1^2 + z_2^2)
\]

Is the Lyapunov function of \( f_0(z) \), and \( W(0) = 0 \).

According to equation (23) derivative of \( W(z) \),

\[
\dot{W}(z) = \frac{\partial W(z)}{\partial z} \dot{z} + \frac{\partial W(z)}{\partial y} \dot{y}
\]

\[
\dot{W}(z) = \lambda_1 z_1 + \lambda_2 z_2 + 1 \quad p(z, y)
\]

According to equation (12), the output of passive control system, \( u(t) \) by,

\[
\begin{align*}
u(t) &= 1^T [(-z_1 + z_1z_2 + ay) - z_1 z_2] \frac{1}{0} - ay + v \\
u(t) &= -2z_1 + z_1z_2 + y(a - \alpha) + v
\end{align*}
\]

According to \( z_1 = x, z_2 = z \) the system,
\( u(t) = -2 \cdot x + x \cdot z + y \cdot (a - \alpha) + v \)  

(27)

A control model had been developed for Shimizu-Morioka chaotic system using passive control method as seen in Figure 5.

Fig. 5. The passive control model developed for Shimizu-Morioka chaotic system

Where, \( \alpha = 1 \) and \( v = 0 \).

Fig. 6. The time response of the states for the control of the Shimizu-Morioka chaotic systems to \( E_0(0,0,0) \) equilibrium point with controllers activated at \( t = 40 \) second for the (a) \( x \), (b)=y, and (c) \( z \) signals

Fig. 7. SMC and passive control which activated with controller at t=30 second for the (a) \( xy \), (b) \( xz \), (c) \( yz \) and (d)= \( xyz \) portraits signals
The state variables can reach an equilibrium point $E_0(0,0,0)$ as the controller activated the system at 30th second as given in Figure 6. This phenomenon is seen most clearly from the phase portraits as seen in Figure 7. In Figure 6, while $x, y$ and $z$ state variables reach a fix point using sliding mode control, very fast reach they fix point very late after decay oscillation using passive control. In Figure 8, the eigenvalues at stable point using a sliding mode controller are -1.442, -0.5393, 0.6916, is given respectively, while passive controller reached -1.422, -0.5384, 0.6687 about 42. second.

The Shimizu-Morioka chaotic systems are tested by controller activation at $t = 50$ seconds, and then sliding mode control and passive control methods performances are investigated. The results of the simulations are shown in Figure 9, 10 and 11. In Figure 10, the simulation graphs of $x, y$ and $z$ state variable error are illustrated $e_1, e_2$ and $e_3$, respectively, or shows the time history of synchronization error $e_1, e_2$ and $e_3$ [4].
VI. CONCLUSION

In this study, the controllers are developed by the sliding mode control and passive control methods for continuous time nonlinear Shimizu-Morioka chaotic system. Numerical simulations of the sliding mode control method are more effective than passive control method. When the chaotic behavior of the system was terminated, the system reached about the equilibrium point. When the changes of eigenvalues are investigated Shimizu-Morioka system is unstable until the controller activated in 50th seconds. When \( x, y, \) and \( z \) state variables are examined time-dependent on changes the sliding mode control shows a better performance than passive control in terms of the performance criteria as maximum overshoot and settling time. On the other hand, according to the steady state error passive control has better behaviors than sliding mode control method.

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