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Forecasting the Number of Muslim Pilgrims Using Fuzzy Time Series

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Abstract – **Pilgrimage of Muslims is considered the largest human gathering all over the world in which more than three millions move together through a very limited space in a short time period. The yearly number of pilgrims coming from outside Saudi Arabia, denoted by NPO for short, is more than two thirds of the total number of Pilgrims. Therefore forecasting the NPO is considered by Saudi Arabia as the most important indicator in determining the planning mechanism for future secure and comfortable pilgrimage seasons. The main objective of this article is to employ the fuzzy time series approach [1] to forecast the yearly series of NPO and to show that it gives better forecasts than BoxJenkins Procedures. In order to achieve our objective, the fuzzy time series method is used to forecast the future five observations and the results are compared with the results given by Shaarawy et. al [2].**

Keywords – **Fuzzy Time Series; Fuzzy Sets; Linguistic Variable; Time Invariant Model.**

I. Introduction

Forecasting is important for any society to effectively control a plan which is often engaged by collecting a large number of time series data in order to be able to determine the trend of the development.

Pilgrimage (or hajj) of Muslims is one of the most important events all over the world. It is considered the largest human gathering in which pilgrims move together through a very limited space in a short period of time. This important event is repeated annually, and the number of pilgrims is increasing year after year. In addition, hajj is one of the main sources of gross national product (GNP) in Saudi Arabia. Therefore, it has received a great attention by the government of Saudi Arabia. Every year, the kingdom of Saudi Arabia spends a great deal of efforts and money to improve the hajj system, which includes security, economy, management of water and electrical resources, services and goods required by the vast number of pilgrims. However, without knowing the number of pilgrims in advance may make the process of improvement very difficult. Therefore, it is very important to have a mechanism for predicting and forecasting the number of pilgrims in order to determine the size and quality of expansions and maintenance needed in the two holy mosques in Makkah and Medina and to avoid any mistakes or disasters that may occur. One of the main components of the total number of pilgrims is the Number of Pilgrims coming from Outside the kingdom (NPO).

The first objective of this paper is to use the fuzzy time series approach to forecast the number of NPO data.

Fuzzy time series have been applied to a diversity of data such as stock exchange, temperature, enrollment, ...etc. In forecasting, statistical methods such as time series models, Box-Jenkins, ...etc. are commonly used techniques. However, if the given data are in linguistic form or very little, the statistical methods will fail [3, 4, 5]. In order to handle such a problem, fuzzy time series models [3, 4, 5, 1, 6, 7] have been developed an applied in practice. The first time series model was introduced by Song-Chisson 1993. And since then, fuzzy time series have drawn attention of many researchers because of its good performance for some kind of forecasting problems.

In [1] they proposed a method for handling forecasting of university enrollments using fuzzy time series.

In this paper, we exploit the fore-mentioned method to forecast NPO.

The remainder of this paper is organized as follows: section (2) presents some basic characteristics of fuzzy time series. A complete description of the algorithm used to predict NPO in section (3). Section (4) is dedicated to evaluate the forecast performance of fuzzy time series procedures and compare their numerical results. Finally, the paper is concluded in section (5).

II. Fuzzy Time Series

In this section we introduce a brief review of fuzzy time series. Let U be the universe of discourse, $U = \{u_1,u_2,\cdots,u_n\}$, a fuzzy set[8] A of U is defined as

$$A = \mu_A(u_1)/u_1 + \mu_A(u_2)/u_2 + \cdots + \mu_A(u_n)/u_n$$

where μ_A is the membership function of A, $\mu_A : U \longrightarrow [0,1]$, $\mu_A(u_i)$ denotes the grade of membership of u_i in A, $\mu_A(u_i) \in [0,1]$. Let $Y(t)(t = \cdots,0,1,2,\cdots)$, be a subset of $\mathbb{R}^1$, be the universe of discourse on which fuzzy sets $\mu_i(t)(i = 1,2,\cdots)$ are defined and let $F(t)$ be a collection $\mu_i(t)(i = 1,2,\cdots)$. Then, $F(t)$ is called a fuzzy time series on $Y(t)(t = \cdots, 0,1,2,\cdots)$. It is obvious that $F(t)$ can be regarded as a linguistic variable, and $\mu_i(t)$ can be viewed as possible linguistic values of $F(t)$, where $\mu_i(t)(i = 1,2,\cdots)$ are represented by fuzzy sets. Furthermore, we can also see that $F(t)$ is a function of time t, i.e., the values of $F(t)$ can be different at different times. If $F(t)$ is caused by $F(t-1)$ only, then this relationship is represented by

$$F(t-1) \longrightarrow F(t)$$

Let $F(t)$ be a fuzzy time series. If for any time t, $F(t) = F(t-1)$ and $F(t)$ only has finite elements then $F(t)$ is called a time-invariant fuzzy time series, otherwise, called time-variant fuzzy time series[1].

Assume that the value of the time series at time t is T_t and assume that the value of the time series at time $(t-1)$ is T_{t-1}, then the variation of the time series between time t and time $(t-1)$ is equal to $T_t - T_{t-1}$. Here, we will introduce some heuristic rules that will be used in the algorithm[1].

Rule 1: The variation of the time series value between times t and $(t-1)$ is related to the variations of the time series values between time t and the past values, and the relationship of the time series between the current value and last value is closer than the one between this value and the other past values.

Rule 2: If the trend of the time series past values is increasing, then the current value of the time series is increasing. If the trend of the time series past values is decreasing, then the current value of the time series is decreasing.

Rule 3: Let the variation of last value be a criterion. Compute the fuzzy relationships between last value and the other past values based on data variations. From the derived fuzzy relationships, we can know the degrees of relationships between the variation of last value and the variations of other past values. The variation of current value can be obtained from the derived fuzzy relationships.

Based on these heuristic rules, firstly we can fuzzify the historical time series data. We use the fuzzified variation of the historical time series data and the linguistic values (big decrease), (decrease), (no change), (increase), (big increase), (too big increase) to forecast time series values. The fuzzified variation of the historical time series between T_t and T_{t-1} can be described as follows:

$$F(t)=u_1/(big\ decrease) + u_2/(decrease) + \cdots + u_i(L) + \cdots + u_m/(too\ big\ increase) \quad (1)$$

where $F(t)$ denotes the fuzzified variation of the time series between t and $(t-1)$, u_i is the grade of membership to the linguistic value L, m is the number of the elements in the universe of discourse, and $1 \le i \le m$.

To forecast the time series value at t, we must decide how many values of the time series will be used, where the number of time series values to be used is called the *window basis*. Suppose we set a window basis to$w$ values, then the variation of last value is used to be a criterion and the other variations of w past values are used to form a matrix which is called the *operation matrix*. The *criterion matrix* $C(t)$ and the operation matrix $O^w(t)$ at t are expressed as follows [1]:

$$C(t) = F(t-1) = \begin{bmatrix} \overset{(big\ decrease)}{C_1} & \overset{(decrease)}{C_2} & \cdots & \overset{(too\ big\ increase)}{C_m} \end{bmatrix} \quad (2)$$

$$O^w(t) = \begin{bmatrix} F(t-2) \\ F(t-3) \\ \vdots \\ F(t-w-1) \end{bmatrix} = \begin{bmatrix} \overset{(big\ decrease)}{O_{11}} & \overset{(decrease)}{O_{12}} & \cdots & \overset{(too\ big\ increase)}{O_{1m}} \\ O_{21} & O_{22} & \cdots & O_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ O_{w1} & O_{w2} & \cdots & O_{wm} \end{bmatrix} \quad (3)$$

Table 1: Number of Pilgrims coming from outside Kingdom

Year	NPO	Year	NPO	Year	NPO	Year	NPO
1390	431270	1401	879368	1412	1012917	1423	1431012
1391	479339	1402	853555	1413	992813	1424	1419706
1392	645182	1403	1003911	1414	995611	1425	1534759
1393	607755	1404	919671	1415	1043274	1426	1557447
1394	918777	1405	851761	1416	1080465	1427	1654407
1395	894573	1406	856718	1417	1168591	1428	1707814
1396	719040	1407	960386	1418	1132344	1429	1729841
1397	739319	1408	762755	1419	1056730	1430	1613965
1398	830236	1409	774560	1420	1267555	1431	1799601
1399	862520	1410	828993	1421	1363992	1432	1828195
1400	812892	1411	720102	1422	1354184	1433	1752932

Table 2: Actual values and variation of NPO

Year	1390	1391	1392	1393	1394	1395	1396	1397	1398	1399	1400
NPO	431270	479339	645182	607755	918777	894573	719040	739319	830236	862520	812892
Variation	-	48069	165843	-37427	311022	-24204	-175533	20279	90917	32284	-49628
Year	1401	1402	1403	1404	1405	1406	1407	1408	1409	1410	1411
NPO	879368	853555	1003911	919671	851761	856718	960386	762755	774560	828993	720102
Variation	66476	-25813	150356	-84240	-67910	4957	103668	-197631	11805	54433	-108891
Year	1412	1413	1414	1415	1416	1417	1418	1419	1420	1421	1422
NPO	1012917	992813	995611	1043274	1080465	1168591	1132344	1056730	1267555	1363992	1354184
Variation	292815	-20104	2798	47663	37191	88126	-36247	-75614	210825	96437	-9808
Year	1423	1424	1425	1426	1427	1428	1429	1430	1431	1432	1433
NPO	1431012	1419706	1534759	1557447	1654407	1707814	1729841	1613965	1799601	1828195	1752932
Variation	76828	-11306	115053	22688	96960	53407	22027	-115876	185636	28594	-75263

The relation between the operation matrix $O^w(t)$ and the criterion matrix $C(t)$ can be calculated, and we can get a relation matrix $R(t)_{w\times m}$ by performing $R(t) = O^w(t) \otimes C(t)$, where

$$R(t) = \begin{bmatrix} O_{11}\times C_1 & O_{12}\times C_2 & \cdots & O_{1m}\times C_m \\ O_{21}\times C_2 & O_{22}\times C_2 & \cdots & O_{2m}\times C_m \\ \cdots & \cdots & \ddots & \cdots \\ O_{w1}\times C_2 & O_{w2}\times C_2 & \cdots & O_{wm}\times C_m \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1m} \\ R_{21} & R_{22} & \cdots & R_{2m} \\ \cdots & \cdots & \ddots & \cdots \\ R_{w1} & R_{w2} & \cdots & R_{wm} \end{bmatrix} \quad (4)$$

where $R_{ij} = O_{ij} \times c_j$, $1 \le i \le w$, $1 \le j \le m$. From the relation matrix $R(t)$, we get the degree of relationships between the last value of time series and the other past values in time series data variations. Then, we can get the forecasting variation of the time series at t, where

$F(t) = [Max(R_{11}, R_{21}, \cdots, R_{w1}) \cdots Max(R_{12}, R_{22}, \cdots, R_{w2}) \cdots Max(R_{1m}, R_{2m}, \cdots, R_{wm})]$ (5)

III. PREDICTION ALGORITHM

In this section, we discuss in more detail the algorithm used to predict the time series of number of pilgrims coming from outside the kingdom of Saudi Arabia (NPO) consists of 44 observations (from year 1390AH # to year 1433 AH) table (1).

Step 1: From the time series data shown in table (1), compute the variations of the time series values between any two continuous values. The variation of time series current value is the time series current value minus the time series previous value as shown in table (2).

To define the universe of discourse U, let v_{min} be the minimum of variation and v_{max} be the maximum of variation. Then the universe of discourse $U = [v_{min} - v_1, v_{max} + v_2]$, where v_{min} and v_{max} are suitable positive numbers. Here, $v_{min} = 369$, $v_{max} = 478$ and U = [– 198000, 311500]

Step 2: Partition the universe of discourse U into several even length intervals $u_1, u_2, \cdots, u_m$. In this paper, we partition the universe of discourse U into six intervals:

$u_1 = [-198000, -113083]$,
$u_2 = [-113083, -28167]$,
$u_3 = [-28167, 56750]$,
$u_4 = [56750, 141667]$,
$u_5 = [141667, 226583]$,
$u_6 = [226583, 311500]$.

Step 3: Define fuzzy sets on the universe of discourse U. First, determine some linguistic values represented by fuzzy sets to describe the degree of variation between two continuous values. Here, we consider six fuzzy sets which are A_1 = (big decrease), A_2 = (decrease), A_3 = (no change), A_4 = (increase), A_5 = (big increase), A_6 = (too big increase).Then, define fuzzy sets $A_1, A_2, \cdots, A_6$ on the universe of discourse U as follows:

$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + 0/u_5 + 0/u_6$
$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + 0/u_5 + 0/u_6$
$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + 0/u_5 + 0/u_6$ (6)
$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4 + 0.5/u_5 + 0/u_6$
$A_5 = 0/u_1 + 0/u_2 + 0/u_3 + 0.5/u_4 + 1/u_5 + 0.5/u_6$
$A_6 = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + 0.5/u_5 + 1/u_6$

Step 4: Fuzzify the values of time series historical data. If the number of variation of the time series at value i is p, where $p \in u_j$, and if there is a value represented by fuzzy set A_k in which the maximum membership value occurs at u_j, then p is translated to A_k. The fuzzified variations of the time series data are shown in table (3).

Step 5: Choose a suitable window basis w, and calculate the output from the operation matrix $O^w(t)$ and the criterion matrix $C(t)$, where t is the year for which we want to forecast the time series value. The fuzzified forecasted variations for all years are given in table (4).

Step 6: Defuzzify the fuzzy forecasted variations derived in the former step as follow:

- If the grades of membership of the fuzzified forecasted variation are all 0, then we set the forecasted variation to 0.
- If the grades of membership of the fuzzified forecasted variation have only one maximum u_i, and the midpoint of u_i is m_i, then the forecasted variation is m_i. If the grades of membership of the fuzzified forecasted variation have more than one maximum $u_1, u_2, \cdots, u_k$, and their midpoints are $m_1, m_2, \cdots, m_k$, respectively, then the forecasted variation is $(m_1 + m_2 + \cdots + m_k)/k$.

Step 7: Calculate the forecasted time series value. The forecasted time series value is forecasted variation plus the true value of last year.

Step 8: Calculate the mean absolute percentage error (MAPE), which is a measure of prediction accuracy of a forecasting method in statistics. It usually expresses accuracy as a percentage, and is defined by formula (7):

$$MAPE = \frac{1}{m}\sum_{t=1}^{m}\left|\frac{true\ value - forecast}{true\ value}\right| \times 100 \quad (7)$$

Table (5) shows time series true values versus predicted values and the error estimate (MAPE) for all years for window base; $w = 5$. Figure (3.1) shows the graph of both true and predicted values.

3.1. Accuracy of Prediction for Different Window Bases

From equation (3) it is clear that the operation matrix $O^w(t)$ size depends on window base w and consequently the relation matrix $R(t)$ which is the kernel of prediction. Table (6) shows prediction for different values of window base, e.g., w = 5,10,15,20,25,30. Also table (8) shows MAPE error for the different values of w.

Table 3: Fuzzified historical values of time series

Year	Variations	Fuzzified Variations
1390		
1391	48069	A_3
1392	165843	A_5
1393	-37427	A_2
1394	311022	A_6
1395	-24204	A_3
1396	-175533	A_1
1397	20279	A_3
1398	90917	A_4
1399	32284	A_3
1400	-49628	A_2
1401	66476	A_4
1402	-25813	A_3
1403	150356	A_5
1404	-84240	A_2
1405	-67910	A_2
1406	4957	A_3
1407	103668	A_4
1408	-197631	A_1
1409	11805	A_3
1410	54433	A_3
1411	-108891	A_2
1412	292815	A_6
1413	-20104	A_3
1414	2798	A_3
1415	47663	A_3
1416	37191	A_3
1417	88126	A_4
1418	-36247	A_2
1419	-75614	A_2
1420	210825	A_5
1421	96437	A_4
1422	-9808	A_3
1423	76828	A_4
1424	-11306	A_3
1425	115053	A_4
1426	22688	A_3
1427	96960	A_4
1428	53407	A_3
1429	22027	A_3
1430	-115876	A_1
1431	185636	A_5
1432	28594	A_3
1433	-75263	A_2

Table 4: Forecasted variations with windows basis $w = 5$

Years	Membership functions of forecasted variations					
	u_1	u_2	u_3	u_4	u_5	u_6
1396	0	0.5	1	0.25	0	0
1397	0.5	0.5	0	0	0	0
1398	0	0.5	1	0.25	0	0
1399	0	0	0.5	0.5	0.5	0
1400	0	0.5	1	0.5	0	0
1401	0.5	1	0.5	0	0	0
1402	0	0	0.5	1	0.5	0
1403	0	0.5	1	0.5	0	0
1404	0	0	0	0.5	1	0.5
1405	0.5	1	0.5	0	0	0
1406	0.5	1	0.5	0	0	0
1407	0	0.5	1	0.5	0	0
1408	0	0	0.5	1	0.5	0
1409	1	0.5	0	0	0	0
1410	0	0.5	1	0.5	0	0
1411	0	0.5	1	0.5	0	0
1412	0.5	1	0.5	0	0	0
1413	0	0	0	0	0.5	1
1414	0	0.5	1	0.5	0	0
1415	0	0.5	1	0.5	0	0
1416	0	0.5	1	0.5	0	0
1417	0	0.5	1	0.5	0	0
1418	0	0	0.5	1	0.5	0
1419	0.5	1	0.5	0	0	0
1420	0.5	1	0.5	0	0	0
1421	0	0	0	0.5	1	0.5
1422	0	0	0.5	1	0.5	0
1423	0	0.5	1	0.5	0	0
1424	0	0	0.5	1	0.5	0
1425	0	0.5	1	0.5	0	0
1426	0	0	0.5	1	0.5	0
1427	0	0.5	1	0.5	0	0
1428	0	0	0.5	1	0.5	0
1429	0	0.5	1	0.5	0	0
1430	0	0.5	1	0.5	0	0
1431	1	0.5	0	0	0	0
1432	0	0	0	0.5	1	0.5
1433	0	0.5	1	0.5	0	0

Table 5: True values versus forecasted values using the proposed fuzzy time-series method with the window basis $w = 5$

Years	Actual Value	Forecasted Value	Errors (%)
1396	719040	908865	26.40
1397	739319	605957	18.04
1398	830236	753611	9.23
1399	862520	929444	7.76
1400	812892	876812	7.86
1401	879368	742267	15.59
1402	853555	978576	14.65
1403	1003911	867847	13.55
1404	919671	1188036	29.18
1405	851761	849046	0.32
1406	856718	781136	8.82
1407	960386	871010	9.31
1408	762755	1059594	38.92
1409	774560	649672	16.12
1410	828993	788852	4.84
1411	720102	843285	17.11
1412	1012917	649477	35.88
1413	992813	1239500	24.85
1414	995611	1007105	1.15
1415	1043274	1009903	3.20
1416	1080465	1057566	2.12
1417	1168591	1094757	6.32
1418	1132344	1267799	11.96
1419	1056730	1061719	0.47
1420	1267555	986105	22.20
1421	1363992	1451680	6.43
1422	1354184	1463200	8.05
1423	1431012	1368476	4.37
1424	1419706	1530220	7.78
1425	1534759	1433998	6.57
1426	1557447	1633967	4.91
1427	1654407	1571739	5.00
1428	1707814	1753615	2.68
1429	1729841	1722106	0.45
1430	1613965	1744133	8.07
1431	1799601	1500882	16.60
1432	1828195	1983726	8.51
1433	1752932	1842487	5.11

Table 6: Predicted values with different window Bases

Years	*different window bases and the coressponding prediction*						
	True values	$w = 5$	$w = 10$	$w = 15$	$w = 20$	$w = 25$	$w = 30$
1396	719040	908864.67					
1397	739319	605956.67					
1398	830236	753610.67					
1399	862520	929444.33					
1400	812892	876811.67					
1401	879368	742267.00	742267.00				
1402	853555	978576.33	978576.33				
1403	1003911	867846.67	867846.67				
1404	919671	1188036.00	1188036.00				
1405	851761	849046.00	849046.00				
1406	856718	781136.00	781136.00	781136.00			
1407	960386	871009.67	871009.67	871009.67			
1408	762755	1059594.33	1059594.33	1059594.33			
1409	774560	649671.67	649671.67	649671.67			
1410	828993	788851.67	788851.67	788851.67			
1411	720102	843284.67	843284.67	843284.67	843284.67		
1412	1012917	649477.00	649477.00	649477.00	649477.00		
1413	992813	1239500.33	1239500.33	1239500.33	1239500.33		
1414	995611	1007104.67	1007104.67	1007104.67	1007104.67		
1415	1043274	1009902.67	1009902.67	1009902.67	1009902.67		
1416	1080465	1057565.67	1057565.67	1057565.67	1057565.67	1057565.67	
1417	1168591	1094756.67	1094756.67	1094756.67	1094756.67	1094756.67	
1418	1132344	1267799.33	1267799.33	1267799.33	1267799.33	1267799.33	
1419	1056730	1061719.00	1061719.00	1061719.00	1061719.00	1061719.00	
1420	1267555	986105.00	986105.00	986105.00	986105.00	986105.00	
1421	1363992	1451680.00	1451680.00	1451680.00	1451680.00	1451680.00	1451680.00
1422	1354184	1463200.33	1463200.33	1463200.33	1463200.33	1463200.33	1463200.33
1423	1431012	1368475.67	1368475.67	1368475.67	1368475.67	1368475.67	1368475.67
1424	1419706	1530220.33	1530220.33	1530220.33	1530220.33	1530220.33	1530220.33
1425	1534759	1433997.67	1433997.67	1433997.67	1433997.67	1433997.67	1433997.67
1426	1557447	1633967.33	1633967.33	1633967.33	1633967.33	1633967.33	1633967.33
1427	1654407	1571738.67	1571738.67	1571738.67	1571738.67	1571738.67	1571738.67
1428	1707814	1753615.33	1753615.33	1753615.33	1753615.33	1753615.33	1753615.33
1429	1729841	1722105.67	1722105.67	1722105.67	1722105.67	1722105.67	1722105.67
1430	1613965	1744132.67	1744132.67	1744132.67	1744132.67	1744132.67	1744132.67
1431	1799601	1500881.67	1500881.67	1500881.67	1500881.67	1500881.67	1500881.67
1432	1828195	1983726.00	1983726.00	1983726.00	1983726.00	1983726.00	1983726.00
1433	1752932	1842486.67	1842486.67	1842486.67	1842486.67	1842486.67	1842486.67

Table 7: Prediction errors with different window bases

Window bases(w)	MAPE(%)
5	9.78
10	8.21
15	6.54
20	4.77
25	2.90
30	1.92

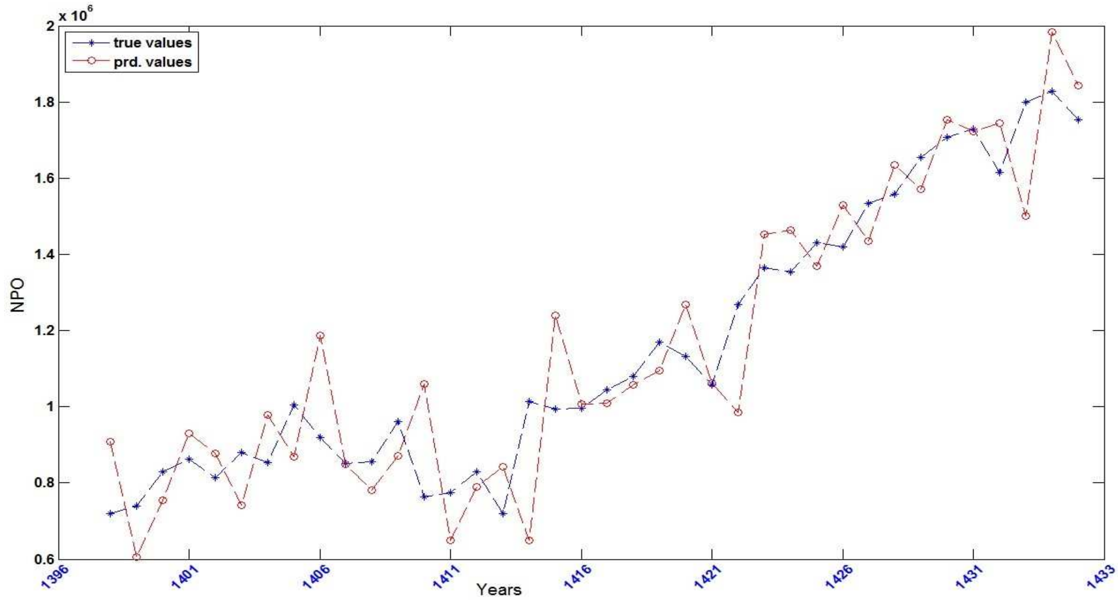

Fig.3.1: The true NPO and forecasts of observations.

IV. Forecasting

In this experiment, fuzzy time series prediction algorithm was deployed to forecast the next five future observations. The point forecasts for these observations are given by table(8) and the result are shown in figure(4.2).

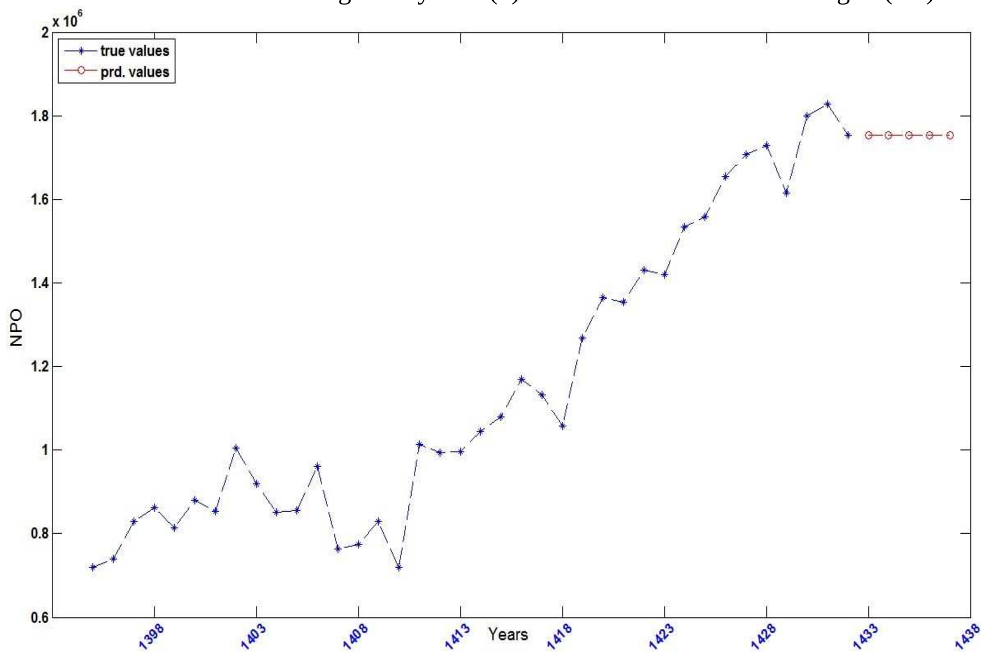

Fig. 4.2: The true NPO and future five observations.

Table 8: Predicted values for the next five years

Year	*Forecasted Value*
1434	1752930
1435	1752890
1436	1752900
1437	1752980
1438	1752950

V. Conclusion

The authors have proposed to use the fuzzy time series approach to forecast the series of number of Pilgrims coming from outside the Kingdom of Saudi Arabia from year 1390 AH to year 1433 AH. Point forecasts for the next five future years are provided by the authors using the proposed approach. The mean absolute percentage error

(MAPE) of the last five observation forecasts achieved by the approach is equal to 7.7%, compared with the results of Box-Jenkins approaches(9.6%)[2]. It has been shown that the proposed approach gives better forecasts than those achieved the traditional Box-Jenkins approaches.

REFERENCES

[1] J.R. Hwang, S.-M. Chen, C.-H. Lee, Handling forecasting problems using fuzzy time series, Fuzzy Sets and Systems, 100 217-228, (1998).

[2] S. Shaarawy, E. Khan, M. Elgamal, Modeling and Forecasting the Number of Pilgrims Coming from Outside the Kingdom of Saudi Arabia Using Bayesian and Box-Jenkins Approaches, (IJACSA) International Journal of Advanced Computer Science and Applications, Vol.5, No. 4, 2014.

[3] Q. Song and B.S. Chissom, Fuzzy time series and its models, Fuzzy Sets and Systems 54, 269-277, (1993).

[4] Q. Song and B.S. Chissom, Forecasting enrollments with fuzzy time series{Part I, Fuzzy Sets and Systems54, 1-9, (1993).

[5] Q. Song and B.S. Chissom, Forecasting enrollments with fuzzy time series{Part II, Fuzzy Sets and Systems62, 1-8, (1994).

[6] F.M. Tseng, G.H. Tzeng, H.C. Yu and B.J.C. Yuan, Fuzzy ARIMA model for forecasting the foreign exchange market, Fuzzy Sets and Systems 118, 9-19, (2001).

[7] W. Wu, Fuzzy reasoning and fuzzy relational equations, Fuzzy Sets and Systems 20, 67-78, (1986).

[8] L.A. Zadeh, Fuzzy sets, Fuzzy sets, Inform. and Control 8 (1965) 338 353.

The years ate written using Lunar Calendar (AH) from 1390(1971) ~1433(2012); one lunar year is shorter than Gregorian year by about 11 days (see http://en.wikipedia.org/wiki/Islamic Calenda).

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