

# Calculation of The Electromagnetic Field of The Microwave of Devices With Use of Method FDTD and Integral Kirchhoff

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**Abstract** – Application of superficial integral Kirchhoff together with method FDTD expands a scope of a method. Such hybrid is effective at calculations of diagrams of an orientation of aeriels, the effective area of reflection and in other cases where the field on some removal from object is required to find. Accuracy of calculations with use of superficial integral Kirchhoff is commensurable with accuracy of method FDTD.

**Keywords** – Method FDTD, Equations Maxwell's, Microwave Devices, Algorithm.

## I. INTRODUCTION

The decision of equations Maxwell's method FDTD enables calculations electromagnetic inside of some limited area of space. Restriction on the sizes of area in which calculations are spent, are imposed limited computing capacity of applied computers, namely three factors: in the limited speed of performance of the arithmetic operations, in the limited volume of operative memory and in the limited speed of data exchange between the processor with memory. However there is a number of problems in which calculation of electromagnetic fields on greater distances from some object radiating an electromagnetic field is necessary. Among them calculation of the diagram of an orientation of aeriels, definition of an effective surface of reflection of the radar-tracking purposes, the decision of problems of diffraction. In given clause the effective way of calculation of fields in the microwave devices is offered by method FDTD. There are some methods of calculation of a field in the microwave devices. All of them include integration on the closed surface which covers radiating or disseminating object. And in the rest there are significant differences. One methods integrate, using superficial equivalent currents (electric and magnetic), others directly use fields E and methods are applied by N.Odni in frequency area, others work in time area.

## II. DEVELOPMENT OF MATHEMATICAL MODELS

For example, to calculation of a field often apply integration on elements into which the closed surface is broken. Each element of a surface is elementary electric and a magnetic field simultaneously. If at this field it is

calculated by method FDTD, following operations are carried out:

- All components E and H fields on a surface on which there is integration, are kept on all steps on time. By the end of calculations the time form of a field in each point of a surface is known.

- Fields E and H on a surface will be transformed to equivalent electric and magnetic currents.

- Transformation of currents on a surface in frequency area is carried out.

- Are calculated making E and corresponding making fields H for all frequencies and from each element on a surface. Elements in this case are usual cells FDTD. The own system of polar coordinates is compared each cell.

- Values E and corresponding components of field H will be transformed to rectangular coordinates (Ex, Ey, Ez). Only on this step there is actually an integration on a surface.

- Return transformation to time area is made. Now the required field is found.

There is a number of updating of the resulted method of calculation of a field, but at application of method FDTD by more logical calculation of a field with use only time area and without transformation of fields to equivalent currents looks. Besides it is desirable to do without transformation to other systems of coordinates. Such method exists are calculations with use of superficial integral Kirchhoff.

The way of application of superficial integral Kirchhoff together with method FDTD is resulted in [1-5]. However in [3] three gross blunders in formulas are admitted. Besides the mess is entered into the order of the integration. It was necessary to deduce anew formulas, having taken advantage of idea Ramahi (Ramahi, the author [3]). The result has turned out good, therefore, despite of annoying mistakes in clause it would be desirable to express the author the gratitude. Integral Kirchhoff is connected with a field inside of the limited volume with a field and its derivatives on a surface limiting volume. This formula is deduced in the middle of 19 centuries by the German physicist Kirchhoff, in an time area looks like [3]:

$$\psi(p, t) = \frac{1}{4\pi} \oint_s \vec{n} \left[ \frac{\nabla' \psi(p', t')}{R} - \frac{\vec{R}}{R^3} \psi(p', t') - \frac{\vec{R}}{cR^2} \frac{\partial \psi(p', t')}{\partial t'} \right]_{ret} dA' \quad (1)$$

where  $p, p'$  - a point of supervision and a point on a surface accordingly;  $\vec{n}$  - an individual vector of a normal to a surface;  $\psi(p, t)$  - scalar function which can be any of six component of a field;  $R$  - distance from a point of supervision up to a point on a surface;  $\vec{R}$  - a vector  $p, p'$ ;  $A'$  - the area of an element of a surface. In the formula (1) *ret* means, that integration is carried out in view of late time  $t' = t - \frac{R}{c}$ . The vector of a normal is directed inside of the closed volume.

The formula (1) expresses principle Hugeness according to which each point on wave front is a fictitious source of an imagined spherical wave. Each site of a surface  $dA'$  radiates a wave which  $p$  comes to a point of supervision with delay  $R/c$ . Thus on each step FDTD on time for surfaces of integration there is a set of fictitious sources, the field from which will come in a point of supervision with different delay, as distance  $R$  for all points variously. It means, that on one time step FDTD from (1) contributions of sites  $dA'$  in different time sites of a target signal in a point of supervision turn out.

He step on time at calculation of integral (1) is closely connected with step on time FDTD and is equal to it. The target sequence in a point of supervision has the same step on time. However delay  $R/c$  can not be multiple to a step on time. Therefore received time of a delay is approximated to the near future, multiple to step FDTD. The mistake arising at it is insignificant, since the step on time in classical FDTD is small in comparison with the period of fluctuations of a calculated signal.

To result (1) in a kind convenient for application together with algorithm FDTD, it is necessary to execute a number of transformations. Thus we shall consider, that all sites of a surface on which integration is conducted, in algorithm FDTD are always perpendicular one of axes of coordinates.

Let's assume, that the surface  $dA'$  is perpendicular axes  $Z$  and the point  $p'$  has coordinates  $(i, j, k)$  (fig. 1).

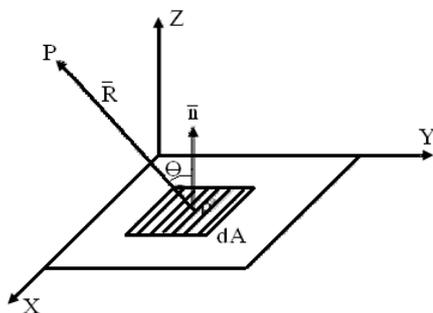


Fig. 1. The surface  $dA'$  is perpendicular axes  $Z$  and the point  $p'$  has coordinates  $(i, j, k)$ .

Let's apply transformation sub integral composed (1) in final differences:

$$\begin{aligned} \frac{1}{4\pi} \vec{n} \frac{\nabla' \psi(p', t')}{R} &= \frac{1}{4\pi R} \frac{\partial \psi(p', t')}{\partial z} \equiv \\ &\equiv \frac{1}{4\pi R} \frac{\psi(i, j, k+1, t') - \psi(i, j, k-1, t')}{2\Delta z}, \\ \frac{1}{4\pi} \vec{n} \frac{\vec{R}}{R^3} \psi(p', t') &= \frac{1}{4\pi R} \frac{R \cos(\theta)}{R^3} \psi(p', t') = \\ &= \frac{1}{4\pi} \frac{\cos(\theta)}{R^2} \psi(i, j, k, t'). \end{aligned} \quad (2)$$

For simplification of the further record we shall enter designations:

$$\begin{aligned} D &= \frac{1}{4\pi R}, \quad E = -\frac{1}{4\pi} \frac{\cos(\theta)}{R^2}, \\ F &= -\frac{1}{4\pi} \frac{\cos(\theta)}{cR}. \end{aligned} \quad (3)$$

Also we shall apply standard for FDTD designations to discrete values of time:

$$t' = n + 1, t' - t = n, t' + \Delta t = n + 2.$$

Besides we shall recollect, that time in a point of supervision  $t = t' + \frac{R}{c}$ . Approximating  $t$  up to the

nearest whole step on time we shall designate it as  $t_n^*$ .

Expression (1) in view of the entered designations (2) and (3) will enter the name for one platform  $dA$  in the form of:

$$\psi(p, t_n^*) = \left[ \begin{aligned} &D \frac{\psi(i, j, k+1, n+1) - \psi(i, j, k-1, n+1)}{2\Delta z} + \\ &+ E \psi(i, j, k, n+1) + \\ &+ F \frac{\psi(i, j, k, n+2) - \psi(i, j, k, n)}{2\Delta t} \end{aligned} \right] A_{ij}. \quad (4)$$

where  $\psi_{i,j} = \Delta x \Delta y$ .

In (4) there is no sense to write down the sum on all sites of a surface since  $(p, t_n^*)$  generally for each site has different value as variously distance  $R$ . The time sequence in a point of supervision  $p$  turns out summation of values  $(p, t_n^*)$  each step on time on all sites of a surface in view of time of delay.

In (4) value of function  $(p, t_n^*)$  it is calculated with use of values on three steps on time. But it does not mean that it is necessary to remember values of two previous steps always. We shall group composed (4) on time steps and we shall write down in the form of:

$$\psi(p, t_n^*) = F_1(n) + F_2(n+1) + F_3(n+2), \quad (5)$$

where

$$\begin{aligned} F_1(n) &= \left[ -A \frac{\psi(i, j, k, n)}{2\Delta t} \right] A_{i,j}, \\ F_2(n+1) &= \left[ D \frac{\psi(i, j, k+1, n+1) - \psi(i, j, k-1, n+1)}{2\Delta z} + \right. \\ &\quad \left. + E \psi(i, j, k, n+1) \right] A_{i,j}, \\ F_3(n+2) &= \left[ F \frac{\psi(i, j, k, n+2)}{2\Delta t} \right] A_{i,j}. \end{aligned} \quad (6)$$

In resulted (5) and (6) there are steps  $n, n+1$  and  $n+2$ . But it is possible to calculate all these values for one step, we shall admit a current step  $n+1$ . Then if to consider, that the step  $F_1(n)$  it is added in  $t_{n+1}^*$  a step of calculated sequence is calculated.  $F_2(n+1)$  it is added in the previous time point  $t_n^*$ , and  $F_3(n+2)$  it is added on one time step earlier in calculated sequence  $t_{n-1}^*$ . In this case

$$F_3(n+2) = -F_1(n).$$

So it is not required to store value of fields on a surface of integration. But for acceleration of calculations it is possible to calculate in advance values of distances and co sinus corners from all sites of a surface up to a point of supervision. Instead of function it is possible to substitute freely  $E_x, E_y, E_z, H_x, H_y$  or  $H_z$ .

All components of cell Yee are in different places of space. But distances and co sinus corners it is quite possible to calculate the general for all a component. The main thing that they belonged to one cell. Then at calculation of a distant field the calculated components also will appear shifted in space that it is convenient to use at testing the program when the field simultaneously, in the same cell, is calculated as directly algorithm FDTD, and the decision of superficial integral Kirchhoff [3-11].

Here the conclusion of the formula only for one surface is resulted. For other five surfaces the conclusion of formulas is similar. Integration should be conducted strictly on the closed surface.

Borders of integration can be approached closely to the object set in calculations FDTD.

At calculation of fields on enough greater distances from the equation (6) it is possible to exclude composed, decreasing as  $1/R^2$ , and a corner of a direction on a point of supervision to calculate one on all side of a surface of integration. This case corresponds to a case of a distant zone in its usual understanding and allows to accelerate calculations and to reduce memory demanded for calculations.

### III. NUMERICAL RESULTS

For testing the way of calculation of integral Kirchhoff resulted in the present clause the test in which calculation of a field in a distant zone is spent for a point which is inside of computing volume is lead. It has allowed to compare the received results to calculation by directly method FDTD.

The problem was solved in a grid  $150 \times 50 \times 50$  cells, a step on space of 1 sm in all directions. Object - well spending ring in diameter of 33 sm and height 33 see Thickness rings 1 see the Axis of a ring is parallel to axis Y. The center of a ring has coordinates (25, 25, 25). On a ring the flat wave in a direction of axis X falls. Vector E is parallel to axis Z. The form of a field - a radio impulse with bearing frequency of 1500 MHz and amplitude 1 V/m. Boundary conditions - eight layers PML c in factor of reflection of 0,001 %. The border of integration is in

three cells from object. In a point of supervision with coordinates (140, 25, 25) the conclusion of all components of vector E for the absent-minded field as directly from calculations FDTD, and by the decision of integral Kirchhoff was carried out. Results were compared among themselves. Following results are received. Component  $E_z$ , the greatest on amplitude (600  $\frac{1}{4}$ O/m), has difference of 1,7 %, component  $E_x$  (3,2  $\frac{1}{4}$ O/m) differs on 0,6 %. Component  $E_y$  it should be equal to zero, but at a finding of integral Kirchhoff the impulse by amplitude of 15  $\frac{1}{4}$ O/m is received, that in 40 thousand times is less, than amplitude components  $E_z$ .

In practice the error of definition components does not play a role. The error of calculation of the main components of a field as has shown a number of test calculations for various, does not exceed 5 % and usually lays in a range (1-3)% for a case of calculation of a field. At calculation of a field on significant removal the error decreases. We shall illustrate it with an example.

Other example - the decision of a problem of definition of the effective area of reflection for a case of a reflecting plate of square section. In this case analytical expression for is known. We shall take a square ideally spending plate. In case the length of a wave is much less than party of a square, is calculated under the formula:

$$\sigma = 4\pi \frac{a^2}{\lambda^2}, \quad (7)$$

where  $a$  - the party of a square,  $\lambda$  - length of a wave.

At the same time, through intensity of field it is expressed as:

$$\sigma = 4\pi R \frac{E_s^2}{E_i^2}, \quad (8)$$

where  $R$  - distance from object up to a point of supervision,  $E_s$  - intensity of the absent-minded field on distance  $R$ ,  $E_i$  - intensity of the field falling on object. The formula (7) is fair for a case when the field falls perpendicularly surfaces of a plate, and (8) - when the point of supervision is in a direction, the return to a direction of distribution of a falling wave.

Substituting (7) in (8) and expressing  $E_s$ , we shall receive:

$$E_s = \frac{aE_i}{\lambda R}. \quad (9)$$

For a square plate in the sizes with the party  $a=1$  m, at length of a wave of  $\lambda=0,1$  m with  $E_i=1 \frac{V}{m}$  on distance

$R=1000$  m the on (9) is received  $E_s = 0,01 \frac{V}{m}$ .

The same problem has been solved by method FDTD with application of integral Kirchhoff for two cases. The step on space is equal the first case 10, 150 steps of the account in other 20, 300 steps of the account (1 sm and 5 mm accordingly). In the first case it is received  $E_s = 0,0097 \frac{V}{m}$ , in the second  $E_s = 0,0101 \frac{V}{m}$ . The error of 3 % and 1 % is within the limits of an error of method

FDTD for a corresponding parity of length of a wave and a step on space. The problem was solved in volume  $20 \times 120 \times 120$  cells, a step of 1 sm and with the double value of volume with step of 5 mm. Borders - 8 layers PML. Borders of integration settled down in 3 cells from a plate. On fig. 2. Intensity of a field on distance of 1000m from a plate depending on a direction, calculated with step 5 o is resulted. The zero of degrees coincides with a direction of distribution of a falling wave.

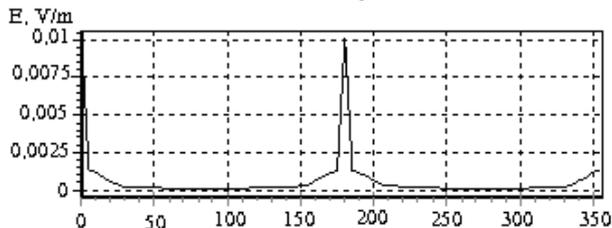


Fig. 2. Intensity of a field on distance of 1000m from a plate depending on a direction, calculated with step 5 o is resulted.

#### IV. CONCLUSIONS

Application of superficial integral Kirchhoff together with method FDTD expands a scope of a method. Such hybrid is effective at calculations of diagrams of an orientation of aerials, the effective area of reflection and in other cases where the field on some removal from object is required to find.

Accuracy of calculations with use of superficial integral Kirchhoff is commensurable with accuracy of method FDTD.

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