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Current Distribution on Wire Antennas with the Approximate Kernel Formulation

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***Abstract* – A conducting device could either radiate or scatter Electromagnetic fields depending on the position of the external source feeding it. In either case, the Engineering parameters of the device can be determined by numerical methods. For linear structures, the Method of Moments is a typical approach that is applied for this analysis and is based on the formulation of the integro/differential equations of Pocklington's or Hallen's equation. The equations are described based on the choice of the Kernel in the numerical solution. In this paper, the implication of the kernel formulation for analysis of wire antenna, with emphasis on the approximate kernel is presented. The result shows the current distribution responsible for Electromagnetic radiation from an antenna and the difficulties associated with it.**

***Keywords* – Approximate Kernel, Integro/Differential Equation, Method of Moment, Thin Wire Formulation.**

I. Introduction

Wire antennas are the most versatile antennas in application. They are used in diverse communication process, ranging from TV, FM Broadcast, satellite, radar and other forms of wireless communication. There are different forms of wire antenna and they are categorized according to the geometry of the linear structure. Some of which include; the diploes, monopoles, arbitrarily bent wires, folded dipoles, linear arrays etc. Wire antennas have good conductivity, unique radiation pattern and low bandwidth application. It is extremely flexible in application because a change in the geometry of the structure results in a device whose performance is acceptable and useful. It is cost effective for practical application. These thin wires when excited by a lumped voltage source at one or more points along the antenna length are capable of radiating electromagnetic (EM) field. The thin wire is characterized by having its radius being far less than its length. As a scatterer, the source is located some distance away from the antenna. In Electromagnetic theory however, there is physically no fundamental differences between the scattering problem and the radiation problems of antenna. [1] The antenna problem is merely an extreme example of near field scattering.

The incident field or source voltage induces current on the antenna. The current in turn generates fields that radiates and propagates into space. The total field becomes the sum of the incident and radiated field which must varnish at the boundary of a perfect electric conductor. This boundary condition is necessary to determine the current distribution induced on the antenna

$E^i + E^r = 0$

With knowledge of the current distribution on the antenna surface, all other parameters of Engineering interest can be obtained by numerically evaluating the formulas. A powerful technique that ensures the computational analysis of wire antennas and scatterers is the Method of Moment. This numerical method for the solution of wire antennas was initially investigated by Ma, Harrington and Richmond [2]. Several works have been done ever since by many authors leading [3]-[6] to the development/implementation of wire antenna analyzer [7] and commercial softwares. The softwares include NEC 2, NEC 4, MININEC, FEKO, HFSS etc.

Solutions arising from the problems of radiation and scattering properties of linear antennas are expressed by two basic integral/integrodifferential equations namely; Hallen's equation and Pocklington's equation. These equations can be analyzed based on two distinct kernel formulations;

- Approximate kernel analysis or thin wire approximation and
- Exact kernel analysis

The approximate kernel is the most popular formulation that finds more application in wire antenna analysis and is discussed in this work. The criterion for its use will be described.

Section 2 & 3 describes the different Kernel formulations for analysis of linear antennas. Section 4 gives the different source model and the implications of exceeding the thin wire formulation. Section v concludes the work.

II. Thin Wire Approximation

Due to the cylindrical symmetry of the wire antennas, its analysis is carried out in the cylindrical coordinate especially in determining the fields near the antenna. At far distances, the radiated fields are best described in spherical coordinate because any finite current source appears as a point source from far distances. For this reason, It is necessary to note that as point sources, the angular distribution of the radiation pattern is independent of the linear distance.

Considering a straight thin wire antenna excited at the mid point by an arbitrary source, the thin wire approximation involves the following conditions;

- The wire radius (a) is far less than the length (L) of the antenna ($a \ll L$). This ensures that the electric field boundary conditions are enforced in the axial direction.
- The wire is directed along the z axis and the wire's material is a perfect conductor $\sigma = \infty$. This conforms well to practical application of antenna materials such as copper, silver etc.
- Integrations over the flat end surface of the wire are neglected since the wire radius is very small.

- The circumferential component of the surface current density is neglected.
- The axial component ($J_{z)}$) is considered to be independent of field in the ϕ direction. This is because, in perfect conductors EM fields vanish inside them and due to skin effect current tends to pile up on conductor surface. This gives rise to thin current coat with depth of penetration δ equal to

$$\delta = \sqrt{2/\omega\mu\sigma}$$

Were

μ = Permeability of the medium

σ = Conductivity of material

ω = $2\pi f$

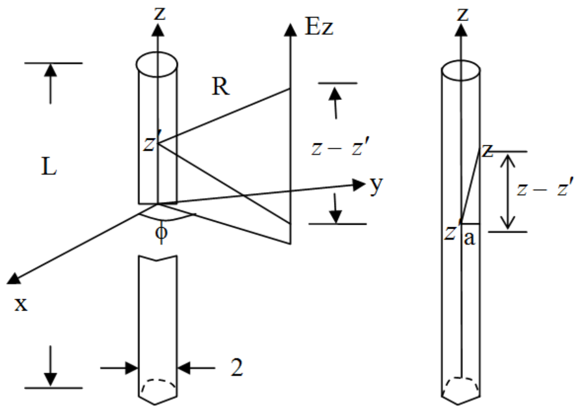

Fig. 1. Geometry of a linear wire antenna

For a perfectly conducting z directed thin wire, an incident field ($E^{i)}$) excites on this wire a surface current density J which depends on the z orientation

$$J = \frac{I_z(z')}{2\pi a} \qquad (1)$$

$$A_z(\rho,\phi,z) = \mu \int_{-l/2}^{+l/2}\int_{0}^{2\pi} \frac{I_z(z')}{2\pi} \frac{e^{-jk\,r}}{4\pi r} d\phi'\,dz' \qquad (2)$$

$$r = |r - r'| = \sqrt{(z-z')^2 + (p-p')^2}$$

$|p-p'| = p^2 + a^2 - 2p \bullet p' = p^2 + a^2 - 2pa\cos$ $(\phi'-\phi)$. $\phi' - \phi$ is replaced by ϕ' since the result is cylindrically symmetric using p = a

$$A_z(\rho,\, z) = \mu \int_{-l/2}^{+l/2} \frac{I_z(z')}{2\pi} \int_{0}^{2\pi} \frac{e^{-jk\,r}}{4\pi r} d\phi'\,dz' \qquad (3)$$

$$r = \sqrt{(z-z')^2 + p^2 + a^2 - 2pa\cos\phi'}$$

$\int_0^{2\pi} \frac{e^{-jk\,r}}{4\pi r} \; d\phi'$ = Approximate cylindrical wire kernel

If a is very small

$$r = \sqrt{(z-z') + p^2}$$

Hence $A_z\ (p,z) = \mu \int_{-l/2}^{+l/2} I_z(z') \frac{e^{-jkr}}{4\pi r} dz' \qquad (4)$

Equation 4 is the vector potential expression for a thin wire using the approximate kernel. It is applied to wires with small dimension compared to their wave length. The radiated field resulting from an incident electric field is given by

$$E_z^r = -j\omega A_z \frac{-j}{\omega\mu\varepsilon} \frac{\partial^2 A_z}{\partial z^2} \qquad (5)$$

Enforcing the boundary condition for a zero tangential electric field in the surface of the wire

$$E_Z^i = \frac{j}{\omega\mu\varepsilon}\left[\frac{\partial^3}{\partial z^2} + k^2\right] A_z \qquad (6)$$

$$\sqrt{r} = (z-z') + a^2$$

$$E^i = \frac{j}{\omega\mu\varepsilon} \int_{-l/2}^{l/2} I_z(z') \left[\frac{\partial^2}{\partial z^2} + k^2\right] \frac{e^{-jkr}}{4\pi r} dz' \qquad (7)$$

From the expression, equation 7 is known as Pocklington's integro differential equation. It belongs to the general class of fredholm Integral equation of the first kind. The latter is characterized by the presence of the unknown function only under the integral whose limits are constant. It is described by a matrix condition number that tends to increase rapidly as the segment size decreases. The approximate kernel is also well defined at z = 0.

Analysis involving the approximate kernel usually gives rise to the presence of oscillation. These oscillations are unphysical and unnatural [8]. A technique has been proposed to handle this anomaly by generating a new current from the magnetic field produced by the original oscillating current.

$I_{new} = 2\pi p \times I_{old}$

This method is also analogous to the method of auxiliary sources which is an approximate method for the solution of electromagnetic scattering problems. Electromagnetic scattering problems are not confined to one dimensional problem alone. They can also be extended to two or three dimensional problems [9].

III. Exact Kernel Formulation

The exact kernel formulation is used where the approximate kernel fails. It is applied principally for wires with large diameter of which the thin wire approximation can no longer be in use. Other method of moment approach can also be applied for the analysis of large diameter antennas [10]. The use of the exact kernel results in well posed solutions devoid of instabilities, though it is not devoid of the existence of singularity at the mid point of the antenna (z = 0) [11].

The electric vector potential is given by

$$\frac{4\pi}{\mu_o} A_z\ (p,z) = \int_{-l/2}^{l/2} k(z-z')\ I(z')\,dz' \qquad (8)$$

With the exact kernel $k(z) = k_{ex}\ (z)$

$$k_{ex}(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{\exp[-jk\sqrt{(z-z')^2 + 4a^2\sin^2\phi/2}}{\sqrt{(z-z') + 4a^2\sin^2(\phi/2)}} \qquad (9)$$

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The resulting electric field expression is thus given as

$$E^{i} = \frac{1}{j4\pi\omega\varepsilon}\left(\frac{\partial^{2}+k^{2}}{\partial z^{2}}\right)\int_{-l/2}^{l/2} k_{ex}(z-z')\,I(z')\,dz' \qquad (10)$$

For $\frac{-l}{2} \le z \le \frac{+l}{2}$, μ and ε are permeability and permittivity of homogeneous space outside the wire. Equation 10 is Pocklington's equation with the exact kernel. It can be seen that a major set back in the use of the exact kernel is the extra numerical effort required to evaluate the unknown current due to the presence of the extra integral in the kernel. It is worth noting also that accuracy should not be given off for mathematical complexity. The solution to the equation will therefore be well defined, unique and devoid of instabilities.

Further analysis involving asymptotic approach can aid in the analysis and understanding of finite antennas [12].

IV. Source Models and Implication

The delta gap model, magnetic frill generator and plane wave incidence are the main source modeling for thin wire antennas [13]. The thin wire approximate kernel results from taking an axis to surface distance. This is because of the difficulty in establishing the far field radiation on the antenna surface. Although with the approximate Kernel, reliable results have been produced so long as the thin wire criteria are not exceeded. The effect of exceeding the thin wire criteria leaves the graph of the current distributions with oscillations. These oscillations are actually due to Round off errors which results when large segmentation is applied on the linear conductor. It is also seen in this work that the thin wire approximation detoriates as the segment length approaches the radius of the wire. These oscillations are not peculiar to Pocklington's equation alone. It is also applicable to its Hallen's counterpart. Another form of oscillation could exist in the current distribution of an antenna [8]. This time it is as a result of the non solvability issues associated with approximate Kernel.

The problems arising from the current distribution are not common to the delta function generator alone, but also apply to other source models. It is therefore imperative to state that the oscillations are not as a result of the type of source model in use.

The thin wire approximations detoriates as the segment length approaches the radius of the wire.

Equations for expressing radiation and scattering analysis are not only expressed in electric field form. They are also expressed with magnetic field integral equation (MFIE). The use of MFIE is associated with well conditioned matrix whose current distributions are devoid of instabilities. This implies that the simulation curves are smooth and insensitive to oscillations. MFIE is not suitable for analysis involving open or thin surfaces but is applied mostly in closed surfaces.

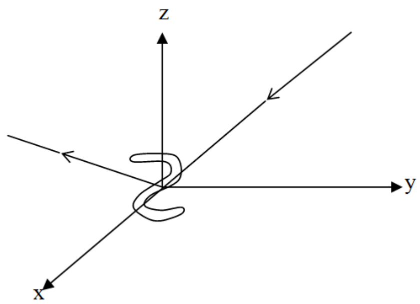

Fig. 2. Scattering property of wire antenna

Figure 2 shows fields scattered by a wire object of plane wave incidence. A vital parameter of interest when considering scattering property is the bistatic scattering cross section. It is defined by that area for which the incident wave contains sufficient power to produce the field by an ominidirectional radiator. It is given by

$$\nabla = 4\pi r^{2}\left|E_{r}\right|^{2}$$

Where;

∇ = Bistatic radar cross section

$E_{r} = Electric field$

r = Far field distance.

It can also be concluded that the angle of scattering is equal to the angle of incidence.

For a z oriented wire illuminated by a theta (θ) oriented plane wave of unit amplitude, the scattered field is given by

$Es = Sin\theta\, e^{j\beta z Cos\theta i}$ It has no azimuthally excited current, a phi (ϕ) polarized incident wave will induce no current on the wire.

V. Results/Discussions

The graph of fig 3 shows large no of subsections in the basis function used in determining the current distribution, applying pocklington's equation. The graph is characterized by the presence of large oscillations. When the thin wire approximation is considered for large antenna radius, the current distribution is irregular and cannot be used to obtain other relevant radiation parameters of the antenna. The result is said to be associated with high level of singularity as shown in figure 4.

Fig. 6 shows oscillations in the current distribution, using Hallen' equation. It justifies the fact that the oscillations are not peculiar to Pocklingon's equation alone

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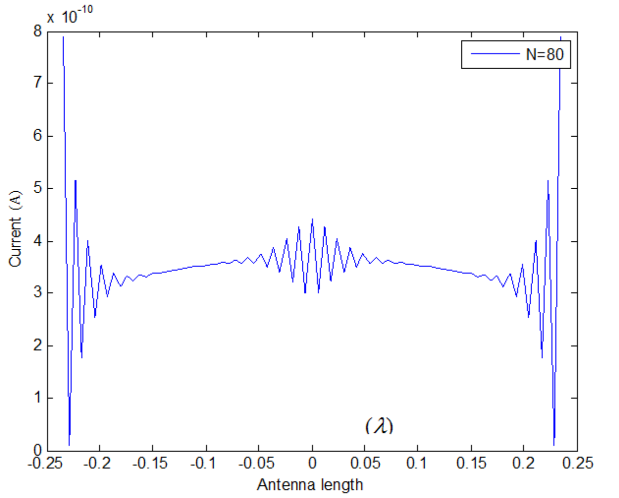

Fig. 3. Current distribution for large number of segments in Pocklington's equation.

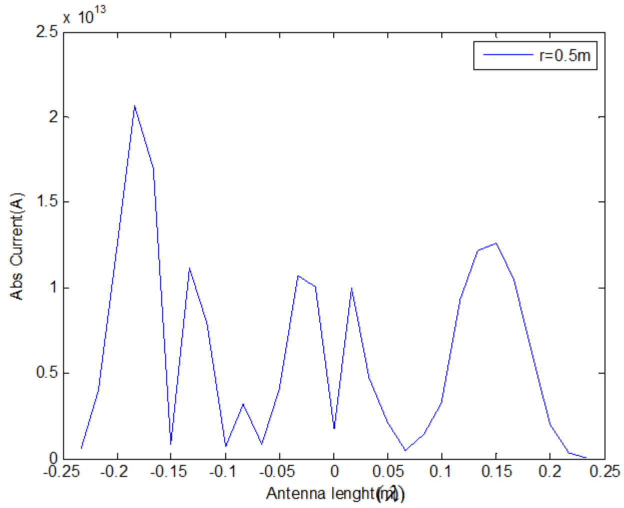

Fig. 4. current distribution with large antenna radius

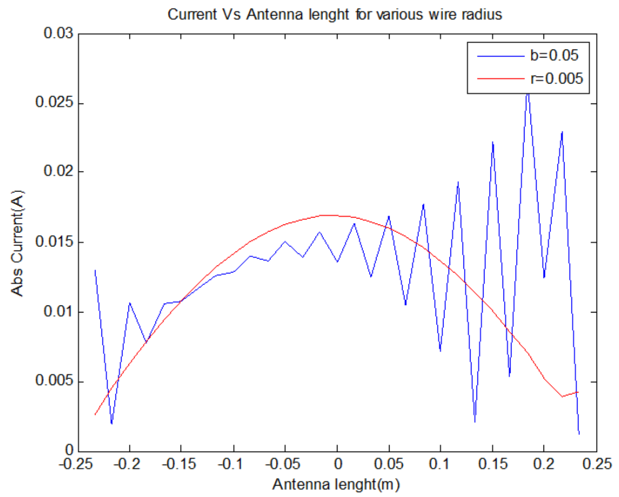

Fig. 5. Current distribution for various wire radius.

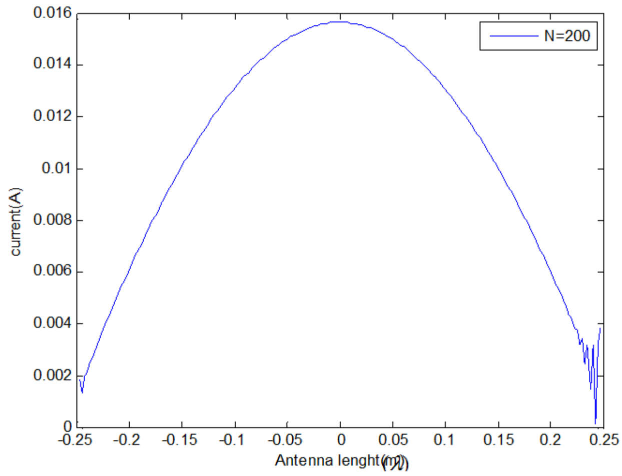

Fig. 6. Current distribution showing oscillations using Hallen's equation.

VI. Conclusion

Radiation and scattering properties of metallic wire structures are described by electromagnetic field theory. The antenna as a scatterer is usually excited by plane waves. While as a radiator, the delta gap or magnetic frill generators are the most applicable feed models. When Pocklington's or Hallen's equation is applied with the approximate kernel, the result of the current distribution is noticed to have oscillations under certain conditions. This oscillation occurs when the number of segments in the basis functions is too large. But With the exact kernel, all thin wire approximations and assumptions does not hold but leaves the solution well defined. This is compensated for with dense mathematical complexity in the analysis of the solution. It is also seen from the result that applying the approximate kernel in the analysis of the current distribution for wires with large radius yields unacceptable plots (fig.4) with high peaks at interval. This result depicts high level of singularity. The approximate kernel formulation is accurate for the solution of wire antennas so long as the thin wire criteria are not exceeded.

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