International Journal of Engineering Innovation & Research
Volume 5, Issue 6, ISSN: 2277 – 5668

Magnetoelastic Deformation of the Current Carrying Shells With The Orthotropy of Conductive Properties

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***Abstract* – Nonlinear deformation of current-carrying orthotropic shells in the magnetic field is studied in ax symmetric statement. Consideration of nonlinearity is studied for its effect when research of influence of external magnetic induction on intense the determining stressed state of current-carrying orthotropic shells. It is shown that with a change in the normal component of the external magnetic induction, there is a significant change in the stress state of the shell and its electromagnetic field.**

***Keywords* – Shell, Magnetic Field, Magneto Elasticity.**

I. Introduction

Increased interest in the problems of mechanics of coupled fields, primarily to electromagnetoelasticity, caused by the needs of today's technological advances in various industries and the development of innovation technologies. The issues of motion of a continuum with electromagnetic effects fill a highly important place in the mechanics of coupled fields. One of the main directions of development of modern solid mechanics is a development of the theory of conjugate fields and, in particular, the theory of the electromagnetic interaction with deformable medium [8, 10-18, 21, 23, 24]

The mechanism of interaction of an elastic medium with the electromagnetic field is diverse and depends on the geometrical characteristics and physical properties of the body under consideration. In particular, this mechanism gets some specifics when considering the problems of thin plates and shells having anisotropic conductivity.

In creating optimal structures in modern engineering, widespread use is made of thin-walled shells and plates as structural elements in which effects of nonlinear electromagnetic interaction with magnetic fields are significant. In studies of nonlinear magneto-elasticity problems of special interest is determination of the stress-strain state of current-carrying plates and shells on exposure to variable electromagnetic and mechanical fields with regard to anisotropic electro-conductivity, magnetic and dielectric permittivity.

Demand of these problems and interest in ones is conditioned by wide application in modern engineering as constructive elements of thin shells and plates, which are exposed to strong magnetic fields. These problems occur in modern technology, where such structures are used as protecting or bearing elements for shielding external fields of strong magnetic equipment. This interest is conditioned by the need to solve problems of electromagnetic compatibility with the development of modern measuring systems, computer devices, measurements of weak pulsed fields on the background of large fields, the development of the protection of personnel from electromagnetic effects, etc.

The electromagnetoelasticity coupled problems of anisotropic plates and shells having anisotropic conductivity are of scientific interest in terms of both theory and applications. The matter is that in the case of thin anisotropic bodies having anisotropic conductivity it is possible to solve optimal problems of magnetoelasticity by the variation of all physical-mechanical material parameters of body. In particular, when mechanical and geometric parameters of the problem are constant, using variation of anisotropic electrodynamic parameters it is possible to obtain constructive elements with qualitatively new mechanical behavior. It should be noted, that recently the materials with new electromagnetic properties were created. These materials can be use in different areas of new appliances at creation of new technologies.

Thin shells are widely used as members of advanced structures. Due to more stringent requirements to the service conditions of such structures, not only rigid but also flexible shells should be used [2, 6, 7, 25] Along with the development of the theory of flexible shells, it is also necessary to develop the theory of flexible anisotropic shells in the nonstationary magnetic fields [3, 19, 20, 24]

Problems interaction between electro-magnetic field and deformed bodies are frequent in advanced technology.

II. Nonlinear Formulation of the Problem. Basic Equations

Flexible current-carrying conical shells of variable thickness, finite conductivity, excluding the effects of polarization and magnetization and thermal stresses are considered. Elastic properties of the shell are considered orthotropic, which main directions of elasticity coincide with the directions of the corresponding coordinate lines. Material obeys the generalized Hooke's law and has a finite conductivity.

Electromagnetic properties of the material of the current-carrying shell are characterized by tensors of electrical conductivity σ_{ij}, magnetic permeability μ_{ij} and dielectric permittivity ε_{ij} $(i,\ j=1,2,3)$. At the same time due to the crystallophysics for the considered class of conducting media with rhombic crystal structure it was considered that the tensors $\sigma_{ij}, \mu_{ij}, \varepsilon_{ij}$ take a diagonal form [4, 15, 22]

Let us define quantities and write equations that describe the electromagnetic field. In an Eulerian coordinate system, the electromagnetic field of the body is characterized by electric-field intensity $\vec{e}$, magnetic-field

International Journal of Engineering Innovation & Research
Volume 5, Issue 6, ISSN: 2277 – 5668

intensity $\vec{h}$, electric-flux density $\vec{d}$, and magnetic-flux density $\vec{b}$.

In Lagrangian coordinate system, the respective quantities a denoted $\vec{E}$, $\vec{H}$, $\vec{D}$ and $\vec{B}$. A vector $\vec{x}$ is carried from the Eulerian coordinate system to $\vec{\xi}$ in the Lagrangian system by the relations:

$$\rho = \Gamma\rho^*;\ \vec{E} = F^T\vec{e};\ \vec{H} = F^T\vec{h};$$
$$\vec{D} = \Gamma F^{-1}\vec{d};\ \vec{B} = \Gamma F^{-1}\vec{b};\ \Gamma\rho = PF^T; \qquad (1)$$
$$\vec{P}_R = P\vec{n}_R;\ R_e = \Gamma\rho_e;\ \vec{J} = \Gamma F^{-1}\vec{j}$$

where $\Gamma = \det\left|\frac{\vec{x}}{\vec{\xi}}\right|$, $F = \frac{\partial x_i}{\partial \xi_j}$ $(i, j = 1,2,3)$.

In this case, the equations of magneto-elasticity for anisotropic bodies in Lagrangian coordinate in the region occupied by the body (the interior of the region) can be written as follows:

$$rot\,\vec{E} = -\frac{\partial\vec{B}}{\partial t};\ rot\,\vec{H} = \vec{J} + \vec{J}_{cm};$$
$$div\,\vec{B} = 0,\ div\,\vec{D} = 0; \qquad (2)$$
$$\rho\frac{\partial\vec{\upsilon}}{\partial t} = \rho(\vec{f} + \vec{f}^{\wedge}) + div\,\hat{\sigma} \qquad (3)$$

where $\vec{J}_{cm}$ – a density of foreign current, $\vec{f}$ – a volume force, $\vec{f}^{\wedge}$ – a Lorentz volume force, $\vec{J}$ – a density of current, $\hat{\sigma}$ – an internal stress tensor.

System of equations must be closed magnetoelasticity relations linking the vectors of the electromagnetic field and induction, as well as Ohm's law defining the conduction current density in a movable medium.

If the body is linear with respect to the anisotropic magnetic and electrical properties, the constitutive equations for the electromagnetic field characteristics and kinematic equations for the electrical conductivity, as well as expressions for the Lorentz forces, taking into account the external current $\vec{J}_{cm}$ into the Lagrangian variables are written respectively as:

$$\vec{B} = \mu_{ij}\vec{H},\ \vec{D} = \varepsilon_{ij}\vec{E}, \qquad (4)$$
$$\vec{J} = \sigma_{ij}\Gamma F^T F^{-1}\left[\vec{J}_{cm} + \vec{E} + \vec{\upsilon}\times\vec{B}\right], \qquad (5)$$
$$\rho\vec{f}^{\wedge} = \Gamma^{-1}F^{-1}\left[\vec{J}_{cm}\times\vec{B} + \sigma_{ij}\left(\vec{E} + \vec{\upsilon}\times\vec{B}\right)\times\vec{B}\right]. \qquad (6)$$

Here $\sigma_{ij}, \varepsilon_{ij}, \mu_{ij}$ are the tensors of electrical conductivity, dielectric and magnetic permittivities of linear current-carrying anisotropic body $(i, j = 1,2,3)$ respectively.

For homogeneous anisotropic media, they are symmetric second-rank tensors.

Thus, equations (2) and (3) together with (4)-(6) are a closed system of nonlinear equations of magnetoelasticity for anisotropic current-carrying bodies with anisotropic electrical conductivity, magnetic and dielectric permittivities in the Lagrangian formulation.

Suppose that the geometrical and mechanical characteristics of the body are such that to describe the deformation process is applicable version of the geometrically nonlinear theory of thin shells in the quadratic approximation.

Also we assume that the relative strength of the electric field $\vec{E}$ and magnetic field $\vec{H}$ are performed electromagnetic hypothesis [1, 3, 5, 15]:

$$E_1 = E_1(\alpha,\beta,t);\ E_2 = E_2(\alpha,\beta,t);$$
$$E_3 = \frac{\partial u_2}{\partial t}B_1 - \frac{\partial u_1}{\partial t}B_2;$$
$$J_1 = J_1(\alpha,\beta,t);\ J_2 = J_2(\alpha,\beta,t);\ J_3 = 0;$$
$$H_1 = \frac{1}{2}\left(H_1^+ + H_1^-\right) + \frac{z}{h}\left(H_1^+ - H_1^-\right); \qquad (7)$$
$$H_2 = \frac{1}{2}\left(H_2^+ + H_2^-\right) + \frac{z}{h}\left(H_2^+ - H_2^-\right);\ H_3 = H_3(\alpha,\beta,t).$$

where the $u_i -$ components of the displacement vector envelope points; $E_i, H_i -$ components of the vectors of the electric and magnetic fields shell; $J_i -$ eddy current components; $H_i^{\pm} -$ the tangential components of the magnetic field on the surface of the shell strength; $h -$ thickness of the shell.

These assumptions are some electrodynamic analog of the hypothesis of nondeformable normals and together with the latter hypothesis magnetoelasticity make subtle bodies.

The adoption of these hypotheses allows us to reduce the problem of the three-dimensional deformation of the body to the problem of deformation of the chosen arbitrarily coordinate surface.

Coordinate surface in the unstrained state we assign to the curvilinear orthogonal coordinate system s and θ, where $s -$ length of the arc forming (meridian) is measured from a fixed point, θ– central angle in a parallel circle measured from the selected plane. The coordinate lines $s = const$ and $\theta = const$ lines are the principal curvatures of the surface coordinate.

Choosing a coordinate ζ coordinate normal to the surface of revolution, we refer to the shell of the spatial coordinate system of coordinates s, θ, ζ. Assume that the surface of the conical shell known magnetic induction, and the surface mechanical strength.

Upon receipt of the resolution of the system in the normal form of Cauchy choose as basic functions $u, w, \theta_S, N_S, Q_S, M_S, B_\zeta, E_\theta$.

By selecting these functions in the future, you can choose different combinations of fixing cone. We assume that all the components of the excited electromagnetic field and displacement field belonging to magneto-elasticity problem equation does not depend on the coordinates θ, and also believe that the elastic characteristics and electromagneto-mechanical shell material does not vary along the parallels.

After some transformations [15], we obtain a complete system of nonlinear differential equations in the form magnetoelasticity Cauchy, which describes the stress-strain state of the current-carrying orthotropic conical shell with an unsteady mechanical and magnetic fields:

$$\frac{\partial u}{\partial s}=\frac{1-\nu_s\nu_\theta}{e_s h}N_s-\frac{\nu_\theta\cos\varphi}{r}u-$$
$$-\frac{\nu_\theta\sin\varphi}{r}w-\frac{1}{2}\theta_S^2;\quad \frac{\partial w}{\partial s}=-\theta_S;$$
$$\frac{\partial\theta_S}{\partial s}=\frac{12(1-\nu_s\nu_\theta)}{e_s h^3}M_s-$$
$$-\frac{\nu_\theta\cos\varphi}{r}\theta_S;$$
$$\frac{\partial N_S}{\partial s}=\frac{\cos\varphi}{r}\left(\left(\nu_S\frac{e_\theta}{e_S}-1\right)N_S+\right.$$
$$\left.+e_\theta h\left(\frac{\cos\varphi}{r}u+\frac{\sin\varphi}{r}w\right)\right)-$$
$$-P_S+hJ_{\theta CT}B_\zeta-\sigma_1 h\left[E_\theta B_\zeta+\right.$$
$$\left.0.5\frac{\partial w}{\partial t}B_\zeta\left(B_S^++B_S^-\right)-\frac{\partial u}{\partial t}B_\zeta^2\right]+\rho h\frac{\partial^2 u}{\partial t^2};$$
$$\frac{\partial Q_S}{\partial s}=-\frac{\cos\varphi}{r}Q_S+\nu_S\frac{e_\theta}{e_S}\frac{\sin\varphi}{r}N_S+$$
$$+e_\theta h\frac{\sin\varphi}{r}\left(\frac{\cos\varphi}{r}u+\frac{\sin\varphi}{r}w\right)-P_\zeta-\tag{8}$$
$$-0.5hJ_{\theta CT}\left(B_S^++B_S^-\right)-\sigma_3 h\left[-0.5E_\theta\left(B_S^++B_S^-\right)-\right.$$
$$-0.25\frac{\partial w}{\partial t}\left(B_S^++B_S^-\right)^2-$$
$$-\frac{1}{12}\frac{\partial w}{\partial t}\left(B_S^+-B_S^-\right)^2+0.5\frac{\partial u}{\partial t}B_\zeta\left(B_S^++B_S^-\right)+$$
$$\left.+\frac{h}{12}\frac{\partial\theta_S}{\partial t}B_\zeta\left(B_S^++B_S^-\right)\right]+\rho h\frac{\partial^2 w}{\partial t^2};$$
$$\frac{\partial M_S}{\partial s}=\frac{\cos\varphi}{r}\left[\left(\nu_S\frac{e_\theta}{e_S}-1\right)M_S+\right.$$
$$\left.+\frac{e_\theta h^3}{12}\frac{\cos\varphi}{r}\theta_S\right]+Q_S+N_S\theta_S-$$
$$-\frac{\sin\varphi}{r}\left(\nu_S\frac{e_\theta}{e_S}M_S+\frac{e_\theta h^3}{12}\frac{\cos\varphi}{r}\theta_S\right)\theta_S+$$
$$+\frac{h^3}{12}\frac{\partial^2\theta_S}{\partial t^2};$$
$$\frac{\partial B_\zeta}{\partial s}=-\sigma_2\mu\left[E_\theta+0.5\frac{\partial w}{\partial t}\left(B_S^++B_S^-\right)-\frac{\partial u}{\partial t}B_\zeta\right]+$$
$$+\frac{B_S^+-B_S^-}{h};$$
$$\frac{\partial E_\theta}{\partial s}=-\frac{\partial B_\zeta}{\partial t}-\frac{\cos\varphi}{r}E_\theta.$$

Here N_s, $N_\theta-$ meridional and circumferential forces; $S-$shear; Q_s, $-$shear force; $M_s, M_\theta-$ bending moments; u, $w-$ displacement and deflection; θ_S $-$the rotation angle of the normal; $P_S, P_\zeta-$ mechanical load components; $E_\theta-$mechanical load components; $B_\zeta-$the normal component of the magnetic induction; B_s^+, B_s^-- known components of the magnetic induction on the surface of the shell; $J_{\theta cm}-$component of the electric current density from an external source; $e_s, e_\theta-$ elastic modules in the directions $s, \theta-$ respectively; $\nu_S, \nu_\theta-$ Poisson's ratio, which characterize the tensile transverse compression in the direction of the coordinate axes; $\mu-$ permeability; $\omega-$ the angular frequency; $\sigma_1, \sigma_2, \sigma_3$ $-$ the main components of the tensor conductivity.

Obtained coupled allowing system of nonlinear differential equations of order eight (8) describes the stress-strain state of flexible current-carrying orthotropic conical shells of rotation having orthotropic electrical conductivity, magnetic and electrical permittivity.

Solving of magnetoelasticity boundary values problems associated with the essential computational difficulties. This is because the resolution of the system of equations (8) is a system of differential equations of hyperbolic-parabolic type of eighth-order with variable coefficients. Components of the Lorentz force consider the speed of shell deformation, an external magnetic field, the size and intensity of the conduction current relatively to the external magnetic field. Accounting for nonlinearity in the equations of motion causes nonlinearity in the ponderomotive force.

The developed methodology for the numerical solution of the new class of related problems of the theory of orthotropic magnetoelasticity conical shells of revolution having orthotropic conductivity, based on the consistent application of the finite Newmark schemes, linearization method and discrete orthogonalization [10, 12-15, 21]

To make effective use of the proposed methods assume that the appearance of an external magnetic field does not appear sharp skin effects on the thickness of the shell and the electromagnetic process in the coordinate ζ quickly enters the mode close to steady. This leads to restrictions on the behavior of the external magnetic field and on the geometric and electrical parameters of the shell

$$\frac{\tau}{h^2\sigma\mu}>1,\tag{9}$$

where $\tau-$ the characteristic time of the magnetic field. In case of failure to do so should be considered only the shell of the equation of motion by the magnetic pressure.

Without going into the details of the calculations, limited to quadratic nonlinearity in the equations and cubic nonlinearity in the Lorentz forces, after the application of the scheme Newmark and linearization method, we obtain a sequence of linear differential equations on the corresponding time level in the form

International Journal of Engineering Innovation & Research
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$$\frac{d\,u^{(k+1)}}{dm}=\frac{1-\nu_S\,\nu_\theta}{\rho\, e_S\, h}N_S^{(k+1)}-\frac{\nu_\theta\cos\varphi}{\rho\, r}u^{(k+1)}-$$
$$-\frac{\nu_\theta\sin\varphi}{\rho\, r}w^{(k+1)}+\frac{1}{2\rho}\left(\theta_S^{(k)}\right)^2-\theta_S^{(k+1)}\theta_S^{(k)};$$

$$\frac{d\,w^{(k+1)}}{dm}=-\frac{\theta_S^{(k+1)}}{\rho};$$

$$\frac{d\,\theta_S^{(k+1)}}{dm}=\frac{12(1-\nu_S\,\nu_\theta)}{\rho\, e_S\, h^3}M_S^{(k+1)}-\frac{\nu_\theta\cos\varphi}{\rho\, r}\theta_S^{k+1};$$

$$\frac{dN_S^{(k+1)}}{dm}=\frac{\cos\varphi}{\rho\, r}\left[\left(\nu_\theta\frac{e_\theta}{e_S}-1\right)N_S^{(k+1)}+\right.$$
$$\left.e_\theta\, h\left(\frac{\cos\varphi}{r}u^{(k+1)}+\frac{\sin\varphi}{r}w^{(k+1)}\right)\right]-$$
$$-\frac{P_S^{(k+1)}}{\rho}+\frac{h}{\rho}J_{\theta CT}B_\zeta^{(k+1)}-\frac{\sigma_1\, h}{\rho}\left[\left(-E_\theta^{(k)}B_\zeta^{(k)}+\right.\right.$$
$$\left.+E_\theta^{(k+1)}B_\zeta^{(k)}+E_\theta^{(k)}B_\zeta^{(k+1)}\right)+$$
$$+0.5\left\{-\left(\dot{w}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k)}+\left(\dot{w}^{(t+\Delta t)}\right)^{(k+1)}B_\zeta^{(k)}+\right.$$
$$\left.+\left(\dot{w}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k+1)}\right\}\left(B_S^{+}+B_S^{-}\right)-$$
$$-\left\{-\left(B_\zeta^{(k)}\right)^2\left(\dot{u}^{(t+\Delta t)}\right)^{(k)}+\left(B_\zeta^{(k)}\right)^2\left(\dot{u}^{(t+\Delta t)}\right)^{(k+1)}+\right.$$
$$\left.\left.+2B_\zeta^{(k+1)}B_\zeta^{(k)}\left(\dot{u}^{(t+\Delta t)}\right)^{k}\right\}\right]+h\left(\ddot{u}^{(t+\Delta t)}\right)^{(k+1)};$$

$$\frac{dQ_S^{(k+1)}}{dm}=-\frac{\cos\varphi}{\rho\, r}Q_S^{(k+1)}+\nu_S\frac{e_\theta}{e_S}\frac{\sin\varphi}{\rho\, r}N_S^{(k+1)}+ \qquad (10)$$
$$+\frac{e_\theta\, h}{\rho}\frac{\sin\varphi}{r}\left(\frac{\cos\varphi}{r}u^{(k+1)}+\frac{\sin\varphi}{r}w^{(k+1)}\right)-$$
$$-\frac{P_\zeta^{(k+1)}}{\rho}-0.5\frac{h}{\rho}J_{\theta CT}\left(B_S^{+}+B_S^{-}\right)-$$
$$-\frac{\sigma_3\, h}{\rho}\left[-0.5\,E_\theta^{(k+1)}\left(B_S^{+}+B_S^{-}\right)-\right.$$
$$-0.25\left(\dot{w}^{(t+\Delta t)}\right)^{(k+1)}\left(B_S^{+}+B_S^{-}\right)^2$$
$$-\frac{1}{12}\left(\dot{w}^{(t+\Delta t)}\right)^{(k+1)}\left(B_S^{+}-B_S^{-}\right)^2+$$
$$+0.5\left\{-\left(\dot{u}^{(t+\Delta t)}\right)^{k}B_\zeta^{(k)}+\left(\dot{u}^{(t+\Delta t)}\right)^{(k+1)}B_\zeta^{(k)}+\right.$$
$$\left.\left(\dot{u}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k+1)}\right\}\left(B_S^{+}+B_S^{-}\right)+\frac{h}{12}\left\{-\left(\dot{\theta}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k)}-\right.$$
$$\left.\left.+\left(\dot{\theta}^{(t+\Delta t)}\right)^{(k+1)}B_\zeta^{(k)}+\left(\dot{\theta}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k+1)}\right\}\left(B_S^{+}+B_S^{-}\right)\right]+$$
$$+h\left(\ddot{w}^{(t+\Delta t)}\right)^{(k+1)};$$

$$\frac{dM_S^{(k+1)}}{dm}=\frac{\cos\varphi}{\rho\, r}\left[\left(\nu_S\frac{e_\theta}{e_S}-1\right)M_S^{(k+1)}+\right.$$
$$\left.\frac{e_\theta\, h^3}{12}\frac{\cos\varphi}{r}\theta_S^{(k+1)}\right]+\frac{Q_S^{(k+1)}}{\rho}+$$
$$+\frac{1}{\rho}\left(-N_S^{(k)}\theta_S^{(k)}+N_S^{(k+1)}\theta_S^{(k)}+N_S^{(k)}\theta_S^{(k+1)}\right)$$
$$-\nu_S\frac{e_\theta}{e_S}\frac{\sin\varphi}{\rho\, r}\left(-M_S^{k}\theta_S^{k}+M_S^{k+1}\theta_S^{k}+M_S^{k}\theta_S^{k+1}\right)-$$
$$-\frac{e_\theta\, h^3\sin\varphi\cos\varphi}{\rho\, r^2}\left[-\left(\theta_S^{(k)}\right)^2+2\theta_S^{(k+1)}\theta_S^{(k)}\right]+$$
$$+\frac{h^3}{12\rho}\left(\ddot{\theta}^{(t+\Delta t)}\right)^{(k+1)};$$

$$\frac{dB_\zeta^{(k+1)}}{dm}=-\frac{\sigma_2\mu}{\rho}\left[E_\theta^{(k+1)}+0.5\left(w^{(t+\Delta t)}\right)^{(k+1)}\left(B_S^{+}+B_S^{-}\right)-\right.$$
$$-\left\{-\left(\dot{u}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k)}+\left(\dot{u}^{(t+\Delta t)}\right)^{(k+1)}B_\zeta^{(k)}+\right.$$
$$\left.\left.+\left(\dot{u}^{(t+\Delta t)}\right)^{(k)}B_\zeta^{(k+1)}\right\}\right]+\frac{B_S^{+}-B_S^{-}}{\rho\, h};$$

$$\frac{dE_\theta^{(k+1)}}{dm}=-\frac{1}{\rho}\left(\dot{B}_\zeta^{(t+\Delta t)}\right)^{(k+1)}-\frac{\cos\varphi}{\rho\, r}E_\theta^{(k+1)},\ (k=0,1,2,\ldots)$$

Further, each of the linear sequence of boundary value problems in the relevant time interval is solved numerically using a sustainable method of discrete orthogonalization.

The developed algorithm for solving a new class of problems magnetoelasticity current-carrying conic orthotropic shells of revolution having orthotropic conductivity, magnetic and dielectric constant allows to obtain solutions in a wide range of geometric parameters of the shell, the mechanical characteristics of the material, surface and contour load, type of fixing the boundary contours, the parameters of the electromagnetic field. The algorithm is constructed in such a way that, on the one hand, he had enough common sense to the physical formulation, on the other hand, has the versatility to solve problems for different types of shells. It also has the property that its structure can be used in case of selecting a different theory of shells. It is also allowed the use of different interpolation formulas for calculating the right side of the system of equations.

III. A Numerical Example. Analysis of Electromagnetic Effects

As an example, we consider the nonlinear behavior of the current-carrying orthotropic conical shell of variable thickness $h=5\cdot 10^{-4}(1-0.5\,s/s_N)\,m$.

We believe that the shell of beryllium is under the influence of mechanical force $P_\zeta=5\cdot 10^3\sin\omega t\; N/m^2$, third party electric current $J_{\theta cm}=-5\cdot 10^5\sin\omega t\; A/m^2$, and external magnetic field $B_{S0}=0.1\,T$, and also that the envelope has a finite conductivity orthotropic $\sigma(\sigma_1,\sigma_2,\sigma_3)$.

We assume that by the electric current in the disturbed state is evenly distributed on the shell, the external current density does not depend on the coordinates. In this case, the combined effect on the shell loading, the ponderomotive force consisting of Lorentz forces and mechanical. We investigate the behavior of orthotropic shell, depending on changes in the external normal component of the magnetic induction $B_{\zeta 0}$.

The problem for orthotropic cone of beryllium variable thickness $h = 5\cdot10^{-4}(1-0.5\,s/s_N)\,m$ designed under the influence of the normal component of the magnetic induction $B_{\zeta 0}$ is amended as follows (8 options):

$B_{\zeta 0} = (-0.3, -1.0, -2.0, -3.0, -4.0, -5.0, -6.0, -7.0)$.

In this case, the boundary conditions can be written as

$$u = 0,\quad w = 0,\quad M_S = 0,\quad B_\zeta = B_{\zeta 0}$$

$$\sin\omega t \ \ (hinge) \quad at \quad s = s_0 = 0,$$

$$w = 0,\ \ \theta_S = 0,\ N_S = 0,\ B_\zeta = 0$$

$$(moving) \qquad at \quad s = s_N = 0.5m.$$

The initial conditions take the form

$$\vec{N}(s,t)\,\big|_{t=0} = 0,\quad \dot{u}(s,t)\,\big|_{t=0} = 0,\quad \dot{w}(s,t)\,\big|_{t=0} = 0$$

The parameters of the shell and the material are:

$$s_0 = 0,\ s_N = 0{,}5\ m,\ h = 5\cdot10^{-4}(1-0.5\,s/s_N)\,m,$$

$$r = r_0 + s\cos\varphi;\ \ r_0 = 0.5m,\ \omega = 314.16\sec^{-1}$$

$$\rho = 2300\ kg/m^3,\ B_s^+ = B_s^- = 0.5\ T.\ \varphi = \pi/30,$$

$$B_{S0} = 0.1\,T,\ \mu = 1.256\cdot10^{-6}\ H/m,$$

$$J_{\theta cm} = -5\cdot10^5\sin\omega t\ A/m^2,\ \sigma_1 = 0.279\cdot10^8(\Omega\times m)^{-1},$$

$$\sigma_2 = 0.321\cdot10^8(\Omega\times m)^{-1},\ \sigma_3 = 1.136\cdot10^8(\Omega\times m)^{-1},$$

$$\nu_S = 0.03,\ \nu_\theta = 0.09,\ P_\zeta = 5\cdot10^3\sin\omega t\ N/m^2,$$

$$e_S = 28.8\cdot10^{10}N/m^2,\ \ e_\theta = 33.53\cdot10^{10}\ N/m^2$$

The solution is found in the time interval $\tau = 0 \div 10^{-2}\sec$ for the integration time step is chosen to be $\Delta t = 1\cdot10^{-3}\sec$. Maximum values obtained at time step $t = 5\cdot10^{-3}\sec$. Consider the case in the anisotropy of the electrical resistance equal to beryllium $\eta_3/\eta_1 = 4.07$.

Figure 1 shows the distribution of shell deflection as a function of changes in the external magnetic induction at $s = 0.4m$ and $t = 5\cdot10^{-3}\sec$ all change options:

$B_{\zeta 0} = (-0.3, -1.0, -2.0, -3.0, -4.0, -5.0, -6.0, -7.0)$.

The maximum deflection is observed at $B_{\zeta O} = -7.0$. From calculation results show that with the increase in the value of magnetic induction shell deflection increases. Figure 2 shows the variation of the magnetic induction inside of the shell, depending on changes in the external magnetic induction at $t = 5\cdot10^{-3}\sec$ and $s = 0.45\,m$ all change options $B_{\zeta 0}$.

In the above range of changes in the external magnetic induction internal magnetic induction reaches its maximum value at $B_{\zeta O} = -4.0$. It is found that increasing the external magnetic field induction internal magnetic field also increases.

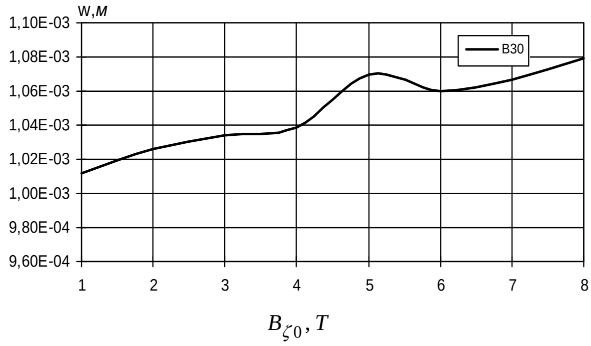

Fig.1. Change the membrane deflection in response to changes external magnetic induction in $t = 5\cdot10^{-3}\sec$ и $s = 0.4m$ all change options $B_{\zeta 0}$.

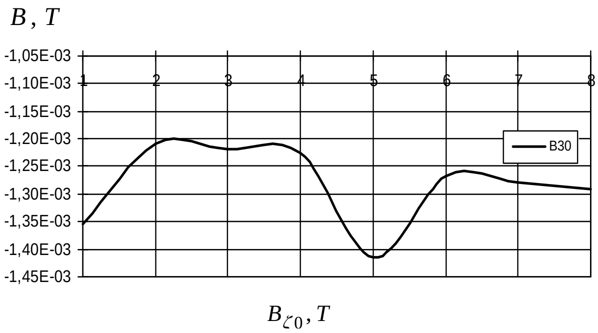

Fig.2. Change the inner shell of the magnetic induction as a function of changes in the external magnetic induction at $t = 5\cdot10^{-3}\sec$ and $s = 0.45\,m$ all change options $B_{\zeta 0}$.

At figure 3 and 4 show the stress changes σ_{22}^+ и σ_{22}^- on the outer and inner surfaces of the shell, depending on changes in the external magnetic induction at $t = 5\cdot10^{-3}\sec$ и $s = 0.4m$.

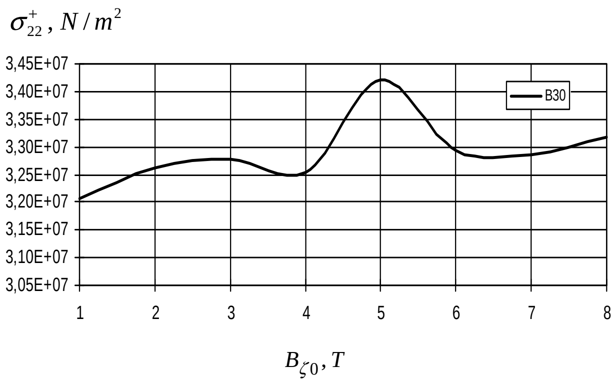

Fig.3. Changing the voltage σ_{22}^+ on the outer surface of the shell in response to changes in the external magnetic induction $t = 5\cdot10^{-3}\sec$ and $s = 0.4m$ all change options $B_{\zeta 0}$.

A value with increasing external magnetic induction voltage on the outer surface of the shell varies depending on the change of direction of the Lorentz force and mechanical load. In the above case, the voltage on the external surface of the shell reaches its maximum value when $B_{\zeta o} = -4.0$.

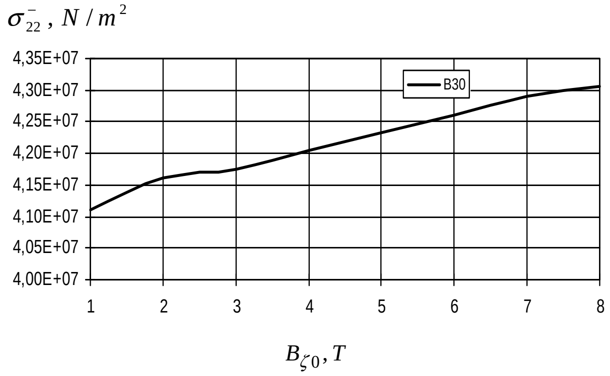

Fig.4. Changing the voltage σ_{22}^{-} the inner surface of the shell in response to changes in the external magnetic induction $t = 5 \cdot 10^{-3}$ sec and $s = 0.4\,m$ all change options $B_{\zeta 0}$.

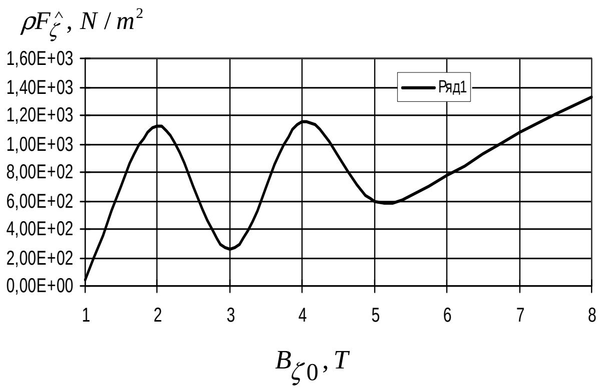

Fig.5. Change the normal component of the Lorentz force ρF_{ζ} depending on changes in the external magnetic induction at $t = 5 \cdot 10^{-3}$ sec and $s = 0.4\,m$ for all change options $B_{\zeta 0}$.

From figure 4 that with increase of magnetic induction value on the outer surface of the inner shell voltage increases. Figure 5 shows the change in the normal component of the Lorentz force ρF_{ζ}, depending on the changes in the external magnetic induction at $t = 5 \cdot 10^{-3}$ sec and $s = 0.4\,m$ for all change options $B_{\zeta 0}$. With increasing magnetic induction value of the normal component of the Lorentz force increases.

IV. Conclusion

In this article, the associated task magnetoelasticity for flexible orthotropic conical shell taking into account the orthotropic conductivity. Get connected resolution systems of nonlinear differential equations describing the stress-strain state of flexible orthotropic conical shells. It was analyzed the influence of external magnetic induction on the state of stress of the orthotropic shell in geometrically nonlinear formulation. It was found that with increasing of magnetic induction the deflection of the shell also increases. Increasing of the external magnetic field induction increases the magnitude of the mechanical stress of the shell.

The change of the magnitude of the internal magnetic field induction of the hell depending on the external magnetic field and the orthotropic conductivity was investigated.

It was established that an increase in external magnetic field induction also increases induction of the internal magnetic field. This corresponds to a real physical processes occurring in the shell and in turn confirms the accuracy of the results.

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