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Traffic Congestion Pricing Combination Design based on Gini Coefficient

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***Abstract* – Pricing is one of the important research topics of road congestion charging policy. Bi-level programming model is highly effective in research traffic congestion charging. Road congestion charging policy and parking charge policy are combined to transform the bi-level programming pricing model, using Gini coefficient which is resource allocation fairness index. Gini coefficient function and parameters for controlling the Gini coefficient were added into the model. The designation of traffic congestion charging and parking charging for different fairness requirements could be gotten through the selecting of different control parameters. This new model could not only make the objective function as large as possible, but also balance the fairness of the system. Examples have shown that the results, the road network resource utilization and system fairness could be improved using the charging model which is considering Gini coefficient.**

***Keywords* – Bi-level Programming Model, Gini Coefficient, Traffic Congestion Charging, Parking Charging.**

I. Introduction

In recent years, urban residents travel demand's rapid growth makes the city traffic congestion problem increasingly serious. Blindly increasing the supply of road facility cannot ease road congestion, but stimulate the traveler increase the amount of travel instead. This method will make roads more crowded [1]. Road congestion charging and parking charge as two means of transportation demand management have important influence on people' s traffic demand. It can change people's travel time, travel modes, travel routes, so as to achieve the aim of relieving urban road congestion.

Bi-level programming model is highly effective in research traffic congestion charging, it can reflect the traffic managers and travelers' interaction game strategy [2-3]. The upper model reflects the strategy which designed by policymakers from the macroscopic level. Its objective function is the maximum total revenue or the minimum total cost. The lower model is used to describe the travelers' decisions following the upper policies. Its objective function is the traveler's maximum own trip profit or the minimum cost [4].

The main problem of congestion charging is the public questioned and resistance instead of technology level difficulty. Cervero points out that, public resistance comes from the highly charge, they think the implementation of congestion charging policy is only benefit to the government managers and a few high-income earners, and they believe that policy is not fair to them [5]. Therefore, the public the sensitive degree of road charging must be reduced, in the guarantee of congestion charge policy's effect, and the fairness level must be improved.

Not like the road congestion charging is not supported by the public, the downtown parking charge is easier for people to accept [6]. Through the analysis of the results of the O-D (ORIGIN-DESTINATION) survey in Sydney city, using Logit model, David came to the conclusion that every 1% increase in downtown parking charge, travelers parking in downtown will reduce 2.04% [7]. But using the parking charge policy to regulate traffic flow will significantly reduce traffic attraction intensity of the town center. And it will reduce the utilization rate of parking facilities in congestion network [8-9].

On the basis of predecessors' research, in this paper, the road congestion charging and parking charging are combined to make up for the inadequacy of their single use, and the established Bi-level programming pricing model using Gini coefficient is used to research the combination pricing considering the network resource fairness.

II. Analysis of Mathematical Model

A. The Calculation Method of Gini Coefficient

In urban life, there are four common modes of transportation, respectively is walking-travel, biking-travel (including aided-bicycles, motorcycles, etc.), bus-travel and driving-travel. The ratio of each mode's occupied road resources and the average traveler resource is fairness index. The road congestion charging and parking charge pricing method is researched to make the Gini coefficient smaller, and get the maximum system total resource utilization, under the condition of the road total resources without increasing.

The calculation steps are shown as following:

1) Identify q^i as the number of traveler which chosen different modes of transportation. Walking-travel, biking-travel, bus-travel and driving-travel are represented by $i = 1,2,3,4$, respectively.
2) According to each means of transportation trips and per capita road area, the resources consumed by each the means of transportation is calculated.
3) Through calculating the road resources consumed and the percentage of the population by each the means of transportation, the cumulative percentage of road resources and the cumulative percentage of population are calculated.
4) Data is fitted according to the allocation of resources. And transformed it into the Lorentz curve $L[x(q)]$ through the linear regression which shown as Fig.1.
5) Draw the Lorentz curve, and Gini coefficient is calculated as $G(q)$ formula (1).

$$G(q) = 1 - 2\int_0^1 L[x(q)]dx(q) \qquad (1)$$

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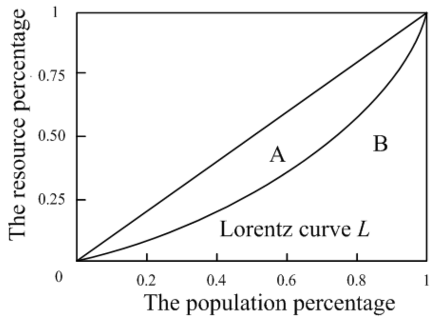

Fig. 1. Lorentz curve L

B. The Establishment of the Mathematical Model

The objective function of upper model is the maximum system total profits. The combination pricing scheme should be designed under the condition of the upper level system overall efficiency, and the network resource allocation fairness also should be considered (the smaller the Gini coefficient).For this purpose, the Gini coefficient and its control parameter are added into the upper model. The traditional Bi-level pricing model is modified, in order to achieve both the goal of efficiency and fairness.

The upper model (P1) is shown in formula 2:

$$\begin{aligned} & max_u\, Z(u,q,x) = [1-\lambda G(q)] \textstyle\sum_i \sum_a x_a^i u_a^i \\ & u_a^i = f(u_{ar}^i, u_{ap}^i) \\ s.t. \quad & u_{ar}^{min} \le u_{ar}^i \le u_{ar}^{max} \\ & u_{ap}^{min} \le u_{ap}^i \le u_{ap}^{max} \\ & 1-\lambda G(q) > 0, \lambda \in (0,+\infty) \end{aligned} \tag{2}$$

In the formula, Gini coefficient $G(q)$ is calculatedby Lorenz curve. λ is the control factor, which could be adjust to satisfy the different equity requirements of road congestion charging and parking charge pricing. The charging strategies made by policy makers are represented as $u = (\cdots, u_a^i, \cdots)$. Road congestion pricing scheme is represented as $u_r = (\cdots, u_{ar}^i, \cdots)$.

The road congestion charging price range can be allowed is represented as $[u_{ar}^{min}\ ,\ u_{ar}^{max}]$.

Parking charge pricing scheme is represented as $u_p = (\cdots, u_{ap}^i, \cdots)$.

The parking charge price range can be allowed is represented as $[u_{ap}^{min}\ ,\ u_{ap}^{max}]$.

Traffic flow made by different modes of transport in the road network is represented as $x = [\cdots, x_a^i, \cdots]$.

From the model we can see that, in the case of constant control coefficient, in order to achieve the maximum objective function, system interest is required to be as large as possible, and the Gini coefficient is required to be as small as possible.

The lower level is the description of different travelers' path choices. Traveler's utility in the OD(r, s) path k is shown in formula 3.

$$U_{rs,k}^i = -\theta c_{rs,k}^i + \varepsilon_{rs}^i \tag{3}$$

In the formula, traveler's cost in the OD(r, s) path k is represented as $c_{rs,k}^i$.

Mutually independent and obey the parameters for θ Gumbel distribution random item is represented as ε_{rs}^i.

The probability of different traveler chooses the OD(r, s) path k as shown in formula 4.

$$p_{rs,t}^i = \frac{exp(-\theta c_{rs,k}^i)}{\sum_k exp(-\theta c_{rs,k}^i)} \tag{4}$$

The attraction of different modes of transportation is represented as h_{rs}^i .

Traveler's cost can be afford in the OD(r, s) path k is represented as $C_{rs,k}^i$.

Traveler's utility in the OD(r, s) path k is shown in formula 5.

$$U_{rs}^i = h_{rs}^i + max_{k\in k_{rs}}(-C_{rs,k}^i) \tag{5}$$

Combined with the formula 2, 3, 4, formula 6 could be gotten.

$$\begin{aligned} u_{rs}^i &= E(U_{rs}^i) \\ &= h_{rs}^i + E(max_{k\in k_{rs}}(-C_{rs,k}^i)) \\ &= h_{rs}^i - S_{rs}^i(C_{rs,k}^i) \end{aligned} \tag{6}$$

Traveler's expected cost in the OD(r, s) path k is represented as $S_{rs}^i(C_{rs,k}^i)$ shown in formula 7.

$$\begin{aligned} S_{rs}^i(C_{rs,k}^i) &= E[min_{k\in k_{rs}}(C_{rs,k}^i)] \\ &= -\frac{1}{\theta} ln \textstyle\sum_k exp(-\theta c_{rs,k}^i) \end{aligned} \tag{7}$$

The random item is represented as ε_{rs}^i , which is mutually independent and obey the parameters for β Gumbel distribution.The probability of traveler chooses different modes of transportation is shown in formula 8.

$$p_{rs}^i = \frac{exp(\beta(h_{rs}^i - s_{rs}^i))}{\sum_i exp(\beta(h_{rs}^i - s_{rs}^i))} \tag{8}$$

The volume of traffic relationship formula between each mode of transportation and network system can be got as shown in formula 9.

$$q_{rs}^i = q_{rs} \cdot \frac{\exp(\beta(h_{rs}^i - s_{rs}^i))}{\sum_i \exp(\beta(h_{rs}^i - s_{rs}^i))} \tag{9}$$

Traveler's cost in the i mode of transportation is shown in formula 10.

$$c_a^i(x_a^i, u_a^i) = t_a^i + u_a^i \tag{10}$$

In the formula, traveler's relative cost of travel time in path a with the i_{th} mode of transportation is represented as t_a^i. The charging strategy made by policy makers in path with the i_{th} mode of transportation a is represented as u_a^i .It can be shown in formula 11

$$u_a^i = f(u_{ar}^i, u_{ap}^i) \tag{11}$$

According to the probability of travelers' choices of different modes of transportation and path, the lower level mathematical model can be set up as shown in formula 12.

$$\begin{aligned} & \min Z(x,f,q) = \\ & \sum_{i=1}^4 \left[\begin{array}{c} \frac{1}{\theta} \sum_{rs} \sum_k \int_0^{f_{rs,k}^i} \ln\omega \\ -\left(\frac{1}{\theta} - \frac{1}{\beta}\right) \sum_{rs} \int_0^{q_{rs}^i} \ln\omega \\ -\sum_{rs} h_{rs}^i \cdot q_{rs} \end{array} \right] \\ & + \sum_{i=1}^4 \sum_a \int_0^{x_a^i} c_a^i(x_a^i, u_a^i) \\ s.t. \quad & \textstyle\sum_k f_{rs,k}^i = q_{rs}^i \quad \sum_i q_{rs}^i = q_{rs} \\ & x_a^i = \textstyle\sum_{rs} \sum_k f_{rs,k}^i \\ & f_{rs,k}^i \ge 0 \quad x_a^i \ge 0 \quad q_{rs} \ge 0 \quad q_{rs}^i \ge 0 \end{aligned} \tag{12}$$

In the formula, the traffic flow in the i_{th} mode of transportation is represented as $f_{rs,k}^i$.

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III. Solution Method for Model

Function of the optimal combination pricing scheme is solved under the consideration of both the upper level system's maximum efficiency and the Gini coefficient as small as possible. Model calculation method is shown as follows:

1) According to the requirements of fairness, select the appropriate control factor λ to decide the Gini coefficient $\lambda G(q)$.
2) Determine the initial value ${u_a}^0$ of combination pricing, and choose the iterations number n.
3) The initial trip volume value q_{rs}^i of each transportation mode and the initial traffic flow value x_a^i of each path can be calculated from the lower level when u^0 was added in.
4) According to the initial data, the resource allocation proportion of each transportation mode can be calculated. Then the Lorentz curve can be fitted to calculate the Gini coefficient.
5) The new value ${u_a}^1$ ofcombination pricing can be calculated from the function of upper level model when q_{rs}^i and x_a^i was added in.
6) The iterative precision is set to k. Stop calculating when $max|u_a^{n+1} - u_a^n| \le k$; otherwise, set $n = n + 1$, continue the iteration.
7) After reaching the iteration precision, the appropriate combination pricing $u_a^i = f\left(u_{ar}^i, u_{ap}^i\right)$ can be got.

IV. Calculation Example of Model

The typical traffic network can be used to validate the effectiveness of the established model.

As shown in the following Fig.2, it contains 4 nodes, one OD path (*r*, *s*) and 5 sections.

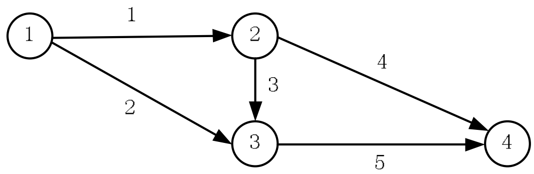

Fig.2 The typical traffic network

The trip time in section A is represented as t_a^i, and the traffic capacity of section A is s represented as c_a .

Walking-travel, biking-travel, bus-travel and driving-travel are represented by $i = 1,2,3,4$, respectively.

The specific values are shown in the following table 1.

Table 1 Details of the path parameters

Path a	traveler's relative cost of travel time				traffic capacity
	t_a^1(min)	t_a^2(min)	t_a^3(min)	t_a^4(min)	c_a(pcu*h^{-1})
1(1,2)	10	7.5	6.2	5	1500
2(1,3)	12	9	7.5	6	1000
3(2,3)	2	1.5	1.2	1	1500
4(2,4)	12	9	7.5	6	1500
5(3,4)	6	4.5	4	3	1000

The specific values of road resources consumed by each mean of transportation in the OD pair are shown in the following table 2.

Table 2 Schedule of road resources consumed by each mean of transportation

Mode	Trip volume (per/h)	Road area occupied (m^2/per)	Travel time (h)	Road resources (m^2*h)	Percentage Resources (%)	Percentage Population (%)
1	350	2	3	2100	2.2	11.7
2	1000	6	4	24000	24.1	33.3
3	750	3	4.5	10125	10.2	25
4	900	20	3.5	63000	63.5	30

The road resources consumed by each mean of transportation can be calculated through formula 13.

$$RE_d = L \cdot T = \sum RE_{di} = (\sum L_i \cdot \eta_i) \cdot T \quad (13)$$

Trip distance is represented as L , travel time is represented as T, and the road area consumed by each mean of transportation is represented as η_i.

The polynomial approximation method is proposed to obtain the Lorentz curve as formula 14.

$$L[x(q)] = 1.961x^4 - 2.405x^3 + 1.388x^2 + 0.05541x \quad (14)$$

The Lorentz curve is shown as Fig.3.

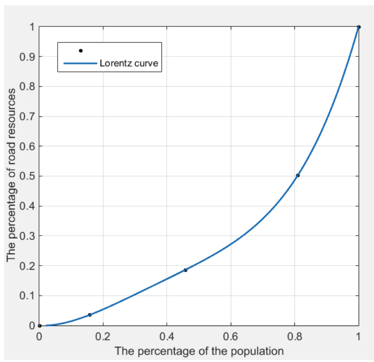

Fig. 3. The Lorentz curve before charging

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Without traffic congestion charging, the Gini coefficient was 0.4374 when OD pair is (1, 4). Its value is bigger than the Gini coefficient watch point value of 0.4. The conclusion shows that, at this time road network resource fairness doesn't stay in high level.

The saturation of traffic flow in each section can be calculated through the calculation of the lower random user model when the initial value $u^0 = 0$ without charging, and it is shown as table 3.

Table 3 Schedule of road resources consumed by each mean of transportation

Path a	x_a^1 (man-time)	x_a^2 (man-time)	x_a^3 (man-time)	x_a^4 (man-time)	C (Pcu)	Road saturation (%)
1(1,2)	224.7	639.3	479.5	575.4	861.5	57.4
2(1,3)	125.5	356.9	267.7	321.2	480.9	48.1
3(2,3)	125.8	357.9	268.4	322.1	482.3	32.2
4(2,4)	98.9	281.4	211.1	253.3	379.2	25.3
5(3,4)	250.6	714.7	536.3	643.2	963.1	96.3

The totalflow volume of each section is represented as C, which is converted to standard value.

As can be seen in the table 3, the saturation value of the section 5 is 96.3%, and it is a relatively congestion path.

Consulting the documents and materials, the system fairness is appropriate when the Gini coefficient in the range of 0 to 0.4.One of the constraints in the model is $1 - \lambda G(q) > 0$, the Gini coefficient is smaller than watch point 0.4 under the condition of $\lambda \geq 2.5$ .Using MATLAB for programming calculation, when $\lambda = 2.5$, the function is convergence at $u^n = 10.2$.At this time, road congestion pricing ratio $u_{ar}^i = 7$, parking charge pricing ratio $u_{ap}^i = 8$, the objective function of upper level model reach the maximum value.

Bring $u^n = 10.2$ into the lower level model, the traffic flow volume in each section can be calculated as table 4.

Table 4 Schedule of road resources consumed by each mean of transportation after charging

Path a	x_a^1 (man-time)	x_a^2 (man-time)	x_a^3 (man-time)	x_a^4 (man-time)	C (Pcu)	Road saturation (%)
1(1,2)	291.1	654.5	556.3	352.3	661.2	44.1
2(1,3)	162.7	372.4	310.7	196.6	371.1	37.1
3(2,3)	162.4	372.1	310.5	196.8	371.2	24.7
4(2,4)	128.6	290.2	245.7	155.6	292.3	19.5
5(3,4)	325.1	745.2	621.3	393.3	742.5	74.3

As can be seen in the table 4, the saturation value of section 5 is decreased from 96.3% to 74.3%, and traffic congestion has been effectively alleviated. The values of road resources consumed by each the means of transportation in the OD pair are shown in the following table 5.

Table 5 Schedule of road resources consumed by each mean of transportation after charging

Mode	Trip volume (per/h)	Road area occupied (m^2/per)	Travel time (h)	Road resources (m^2*h)	Percentage Resources (%)	Percentage Population (%)
1	455.1	2	3	2730.6	3.5	15.7
2	1023.3	6	4	24559.2	31.7	35.3
3	869.7	3	4.5	11740.9	15.1	30.0
4	550.1	20	3.5	38507.0	49.7	19.0

The Lorentz curve can be fitted as formula 15.

$$L[x(q)] = 3.047x^4 - 4.35x^3 + 2.355x^2 + 0.05129x \qquad (15)$$

At this time, the Gini coefficient is 0.3541, and the Lorentz curve is shown as Fig.4.

V. THE MODEL SOLUTION OF DIFFERENT CONTROL FACTORS

The parameter λ is selected according to different requirements of traffic equity, and the road congestion pricing u_{ar}^i and the parking charge pricing u_{ap}^i can be calculated by MATLAB compiler as table 6.

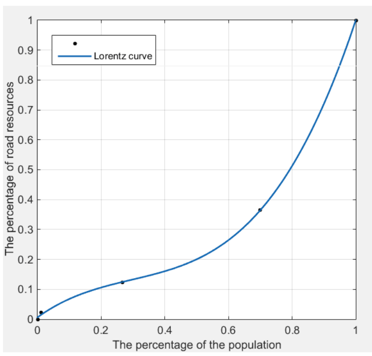

Fig. 4. The Lorentz curve after charging

Table 6 Charge pricing of different requirements of traffic equity

	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1$	$\lambda = 1.5$	$\lambda = 2$	$\lambda = 2.5$	$\lambda = 2.8$
Road congestion charging (unit cost of travel-time)	2.6	2.7	2.9	3.3	5.9	7	11.5
Parking charge (unit cost of travel-time)	2.3	2.4	2.5	2.8	5.1	8	10
Gini coefficient	0.4382	0.4367	0.4321	0.4264	0.3921	0.3541	0.3378
The objective function	987.4	594.3	511.4	354.9	181.4	53.7	17.4

When the value of parameter λ is 0, the function is conventional Bi-level pricing model without considering Gini coefficient. When road congestion pricing $u_{ar}^{i} = 2.6$ and parking charge pricing $u_{ap}^{i} = 2.3$, the objective function of upper level model reaches a relative large size of $Z = 987.4$, but the Gini coefficient value is higher than the alarm value 0.4. Based on the table 5, one can draw a conclusion that, both Gini coefficient and system total revenue is decreased, along with the increase of parameter λ. Different requirements of traffic equity can be achieved by changing parameter λ in this way.

VI. Conclusion

The calculation result shows that, traffic congestion is alleviated as expected, and conditions of system income as well as traffic equity are improved in virtue of traffic congestion combination charging design based on Gini coefficient.

The combination charging is analyzed with measuring the fairness by taking the road resources as the index, under fixed demand of traffic assignment. In following studies, under elastic demand of traffic assignment, with the objective function of maximizing the user's benefits, aiming at different modes of transportation or different time value of travelers, the combination charging strategy with the considering of fairness could be researched.

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