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Modal Analysis of a Non-rotating Inflated Tire using Experimental and Numerical Methods

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***Abstract* – A typical automotive wheel transmits noise and vibration from road to the automotive body structure, and considered as an important parameter in automotive NVH performance optimization. Having a reliable and simple finite element model of the wheel structure can significantly reduce time and cost of design. In this paper a finite element model of an automotive wheel structure validated by experimental modal analysis is presented. Validation is conducted via frequency response functions and resonance frequencies. The shift of resonance frequencies in FRF diagram is also studied and discussed for different conditions. FEM model of the tire is also developed consisting of a Neo-Hookean model for the tire and linear elastic model for the rim. Finally the results are analyzed and compared with the FEM model. Comparison between FEM and experimental modal analyses shows an acceptable accuracy (less than 10 % error). The validated model can be used for other Computer Aided Engineering (CAE) analysis purposes to modify the wheel structure in early design phase and reduce cost and time of design.**

***Keywords* – Modal Analysis, Computer Aided Engineering, Structural Dynamics, Wheel FE Analysis.**

I. Introduction

Pneumatic tires greatly influence the riding comfort and noise level in cars. Tire/road interaction is a significant operative parameter for vehicle manufacturers, because tires are the only member of an automotive having contact with road. The undesirable transmitted vibrations negatively influence on passengers and would be annoying in long term. The previous studies show that in vehicle dynamics interaction of tire resonance frequencies with road disturbance and surrounding air can easily magnify the level of interior noise and vibrations [1]. Hence, the NVH (Noise, Vibration and Harshness) performance of a vehicle has become an important parameter in automotive structural dynamics that need to be carefully designed and optimized in early phase of design [1-3].

The structural vibrations of the tire are generated by the interactions between tire and road that are created by road irregularities and brake torque changes. Vibrations would be transmitted from tire to the automotive body through suspension system [4]. Type of vibration depends on the plane that vibration is studied on and it could be bending, longitudinal and torsion. Tire is made of different materials with complex connectors. These ingredients cause tire behaves as a non-linear structure which makes hard to predict its structural behavior. However it has been shown that at frequencies below 500 Hz the tire shows modal behavior [1]. Therefore modal analysis can be performed to extract its dynamic properties such as mode shapes and natural frequencies.

Experimental modal analysis is conducted for a typical tire, rim and wheel. Different physical and statistical models have been considered to model tire [5].

Due to intrinsic complexity of a wheel structure, its dynamic modeling is challenging [6]. So far, there are a variety of models developed with different levels of complexity to simulate the tire dynamic behavior such as simple spring damper element (single point contact) [7], through a flexible ring model [8], or detailed finite element simulations [9].

The dynamic response of tires, hence, has been investigated since the early 1960s. Experimental natural frequencies and modes of tires have been studied by Chiesa [10], Bohm [12], Potts [11], and Guan [13-14]. Theoretical efforts encompass approximation of the tire as a tension band, through treating it as a ring on an elastic foundation, to a finite element approach [15–17]. Of these, the ring on elastic foundations (REF) model has been the most frequently adopted [15–17] because of the completeness and simplicity of ring theory with less sacrifice of result accuracy. Recently a concept modeling method has been proposed for a seat structure [18], wheel and whole automotive Body-In-White (BIW) structure [19] to predict and analyze their dynamical properties. It has been investigated that even the interior cabin noise can be reduced with some structural modifications [18, 20] to prevent mode interaction.

Here, an experimental modal analysis is conducted to verify the developed finite element model of a wheel structure. Modal analysis has been performed via hammer test. Vibrations have been measured using a 3 directional accelerometer. The validated FE model can be used to predict dynamical properties of the wheel after structural modifications.

II. Experimental Modal Analysis

Modal analysis is the study of the dynamic properties of structures under vibrational excitation. This method is for measuring and analyzing the dynamic response of structures when they are excited by an input. Typical input excitation signals can be classified into impulse, sweep sine, broadband, chirp, etc. and each of them has advantages and disadvantages. The analysis of the signals typically relies on Fourier analysis. The resulting transfer function will show resonances and can estimate characteristic mass, frequency and damping properties. It can also determine structural mode shapes that are very useful to NVH (noise, vibration, and harshness) analysis [2]. One of the important applications of this technique is measuring the vibration of an automotive body structure.

Experimental approach is applied to study dynamic behavior of a tire, rim and tire-rim assembly as entire wheel configuration. Equipment used for modal tests are a hammer for excitation, one and three dimensional accelerometers, an analog-to-digital converter frontend to digitize analog instrumentation signals, Pulse software (signal processing) and a host PC to view the data and analyze it, cables, accessories and adaptors.

The modal test is performed separately on tires and rims, and then on the whole wheel (set of tire and rim). Rim is a metallic component including rim center and ring. Tire is a rubber made of different components including tread band, sidewall and etc. which locates around rim. The assembly of rim and tire makes the whole wheel structure.

Tire modal test is performed by exciting tire in tangential direction and measuring acceleration in radial direction. Rim modal test is conducted and analyzed in a cylindrical coordinate system as shown figure 1. Z is axial direction that is along wheel axis. r and θ are radial and tangential direction respectively.

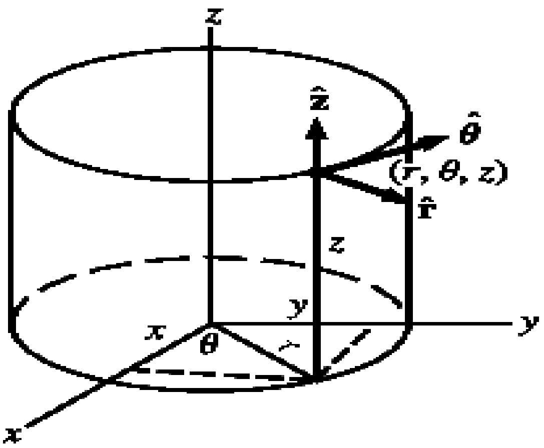

Fig. 1. Applied cylindrical coordinate system for modal analysis

III. Modal Testing of the Tire

In this study all of the excitation signals for modal testing are generated by hammer test because impact excitation is the simplest, quickest and most widely used excitation technique for modal analysis. Impulse input sweeps all frequencies from zero to a few KHz depending on the hammer tip. Tire is excited by an impulse force generated by a hammer. The excitation is evaluated by the frequency range of the input signal which is made by the hammer impact. If energy is allocated equally in the frequency range then assessment of the FRF will be accurate.

IV. Experimental Setup

The tire was hanged from a metallic structure in free-free condition by a flexible sling. In order to hit the tire tangentially as shown in Figure 2, a screw is attached to the tire at center of the width.

Fig. 2. Experimental setup: the screw is attached for tangential excitation and the small aluminum plate is bonded on the tire for radial excitation.

A plastic tip on the hammer head is used to guarantee excitation frequency range of 0-400 Hz which is considered as appropriate frequency range. A small aluminum plate mounted on the tire was also allocated to decrease non-linear effects at the point of impact for radial excitations. As shown in Figure 3 tire is excited in point 1 in the tangential direction and accelerometer is installed at 16 points on tire circumference by heated beeswax. The responses were measured only in radial direction.

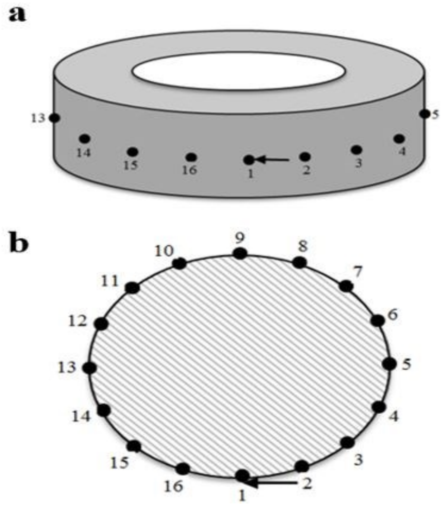

Fig. 3. Experimental setup of the tire showing position and orientation of the force excitations: a) 3D view b) top view.

This methodology is a SIMO (single-input, multiple-output) approach, because one point is excited and then the response is measured at some points. In order to improve the signal to noise ratio each measurement was repeated three times. Coherence functions were employed to evaluate each measurement since impact is applied manually; its direction and intensity are varied.

Since principle mode shapes usually occur at in-plane vibration due to their higher amplitudes, out of plane mode shapes are not taken into account. Figure 4 shows the frequency response function measured at point 7 in radial direction. Values of frequency at peaks are the natural frequencies of the system. Damping properties can be obtained using width of the peaks. Height of the peaks shows the amplitude of the vibration at dB basis. The FRF shows eight resonances in the frequency range 0-400 Hz.

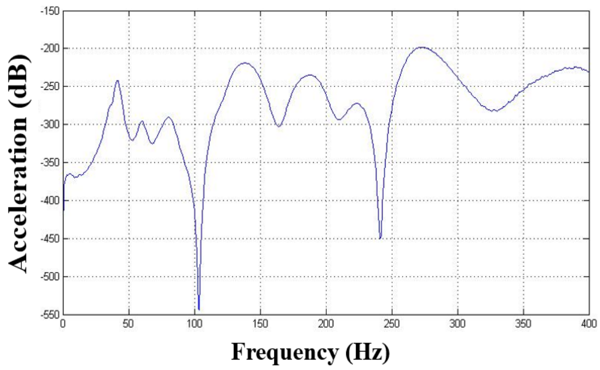

Fig. 4. Measured FRF for tire in radial direction at point 7 due to a radial excitation at point 1.

Sharp peaks in FRF diagrams proof that the tire has negligible damping properties. To obtain tire mode shapes responses of all 16 measuring points is needed. All of them have equal resonance frequencies but different amplitude values. The mode shapes are acquired by these amplitudes. It can be assumed that mode shapes of vibration of the tire do not influence each other. Therefore the peaks in the FRF are matched with individual mass-spring systems with natural frequency ω_n and relative damping c_n. Determination of natural frequencies and mode shapes are two significant outputs of the experimental modal analysis.

V. Modal Testing of the Rim

Since the rim is a metallic member, so its behavior is quite linear and peaks of FRF are very sharp. Rim is excited by an impulse force generated by a hammer excitation. Rim has higher resonance frequencies. In order to provide higher excitation frequency range (up to 1600 Hz) which is desired for rim modal test, metallic hammer head is used.

VI. Experimental Setup

The rim was hanged from a metal structure in free-free condition by a flexible sling. This experiment is performed twice in to different condition with two accelerometers. In the first condition an excitation point is fixed and a three-dimensional accelerometer moves, so it is a SIMO (single-input, multiple-output) approach. Impact force is applied in one point in radial direction and accelerations are measured in three directions at sixteen points (eight points on the rim center and eight points on rim perimeter. In second condition a one-dimensional accelerometer is fixed and thirty two points are considered for excitation. As shown in figure 5, rim is excited in points 1 to 24 in radial direction and in points 25 to 32 in vertical direction and accelerometer is installed at point 8 by beeswax. So acceleration is measured only in radial direction. This is considered as a MISO (multiple-input, single-output) analysis.

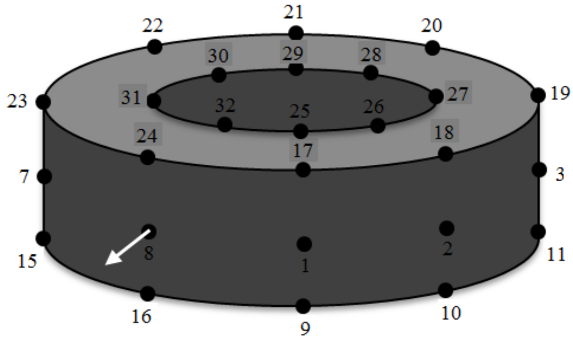

Fig. 5. Experimental set-up of rim: the 3D view of positions and orientations of the force excitations.

Figure 6 shows the frequency response function computed by radial acceleration measuring at point 8 when rim is excited in point 24 at radial direction. The FRF shows five resonances in the frequency range 0-1600 Hz. The first six resonances of the rim are related to rigid modes that in theory they have to be equal to zero Hz, but in experimental condition since the rim is not fully suspended, those occur in about 10 Hz. First resonance frequency of flexible modes is related to bending mode that occurs in 245 Hz. Second resonance frequency is related to torsion mode and happens in 671 Hz, and fifth resonance frequency is an axial mode that occurs in 1390 Hz.

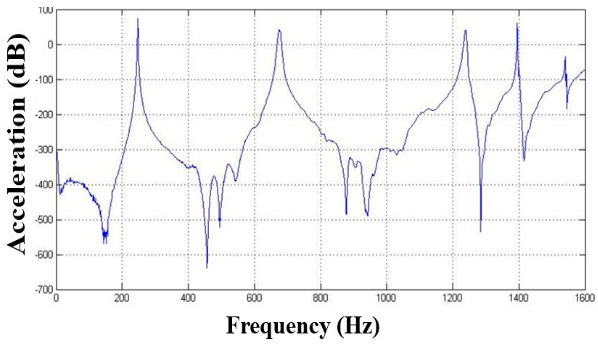

Fig. 6. Measured Rim's FRF in radial direction at point 8 due to a radial excitation at point 24 (Free rim)

VII. Modal Testing of the Wheel

Wheel is composed of two members including the rim and the tire. First one has fully linear behavior and the other one has almost linear behavior in low frequencies. These two members are assembled and this assembling cause that wheel shows linear behavior in 0-500 Hz frequency band. In this section the effect of frequency response of the tire and the rim on the wheel frequency response is studied and analyzed.

VIII. Experimental Setup

The Wheel was hanged from a metal structure in free-free condition by flexible sling as shown in figure 7. Because wheel is relatively heavy, flexible slings behave as less rigid components and can't fulfill free-free condition. Thus in addition to flexible sling soft plastic like cushion is located under the tire for fulfilling free-free condition. In this case also a plastic tip on top of the hammer and screw and small aluminum plate were mounted on the tire to decrease non-linear effects at the point of impact.

Fig. 7. Experimental set-up of the wheel. Excitation in tangential direction.

The rim is expected to behave almost as a rigid body in this frequency range. Therefore, the responses of the rim are measured only at eight points. As shown in Figure 8 wheel is excited at points 1 on the tire in radial and tangential directions and at point 17 on the rim (in axial direction), and an accelerometer is installed at 24 points of wheel, 16 points on perimeter of the tire and 8 points on the rim center. The responses were measured in radial, tangential and axial directions. Accelerometer used for the response measurement is a three-directional accelerometer. Excitation frequency is considered in the range of 0-400 Hz. This is a MIMO (multiple-input, multiple-output) approach, because the structure is excited at two points, and then the response is measured at some other points.

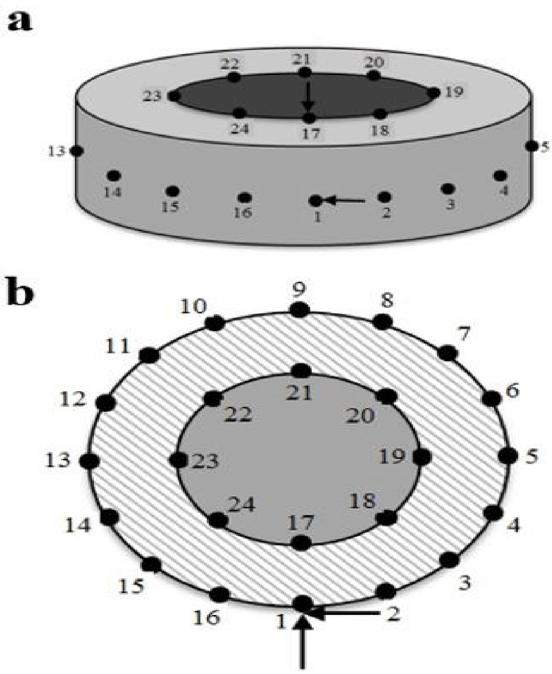

Fig. 8. Experimental setup of the wheel showing position and orientation of the force excitations: a) 3D view b) top view.

Figure 9 shows the frequency response function at point 1 in radial direction, when excitation occurs at point 1 in tangential direction. The FRF shows seven resonances in the frequency range 0-400 Hz. All of clear resonances depicted in figure 8 are related to mode shapes of in-plane vibration. Therefore other resonances are not obvious in this figure. For observing lateral mode shapes FRF diagram in axial and tangential directions are also studied.

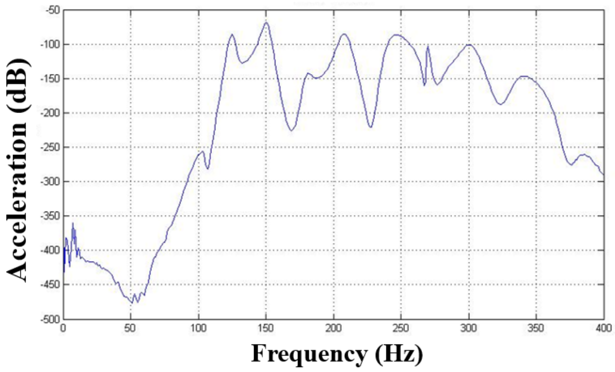

Fig. 9. Measured wheel's FRF in radial direction at point 1 due to a tangential excitation at point 1 (Free wheel)

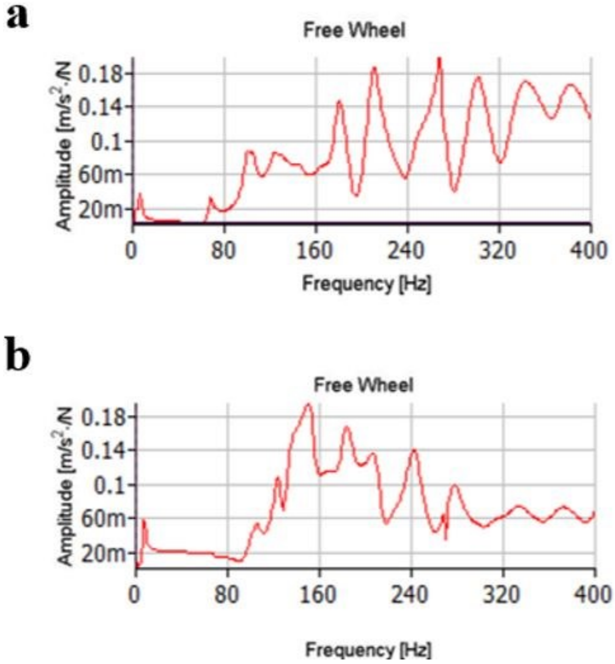

Fig. 10. Measured wheel's FRF at point 3 due to a tangential excitation at point 1 (a) Measured in axial direction (b) Measured in tangential direction.

Second resonance occurs at 105 Hz is related to a torsion mode. Amplitudes of first and second modes are much less than other modes.

IX. Numerical Model

In this section finite element modeling of the ring, tire and wheel which can accurately predict dynamic response of the car wheel in frequency range of 0-500 Hz are introduced. The finite element (FE) model is made using Hyper Works and FE analysis is performed using MD-

Nastran. Model consists of three components namely rim circumference, rim center and tire.

Rim model is made from steel with Young's modulus of 210 GPa and Poisson's ratio of 0.3 and density of 7900 kg/m^3. The model does not include a damping definition. Modeled rim mass equals 6.1 kg which equals to the measured total mass with 100 gr tolerance. Rim is made of two bi-directional PShell elements. Circumferential element has 2.5 mm thickness and the rim center thickness is 3.5 mm. Geometry of the rim is modeled in detail since it significantly affects dynamical behavior of the structure. Geometry and shape of the rim center is more important than the rim circumference on hardness of the model. Appropriate constraints are considered in order to model connections between the rim circumference and rim center.

A radial tire is consisted of radial plies that are supported by a steel reinforced belt and tread layer. Since tires are fabricated by non-linear material and they are bonded together using different methodology like pressing or adhesives, providing a simple numerical model which can accurately predict its dynamical behavior is difficult. Moni-Ryolin or Neo-Hookean material models can be used to describe the tire material model because tire excitation frequency is considered in the range of 0-400 Hz. In this study Neo-Hookean model is applied such that the material behavior is independent of frequency. Neo-Hookean hyperplastic model explains the non-linear materials behavior. Young's modulus for this material is defined as follow:

$$E'' = E'(1+i\eta) \quad (1)$$

E' is referred to the storage modulus, is the component of the stress-strain ratio in phase with the applied strain. η is loss factor and its value for tire is 0.15 [1]. The elastic storage modulus equals 32 MPa.

Density of tire is 650kg/m3. Mass of the modeled tire is equal to 6.14 kg with 100 gr tolerance. Its Poisson's ratio is 0.45 [1]. Three-directional solid elements are used to develop the tire model. Figure 11 shows final model of the tire with tetrahedral and hexagonal elements. Average size of the elements is 10 mm. The tire model has acceptable accuracy in the low frequency range.

Wheel numerical model is prepared by assembling rim and tire models. Appropriate connections are considered between the rim and tire to model the whole wheel structure accurately. Total weight of the wheel model equals 12.34 kg.

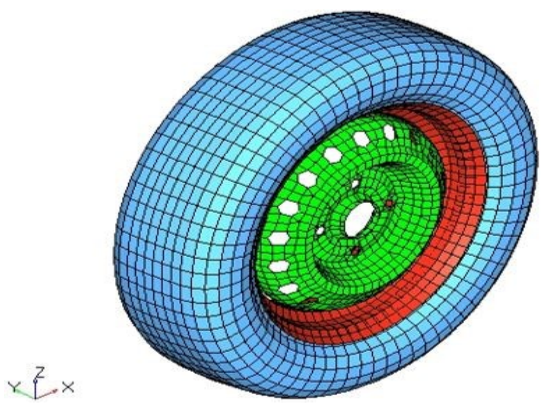

Fig. 11. Finite element model of wheel

X. Results and Discussion

A system of mode shapes requires a matrix with two arrays (n, a) where "n" is circumference index and "a" is belt cross sectional index like what is shown in figure 12, where n equals half of the number of nodes in the mode shape.

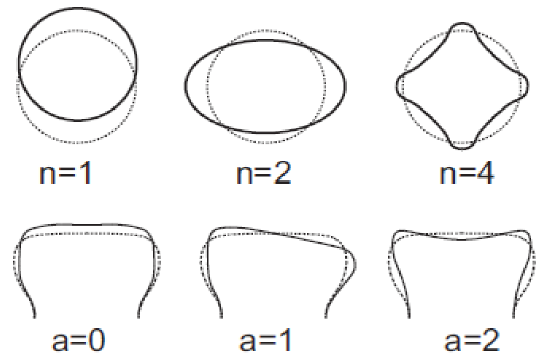

Fig. 12. Naming convention for tire structural modes (n, a) [1]

First six rim modes are related to rigid modes, three of them are related to movement along coordinate axes and three others to rotation around coordinate axes. In these modes, frequency is very low and negligible. Comparing seventh to thirteen mode shapes of rim shows that numerical model resonance frequencies and experimental results have an error of less than 5%. Resonance frequency of the rim, tire and wheel are listed in Tables 1-3 respectively.

Table 1. Comparing the rim's experimental and numerical results

Mode number	Resonance frequencies experiment	Resonance frequencies of model	Mode shape	Error percent
seventh	245	245.9	bending	0.3
eighth	251	257.1	torsion	2.4
ninth	671	692	(3,0)	3.1
Eleventh	1210	1178.1	(4,0)	2.6
thirteenth	1390	1328.3	axial	4.4

Table 2. Comparing the tire's experimental and numerical results

Mode number	Experimental esonance frequencies (Hz)	Resonance frequencies of model (Hz)	Mode shape	Error percent
seventh	41	43.2	(2,0)	5.3
eighth	60	57.5	(3,0)	4.1
ninth	80	84.7	(4,0)	5.8
tenth	138	145.3	(5,0)	5.2
eleventh	190	202.1	(6,0)	6.3

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Table 3. Comparing the wheel's experimental and numerical results

Mode number	Resonance frequencies experiment	Resonance frequencies of model	Mode shape	Error percent
seventh	69	69.5	axial	0.7
eighth	99	91.2	bending	7.8
ninth	101	92.8	torsion	8.1
tenth	124	124.2	(2,0)	0.2
Eleventh	151	154.3	(3,0)	2.1
Twelfth	180	185.6	(4,0)	3.1
Thirteenth	213	210.1	(5,0)	1.3
Fourteenth	246	265.4	(6,0)	7.8

Now mode shapes of the rim, tire and wheel that are computed by the numerical model need to be discussed. Figure 13 shows mode shapes of the rim. First flexible mode shape of the rim is a bending mode and second mode is torsional. These two modes are not in-plan modes, but third to sixth modes occur in circumference plan. Seventh mode is an axial mode where the rim center moves faster than the rim in lateral direction.

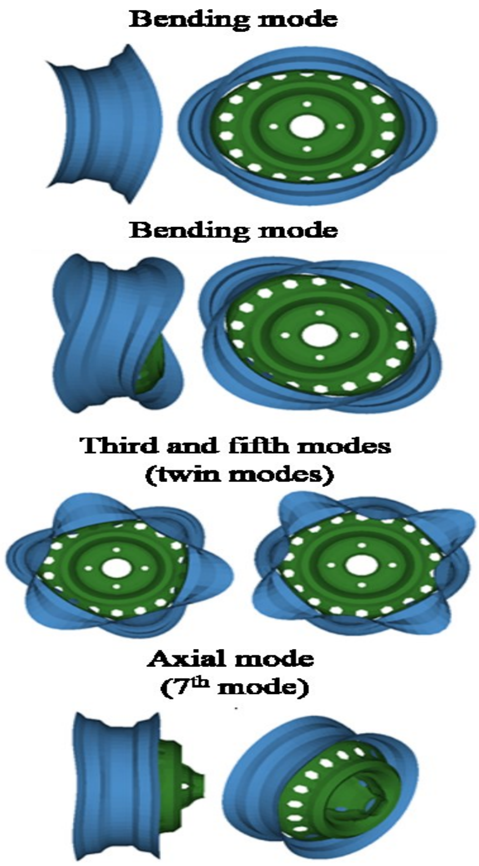

Fig. 13. Mode shapes of rim numerical model.

Figure 14 shows the mode shapes of the tire. Due to the use of one-dimensional accelerometer, responses are measured only in radial direction. Therefore just in-plan vibrations are studied for the tire. Its means that only (n, 0) modes are considered. Since their amplitudes are bigger than other modes and so they are principle modes of the tire.

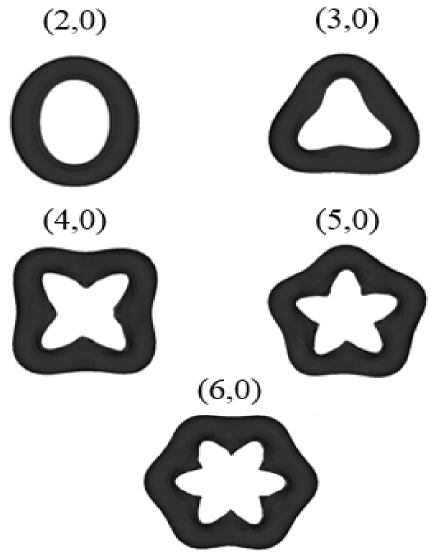

Fig. 14. Mode shapes of tire numerical model.

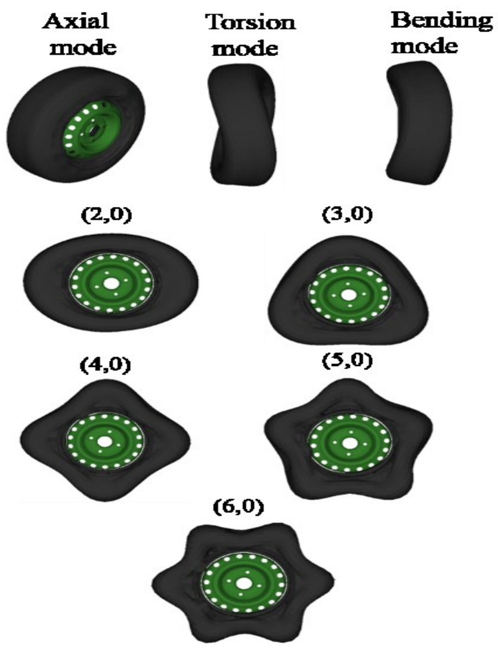

Fig. 15. Mode shapes of wheel numerical model.

As observed in figures 13 to 15 assembling ring and tire produce mode shapes which comparing to those of tire they are shifted to the higher frequencies. As shown in figure 15 first mode shape is axial mode, which it is not belong to tire mode shapes or rim mode shapes. In fact it happens for rim lateral vibration of the tire. So if the connection between tire and rim is much stronger, this mode will occur at higher frequencies. In frequencies studied in this paper rim behaves like a rigid body, therefor indeed wheel mode shapes are like tire mode shapes.

Second mode is a torsion mode and third mode is a bending mode. In these two modes, tire assembling on the rim has led to that first mode of tire increased amount to 60 Hz in wheel because tire assembling on the rim and inflate this collection increase stiffness. Fourth to eighth modes occur in circumference plane. Fourth mode increases from 41 Hz in tire to 124 Hz in wheel, because rim stiffness is more than tire which delays resonance to higher frequencies. This mode is a double mode with little difference in frequency. It is the first double mode of the tire. Similarly all of resonance frequencies are shifted to 80 to 100 Hz higher than tire. All higher order modes are double; they have similar shapes and frequencies.

XI. Conclusion

In this paper an automotive wheel vibration behavior which is very important in NVH design is studied using experimental and numerical modal analysis on a typical rim, tire and wheel. If the wheel structure is designed appropriately in early design phase, the cost and time of other design process would be significantly reduced. A facile and reliable FE model is proposed to analyze and predict NVH performance of a typical car. Neo Hookean model is used to model dynamical properties of the tire. The developed finite element mode is verified with experimental results via comparison of resonance frequencies and mode shapes. The amount of error is less than 5%. Comparison between the model and the experiment has been done via resonance frequencies and frequency response functions. In frequency range 0 – 400 Hz for tire and wheel and 0 – 1600 Hz for rim FRFs are exhibited Smooth, accurate and clear, therefore mode shapes have been obtained by measuring the responses. This validated model has a lot of application in NVH optimization of the automotive body structure in early design phases.

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