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A Bayesian Approach to Process Model Evaluation in Short Run SPC

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***Abstract* — With the availability of Big Data in manufacturing, historical data to initially characterize a process is available in abundance. In fact, evaluating and selecting the best-fitted data set replaces data availability as major concern for setting up a short run SPC. We argue that due to the constant rise in computing power, it might not always be necessary to decide on one specific data set for a priori process characterization and modelling, but instead do most of the evaluation a posteriori. Thus, we introduce a new method to combine expert knowledge and Bayesian statistics for short run SPC in data-rich manufacturing environments. After a discussion on the methodology, its applicability and convergence, its application to turbine blade manufacturing is presented.**

***Keywords* — Bayesian Statistics, Big Data, Quality Assurance, Statistical Process Control, Short Run SPC.**

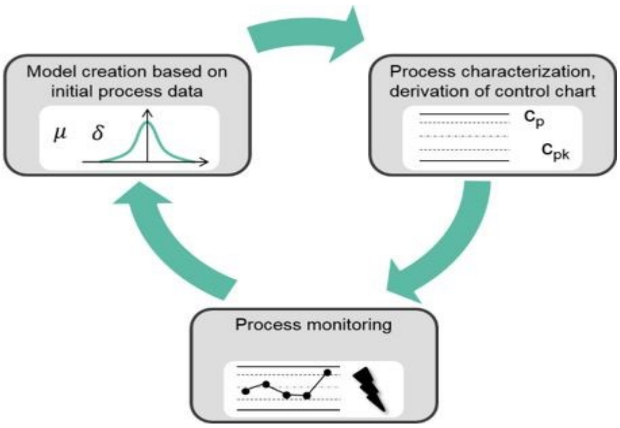

Fig. 1. The three major steps for Statistical Process Control SPC

I. Introduction

Statistical Process Control (SPC) was developed by Walter Shewhart in the 1930s and has since become one of the most important and commonly used tools in quality management. Applications vary widely and can be found from manufacturing [1] to service [2] or even food production [3]. Small batches and lot sizes depict one of the boundaries for SPC as there is only little data which can be used to gain statistical insights into the process. Due to an increasing digitalization and datafication in manufacturing, big data approaches can help to overcome this information gap [4]. Still, when too many data sets are available or might even support contradictory hypothesis, a way to distinguish the most useful data set needs to be identified. This contribution thus introduces a new methodology to evaluate differing data sets and process models for SPC for small batch a posteriori using a Bayesian approach.

II. Statistical Process Control

Statistical Process Control has been described widely in the scientific literature. Thus, only a very brief introduction will be given here to familiarize the reader with the necessary background on which the following argumentation is based. For a detailed introduction to SPC, please refer to e.g. [5,6] or [7] and [8].

The main goal of SPC is to find special causes disturbing the normal process behavior and resulting in non-conforming parts. Typically, three major steps can be distinguished for the application of SPC, as depicted in figure 1:

1) First of all, data of the process needs to be acquired and transformed into a process model specifying how the process looks like under statistical control. This is typically done using distribution functions (e.g. standard normal, F-, or Chi-squared distribution) or just by using standard parameters such as process mean and standard deviation to describe the normal process location and the process variation.
2) Based on these findings, the process is characterized with respect to its capability, expected failure rate, failure costs etc. A control chart is set up using control limits and additional out-of-control indications like runs or trends to detect special causes like shifts in the process location or variance.
3) The process is monitored with respect to these control limits and to these control criteria.

Depending on the cited source, a typical lot size for step (1) is 50 to 200 measurements. With growing lot size, the uncertainty in the estimation of the process parameters e.g. the mean and standard deviation decreases. When only small sample sizes are available, e. g. in small batch or short run production or in destructive testing, the number of available information is not enough to derive a good-enough process model as described in step (1). Therefor, if it is not possible to determine for certain if a process is under statistical control and behaves normally, it cannot be defined and recognized when special causes affect the process. Numerous different approaches have been introduced to overcome this problem, which will be discussed in the following chapter.

III. Literature Review

Three main different approaches to apply SPC for short run production have been identified in the literature review, each again with several differing concretizations – The Increase of the sample size using data from similar processes, the integration of uncertainty in the control procedure and the narrowing of the control focus on remarkable deviations. Each will be presented briefly in this chapter.

As pointed out, the number of available data sets for short run production is too small. On the other hand, comparable processes will have been carried out e.g. on the same machine or by the same technology. Thus, one solution is to focus on the process and not the product and to combine data sets from comparable processes as already stated in [7]. Basically two different approaches can be distinguished here – the *deterministic* and the *statistic* way of selecting data. Several authors presented deterministic approaches as how to select data based on expert knowledge and classification of processes in order to define "similarity" of processes. E.g., [9] used the Opitz-scheme to build subgroups of data. A similar example can be found in [10]. Instead of relying on deterministic classification, data sets can also be selected using statistical methods like cluster analysis, ANOVA or based on their statistic characteristics, e.g. their standard deviation or the average run length, as demonstrated by [11], [12] or [13].

a) *Integration of Uncertainty into the Process Control*

One of the first authors to focus on the problem of short run SPC was Hillier, who suggested to accept uncertainty caused by very small data sets in the beginning of the control process by allowing for much wider control limits. In the course of the process, more data points are generated und thus the control limits tightened [14] and [15]. A similar approach has been presented by [16] with a case study for nuclear weapons. An analysis of the risk involved with this uncertainty can be found in [17] or [18].

b) *Focus on Noticeable Deviations*

Instead of initially building a process model, a simplified process control could also focus on outstanding deviations between two process steps. E.g. if the deviation between process data point five and six has been three times as high as the average deviation from product one to five, a problem in the process could be the root cause. [19] was the first to introduce this kind of approach, using self-defined Q-values (see also [20]). Comparable approaches such as EMWA – exponentially weighted moving average – have since been introduced by [21], [22] and [23], with a good overall overview provided by [24]. [25] used an Artificial Neural Network for this kind of deviation detection, [26] combined the mean and variance into one value introducing a new method called ABS Sequential Probability Ratio Test (SPRT).

It should be pointed out that approaches focusing on noticeable deviations (group "c") do not provide a sufficient process model needed to e.g. estimate a process capability or detect patterns such as shift in mean and others. Thus, this class of approaches helps only to indicate faults in processes that lead e.g. to a shut down. Group b) provides a full SPC, but only with high uncertainties that make a control strategy in the first instances (e.g. product number four or five) almost impossible. In contrast to that, group a) provides a full process model already at hand from the first product onwards. On the other hand, the reliability of the control and failure detection highly depends on the quality of the data used to initially characterize the process and design the process model and therefore on the quality of the grouping procedures. Nevertheless, more and more data becomes available today due to a new trend in manufacturing: big data.

In Germany, approximately 98% of all manufacturing companies are using digital systems e.g. for Enterprise Resource Planning (ERP). About 80% of these systems are used in the area of manufacturing, recording data on processes, quality and many more. As pointed out in a study by McKinsey [4], approximately two per cent of all data worldwide are created in the area of manufacturing. Thus, the basis for process model development for small series production is growing rapidly due to better data availability and exchange. Numerous authors already pointed out the extraordinary value of data for manufacturing in the near future, as can be found in [27], [28] or [29].

On the other hand, big data is characterized by the so-called 5 Vs: volume, variety, velocity, veracity and value [30]. In the case of mining big data sources for SPC in short run production, veracity depicts a severe thread to the model development: What needs to be done if data from source A indicates a different process model (e.g. with respect to mean and capability) than data from source B?

IV. Bayesian Statistics for Short - Run SPC

In the following sub-chapter, we would thus like to introduce a Bayesian approach to overcome conflicting data for short run SPC. After a brief introduction to the basics of Bayesian statistics and its differences to classical approaches, the current problems in model selection for short-run SPC in a big data environment are presented. In a last step, an approach to overcome these will be introduced.

$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)} \tag{1}$$

A. Introduction to Bayesian Statistics

Classical statistical approaches create assumptions based on all available data a priori to a certain event. Knowledge is gathered up front and transferred into a model of the investigated process, e.g. using a standard distribution function. In the subsequent steps, new data points are interpreted according to this a priori model.

In opposition to that, the cleric Thomas Bayes developed a different approach to estimate likelihoods in his 1763 "An Essay towards solving a Problem in the Doctrine of Chances": Based on some (probably imperfect) prior knowledge, an a priori likelihood is estimated. Afterwards, taking in new observations, this distribution or likelihood estimation is refined with every piece of information (new observations) that becomes available in the course of an experiment. Thus, the a posteriori likelihood of observation B under the assumption of hypothesis A is calculated using the it's a priori estimation P(A), the likelihood of B if A is correct P(B|A) and the sum of all probabilities of the observation of B P(B) (see formula 1).

Bayes used this approach to iteratively calculate the likelihood of the landing of a ball on a billiard table starting

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with a 50-50 estimation, refining this mode with every new throw. Few years later, Laplace developed a similar method without knowing Bayes while working his way through the French birth registers. Today, the approach is used in a wide array of applications from google's phrase completion to search algorithms and game theory [31]. In the following chapter, we would like to introduce a method of using Bayesian statistics for model selection for production control in big data environments.

B. Problem Description

For short-run SPC, the usage of trial runs to gain enough insights on a process to state control limits and capabilities (step 1 and 2) is not feasible due to too little overall measurements. Hence, historical data is used very often to gain an a priori process model (compare (a) in the state of the literature). As described, the availability of historical data to derive a process control strategy depicts only a minor problem in todays' manufacturing environment. Instead, a multitude of data points could be used to characterize a process.]

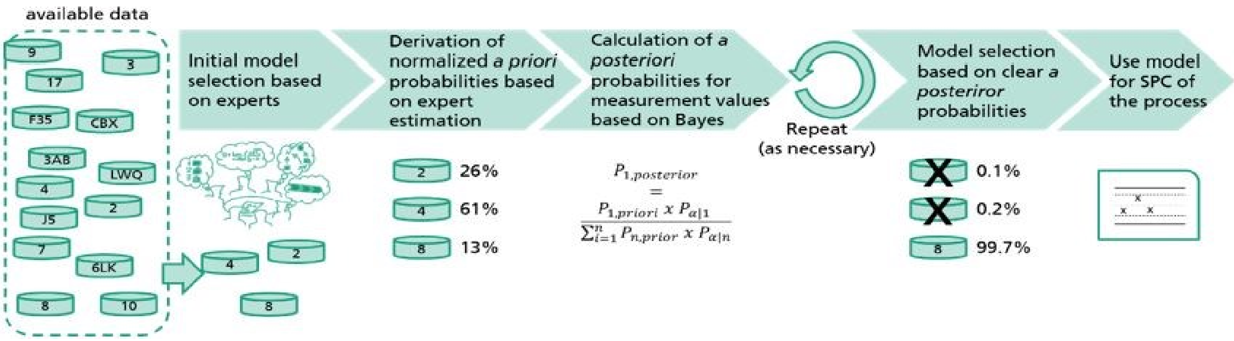

Fig. 1. Proposed methodology.

As seen in the literature review, experts are very often consulted to decide which data sources and groups to include into the characterization of a process. Based on technical similarity, machine characteristics, tools in use, materials and many more, this data selection can already be achieved today. Especially for Computer Aided Quality Management Systems, such a pre-selection of historical data points is common practice [32].

Still, when gathering historical data sets from which to derive a process model for a short-run SPC, conflicting predictions towards the process capability, mean variation etc., might arise. More often than not, several different data sets could be used to characterize a process, but are not compatible from an expert point of view. Typically, two solutions come to mind:

1) Focus on one group a data that seams most plausible or
2) Merge data from different runs and groups.

While (1) inherits the danger of leaving out important data due to an overconfident expert, (2) typically widens the expected mean variation and in the worst case lease to comparing pears to apples. Instead, we propose a different approach, depicted in the next sub-chapter.

C. Methodology

Instead of either merging a larger sub-set of probably comparable data sets or choosing one plausible group, we propose the procedure introduced in figure 2: Based on a wide range of historical data, all plausible sub-groups should be considered for the short-run SPC, but kept in their own sub-groups. Afterwards, the comparability / similarity of these subgroups to the process in question should be estimated using expert knowledge. As depicted in [9], this can e.g. be based on the Opitz-scheme as well as technical characteristics such as machines, tools, machine parameters etc.

Applying this "expert filter" will lead to a smaller sub-group of data sets, each reflecting a different process model with respect to e.g. mean value and variation. All of these different models should subsequently be used for the control of the process. The initial technical similarity from the expert evaluation should be used to calculate the a priori likelihood of the particular model. These reflect the initial probability that the particular model is the correct one to describe the behavior of the process currently under surveillance.

Regrading the calculations, it is assumed that we wish to select the model with the highest probability out of a class of models M_j $(j = 1, \dots, N)$, which are expressed as probability density functions or probability mass functions (pdf, pmf) $f_j(x)$. The pdf or pmfs are estimated from the data-sets of existing "similar" processes. On the other hand an experts judgement U on the probability of each model is given, which is expressed as a prior probability $P(M_j|U)$. Note that the prior probabilities have to be normalized, satisfying the condition

$$\sum_{j=1}^{N} P(M_j|U)\, p(D|M_j,U) = 1.$$

If we now observe data D from the new production process, Bayes' theorem can be used to obtain the probability of the model conditional to the observed data and the experts judgement $P(M_j|D,U)$ as described by Beck and Yuen [36]:

$$P(M_j|D,U) = \frac{P(M_j|U)\, p(D|M_j,U)}{\sum_{j=1}^{N} P(M_j|U)\, p(D|M_j,U)} \quad j = 1,2,\dots,N \qquad (2)$$

where the factor $p(D|M_j,U)$ is the evidence or likelihood of the model M_j given the data D. Note that likelihood is independent of U [34] and therefore $p(D|M_j,U)$ reduces to

$p(D|M_j)$. If D consists only of one observation x_i, then $p(D|M_j) = f_j(x_i)$.

This means the a posteriori probability $P(M_j|D,U)$ of a particular model M_j for being the correct one to predict the behavior of the process under the new surveillance is now known. Note that, if one wishes to select the model with the highest probability only the term $P(M_j|U)\, p(D|M_j,U)$ is relevant. The procedure can be iterated for every new observation $D_i = x_i$ by setting $P_i(M_j|U) = P_i(M_j|D_i,U)$, resulting in:

$$P_{i+1}(M_j|D_{i+1},U) = \frac{P_i(M_j|U)\, p(D_{i+1}|M_j,U)}{\sum_{j=1}^{N} P_i(M_j|U)\, p(D_{i+1}|M_j,U)} \quad (3)$$

The convergence of the posterior depends on the model assumptions and the distribution of the data. Under regular conditions, the different probabilities of the particular models for matching the observed behavior should merge towards favoring one particular model according to the Bernstein-von Mises theorem. For a detailed discussion of the conditions and assumptions for convergence, see [35] or [36].

V. Validation Example

The machining of blades for stationary gas turbines is a typical example for short-run SPCs: A standard set of blades consists of 30 to 50 units. Due to low overall production volume and high variety in size and performance, the same machines are utilized for a large variety of parts, which in turn are only produced once every few weeks or even months. Especially for low-running gas turbine designs, blades might only be machined once in every few weeks. Thus, classical SPC has not been applied yet, but some kind of process control would benefit the overall process reliability and stability: As one single blade can cost more than 10.000 €, losing parts due to a process out of tolerance is not acceptable.

For certain types of probably similar blade types, the manufacturing values for the characteristic of the so-called “assembly platform” have been obtained. For the critical characteristic “interlocking foot length” of one certain blade size and type with a design value of 6.77 mm and a rather narrow tolerance width, the comparability of four other product data sets have been discussed with milling process experts. Based on the factors »location and manufacturing system«, »process«, »machine type«, »blade material«, »tool«, »design value of the characteristic«, »tolerances«, »cutting speed / chipping volume« as well as »clamping«, an initial comparability was determined, based on interviews and discussions with these experts. The relative comparability of data sets to the one that should be explained ranged from 50% to 89% (see figure 3), the fourth data set was excluded due to lower expected explanation power.

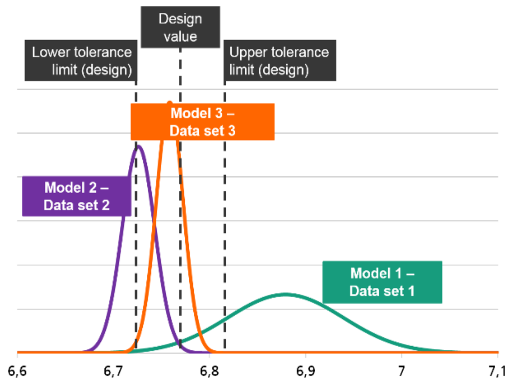

Fig. 3. The three exemplary process models based on the expert selection.

Each data set was transferred into a process model, using the assumption of normal distributions. Based on these discussions, three data sets (comparability ranging from 55% to 89%) were chosen to be used as possible process models. In a first step, these have been normalized, leading to the a priori probabilities $P_0(M_j|U)$ in table 1. Afterwards, the measurement values of the machining of the interlocking foot length have been obtained and used to calculate the posterior probabilities of the three process models.

Table 1. A posteriori likelihood of the three models for the first three observations.

model/ data set M_j	$P_0(M_j\|U)$	Measurement values / observations x_i			
		Number i	1	2	3
		Real value (mm)	6,767	6,758	6,765
1	26,3 %	$P_i(M_j\|D_i,U)$	10,9%	1,65 %	0,3 %
2	31,6 %		4,1 %	0,8 %	0,06 %
3	42,1 %		85 %	97,5 %	99,6 %

As can be seen, the Bayesian evaluation of the a priori models with respect to the observed values from the process quickly converged towards one distinctive model: Already after three real measurement values, it became clear that model 3 showed the highest explanation power towards this new – unknown – process and should thus be used for initial estimation of the process control chart. Twenty measurement values were used in total, but did not change anything with respect to model probability.

After 20 observations from the real process, a normal distribution from these data points had been deducted, showing a very high overall overlap with the values from model 3. Had it been used to deduct a control chart, no failure criterion would have been met in the course of the observations, which again matches the results from the measurements: All 20 blades have been produced in the tolerance range. Based on feedback from process experts, it can be deducted that this Bayesian criterion to evaluate the a posteriori likelihood of a certain process modell after few observations would have helped them to set up a control chart earlier and with more certainty than it is done today.

VI. Discussion

Some restrictions and remarks need to be made on the usage of Bayes in this domain:

1) The a posteriori probabilities will converge towards the best fitting model or data set out of the whole. This also applies for data completely out of range. Even if none of the models really fits, the closest one will have the highest *a posteriori* probability. This means that a correct initial model selection is crucial.
2) The *a priori* and *a posteriori* probabilities will always sum up to 100 % and thus provide only a relative distinction between the data sets or process models.
3) The convergence of the a posteriori probabilities has been observed to be slow for two models with a high overlap and statistical similarity (e.g. almost identical mean and standard deviation).

VII. Conclusion

Using the Bayesian approach for model evaluation can be a helpful tool to decide upon a certain data set for statistical process control in short run production. Its applicability has been discussed in theory as well as shown with the validation case of turbine blade manufacturing. This means that even though Bayesian statistics can be a helpful tool to quickly identify the best-fitted data set for the initial estimation of a control chart in sort run SPC, the involvement of technical experts is crucial – in the initial data selection as well as in the interpretation of the intermediate results.

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Eike Permin, born in 1986, earned a PhD in production engineering, a diploma in mechanical engineering and a B.Sc. in business engineering at the RWTH Aachen university. He is currently working as the head of a research department with focus on production quality at the Fraunhofer-Institute for Production Technology IPT in Aachen, Germany. In addition, he is a lecturer for quality management at the RWTH Aachen university. His major research interests are digitalization of the shopfloor, data analytics for production processes and artificial intelligence for production control.

Christoph Voigtmann, born 1985, completed his studies in Mathematics at the University of Stuttgart. He is currently working as research associate at the Laboratory for Machine Tools and Production Engineering WZL of RWTH Aachen, with the chair of Metrology and Quality Management. He is Chairmen of VDI (German Association of Engineers) working group 1.12 – Capabi--lity of inspection processes as well as member of ISO TC 69/ SC4 - Applications of statistical methods in process management. His main area of work is focusing on statistical methods for process evaluation and capability of inspection processes.

Robert H. Schmitt, born in 1961, completed his studies of Electrical Engineering with the specialisation on Communications Engineering at the Technical University of Aachen and became research associate at the Chair for Metrology and Quality Management. His work there focused on production-related Metrology and Communications Engineering in an automated environm--ent. In 1997 Professor Schmitt moved on to MAN Nutzfahrzeuge AG (commercial vehicles) in Munich where he took on leading positions in the fields of Quality and Production. In 2002 he assumed responsibility for the commercial vehicle production in Steyr, Austria. On July 1st, 2004 he was appointed as professor at the Technical University of Aachen. As head of the Chair for Metrology and Quality Management at the Laboratory for Machine he serves as a member of the Board of Directors at the Laboratory for Machine Tools and Production Engineering (WZL) and the Fraunhofer Institute for Production Technology IPT.