# The Prediction on Beijing Rail Transit Passenger Capacity Based on Grey GM (1,1)

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*Abstract* – In the last few years, Chinese rail transit develops rapidly and the passenger capacity is the basement of the rail transit scheme, design and operation so that pressing ahead the passenger capacity prediction plays a significant and critical role. In accordance with the Beijing rail transit passenger volume data from 2008 to 2016, the paper preprocesses the statistics by moving average, launches the grey GM (1,1) prediction model and then judge the model to be high accuracy, practical and reliable through the mean relative error, mean square error ratio, little probability error.

*Keywords* – Rail Transit, Passenger Capacity Prediction, Grey GM (1,1) Model, Moving Average.

## I. INTRODUCTION

With Chinese economic and social growth, urban rail transit has already become one of the important ways for resident trip. The scheme, design, operation, management and many other decisions of the urban rail transit shall rely on the data of the passenger capacity [1]. Hence, it is quite urgent for forming a kind of scientific and rational rail transit passenger capacity prediction model to explore rail transit development law and seize the passenger transport development trend. At present, as for the passenger capacity, the prediction approaches include the regression analysis method, exponential smoothing method and neural network prediction method [2]-[3]. However, due to be affected by lots of factors, the rail transit passenger capacity prediction encounter lots of indeterminacy factors[4] so that the passenger capacity prediction could not be totally accurate and might encounter large risks. The grey prediction model could cope with the systems which are featured by the short time series, small size sample and incomplete information. Thus, based on Beijing rail transit passenger capacity data from 2008 to 2016, the paper adopts moving average to preprocess the data and reduce data's volatility influence. On this basis, the paper also builds up the grey GM (1,1) prediction model to judge model prediction precision level through the mean relative error, mean square error ratio, little probability error.

## II. THE ESTABLISHMENT & SOLUTION OF THE GREY PREDICTION MODEL

### A. Model Principle

The grey prediction theory was proposed by Professor Deng Julong in the 1990s. In line with the grey prediction theory, any systems including the system with incomplete information and small size features must contain certain specific function. And the whole system shall be ordered as well as be equipped with certain insignificant rules. Grey prediction model is launched on the grey prediction theory. Through coping with the original data, the model seeks the system internal connection and development rules, establishes the prediction model about the grey system with uncertain information so that it is widely used in various fields. Grey prediction model enjoys the unique effects to treat the systems which are featured by the short time series, small size sample and incomplete information, could reduce the influence of the randomness effectively, being easy to calculate.

### B. Model Establishment and Solution

The establishment and solution process of the grey prediction model as follows [5]:

(1) Setting $X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), \cdots X^{(0)}(n)\}$ is the nonnegative original sequence, pressing ahead the accumulation on $X^{(0)}$ could gain 1-AGO sequence:

$$X^{(1)} = \{X^{(1)}(1), X^{(1)}(2), \cdots X^{(1)}(N)\}$$

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), k = 1, 2, 3 \cdots n$$

(2) Construct coefficient matrix $B$ and data vector $Y$

$$B = \begin{bmatrix} -\frac{1}{2}(X^{(1)}(1) + X^{(1)}(2)) & 1 \\ -\frac{1}{2}(X^{(1)}(2) + X^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(X^{(1)}(n-1) + X^{(1)}(n)) & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \cdots \\ X^{(0)}(n) \end{bmatrix}$$

(3) Building GM (1,1) grey differential equation

$$\frac{dx^{(1)}}{dt} + a x^{(1)} = u$$

$a$ is the unknown parameters, also known as development coefficient; $u$ is the unknown endogenous variable, also known as grey action.

(4) Calculating parameter

$$\hat{U} = \begin{bmatrix} \hat{a} \\ \hat{u} \end{bmatrix} = (B^T B)^{-1} B^T Y$$

(5) Discrete Time Response Function of Grey Prediction

$$\hat{X}^{(1)}(k+1) = \left[ X^{(0)}(1) - \frac{\hat{u}}{\hat{a}} \right] e^{-\hat{a}k} + \frac{\hat{u}}{\hat{a}}$$

(6) Pressing ahead inverse accumulated generating operation (IAGO)

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(k)}$$

(7) Accuracy Test
Residua Error:

$$E(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), k = 2,3, \cdots n$$

Relative Residue:

$$[x^{(0)}(k) - \hat{x}^{(0)}] / x^{(0)}(k), k = 2,3, \cdots n$$

Mean Value of $x^{(0)}$:

$$\overline{X} = \frac{1}{n} \sum_{k=1}^{n} x^{(0)}(k)$$

Variance of $x^{(0)}$:

$$S_1 = \sqrt{\frac{1}{n} \sum_{k=1}^{n} [x^{(0)}(k) - \overline{X}]^2}$$

Mean Value of Residual Error:

$$\overline{E} = \frac{1}{n-1} \sum_{k=2}^{n} E(k)$$

Variance of Residual Error:

$$S_2 = \sqrt{\frac{1}{n-1} \sum_{k=2}^{N} [E(k) - \overline{E}]^2}$$

Posterior Error Ratio:

$$C = \frac{S_2}{S_1}$$

Little Error Probability:

$$P = P\{|E(k) - \overline{E}| < 0.6745 S_1\}$$

Correlative verification precision refers to Tab. 2.1

Tab.2.1 Grey Prediction Verification Precision

| Prediction Precision Grade | P | C |
|---|---|---|
| Good | >0.95 | <0.35 |
| Qualified | >0.80 | <0.45 |
| Satisfactory | >0.70 | <0.55 |
| Failed | ≤0.7 | ≥0.65 |

## III. EXAMPLE ANALYSIS

A. *Pretreatment of Data*

Due to the fluctuation of the data, in order to reduce the influence of fluctuation, increase smoothness, enhance sample trend, and decrease errors, it adopts the moving average to preprocess the sample data. The data processing formula is as follows [6]:

$$x'^{(0)}(t) = \frac{x^{(0)}(t-1) + 2x^{(0)}(t) + x^{(0)}(t+1)}{4}$$

$$x'^{(0)}(1) = \frac{3x^{(0)}(1) + x^{(0)}(2)}{4}$$

$$x'^{(n)}(t) = \frac{x^{(0)}(n-1) + 3x^{(0)}(n)}{4}$$

The treated data refer to Tab 3.1.

Tab. 3.1 Pretreatment Results of Beijing Rail Transit Passenger Capacity[7]

| Year | Rail Transit Passenger Capacity | Rail Transit Passenger Capacity Pretreatment |
|---|---|---|
| 2009 | 142268 | 152860 |
| 2010 | 184645 | 176700 |
| 2011 | 219280 | 208630 |
| 2012 | 246162 | 236780 |
| 2013 | 320469 | 299550 |
| 2014 | 338668 | 328890 |
| 2015 | 332381 | 331510 |
| 2016 | 365934 | 357330 |

B. *Passenger Capacity Prediction & Analysis*

Applying the prediction data in Tab.3.1, the paper adopts the grey prediction method to treat and predict Beijing rail transit passenger capacity. The specific prediction results refer to Tab. 3.2 and Fig. 3.1.

Tab. 3.2. Prediction Value of Beijing Rail Transit Passenger Capacity.

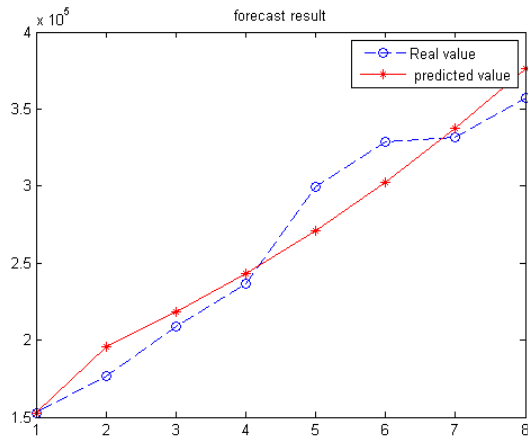| Year | Passenger Capacity | Grey Prediction Value | Residual Error | Relative Residual Error |
|---|---|---|---|---|
| 2009 | 142268 | 152860 | -10592 | -7.45% |
| 2010 | 184645 | 195490 | -10845 | -5.87% |
| 2011 | 219280 | 218010 | 1270 | 0.58% |
| 2012 | 246162 | 243130 | 3032 | 1.23% |
| 2013 | 320469 | 271140 | 49329 | 15.39% |
| 2014 | 338668 | 302370 | 36298 | 10.72% |
| 2015 | 332381 | 337210 | -4829 | -1.45% |
| 2016 | 365934 | 376050 | -10116 | -2.76% |

Fig. 3.1. The True Value and Prediction Value of Beijing Rail Transit Passenger Capacity

(1) In accordance with the residual error data in Table 3-2, it could calculate the mean relative error $\overline{E}$ and confidence precision $P$.

$$\overline{E} = 0.0032 < 0.01, P = 99.68\%$$

(2) Computing mean square error ratio $C$

$$C = \frac{S_2}{S_1} = 0.0559 < 0.35$$

(3) Calculating little probability error P

$$P = P\{|E(k) - \overline{E}| < 0.6745S_1\} = 1 > 0.95$$

In conclusion, the model's prediction precision is good so that it could be used to predict Beijing rail transit passenger capacity. It could calculate $a = -0.1090$ and $u = 168360$ through MATLAB and gain the GM (1,1) model:

$$\hat{X}^{(1)}(k+1) = 1697447e^{0.109k} - 1544587.156$$

## IV. CONCLUSION

Through Beijing rail transit passenger capacity from 2008 to 2016, the paper makes moving average and then launches grey GM (1, 1) prediction model which is featured by high prevision. With making full use of the original data, it shows that the model could be suitable to forecast rail transit passenger capacity. In line with the data of the model prediction, it could provide necessary support for Beijing rail transit scheme in the future, being practical significance.

Through analyzing the Beijing rail transit passenger capacity by applying grey GM (1, 1)，Beijing rail transit passenger capacity maintains the good growth situation. It is expected that the rail transportation would be one of Beijing major transportation means in the future. The related planning departments shall combine Beijing's prospect urban planning to manufacture a rational construction scheme for rail traffic lines. They could adopt this module to forecast Beijing's future rail transit passenger capacity as well as take land use planning into full consideration so as to guarantee the exploitativeness of the scheme. The rail transit network plan shall be featured by the clear hierarchy, demonstrating the develo--pment trend of passenger capacity, rationally developing the function of transportation junctions as well as improving the passenger capacity ability of urban rail transit system. Making use of transit-oriented development (TOD) plan to drive the rail transit to serve the Beijing residents well.

In general, the combination prediction module with quantitative analysis and qualitative analysis enjoys the better prediction precision and wider prediction application scenarios [8]. This module is full of practical significance so that it could be widely applied into other fields.

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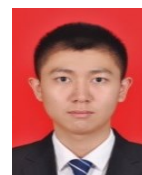
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