International Journal of Engineering Innovation & Research
Volume 8, Issue 1, ISSN: 2277 – 5668

On the Reliability-Based Design Optimization (RBDO) of A Speed Reducer

Muhammad Adib Uz Zaman
Corresponding author email id: a_u_z_ipe@yahoo.com

Date of publication (dd/mm/yyyy): 15/01/2019

***Abstract* – This paper proposes a modified probabilistic procedure for deriving an ultimate strength and strain design model for a speed reducer. Reliability-Based Design Optimization (RBDO) techniques have been used. A prominent decoupled method for RBDO is presented in this paper. To test the proposed method, a mathematical RBDO formulation of a speed reduced was studied and solved. Considering the complex nature of the problem, some constraints were simplified. Two popular solution methods known as Sequential Quadratic Programming (SQP) and Interior Point (IP) were implemented and compared against each other to check for consistency and competitive advantages. This study can be further extended to other steel structures that require robustness in their design parameters to ensure safety and reliability.**

***Keywords* – RBDO, Reliability, Robustness, SQP, IP, Speed Reducer.**

I. Introduction

Structures in areas of high seismic risk may be susceptible to severe damage in a large earthquake. Older structures designed to meet the code criteria of the time of their construction may be at even greater risk. In many cases, these structures have deficiencies in lateral strength and/ or ductility when evaluated with respect to current code criteria. Life safety and financial considerations make many of these structures viable candidates for retrofit seismic strengthening. The buildings often had perimeter frames to resist lateral forces and a light interior slab system to carry gravity forces. The perimeter frames often had deep, stiff spandrel beams and short, flexible columns with relatively small cross sections. Such a system was popular because the deep spandrels became the exterior walls and directly formed the window openings. Due to the relative capacities of the spandrel beams and short columns, the governing failure mode under lateral load is likely to be shear failure of the columns. This failure mode is accompanied by little warning and endangers the gravity-load-carrying system. Compounding the problem, the lateral forces for which older structures are designed may be as little as half those calculated using current seismic codes. A number of researchers have investigated various techniques such as infilling walls, adding walls to existing columns, encasing columns, and adding steel bracing to improve the strength and/or ductility of existing buildings (See [1] - [3]). In the present paper, results of tests of a two-thirds-scale frame strengthened by adding an exterior structural steel bracing system are presented. The bracing system was designed to improve the seismic performance of the frame by increasing its lateral stiffness and capacity. Through addition of the bracing system, load could be transferred out of the frame and into the braces, bypassing the weak columns while increasing strength.

It was desirable to construct and test a large-scale model to evaluate connection behavior and practical aspects of construction. Connections of strengthening elements to the existing structure have an important influence on the behavior of the strengthened system (See [1]). Many seismic strengthening tests have been performed on small scale models; however, concerns exist over whether connection behavior is adequately modeled at small scales.

II. Literature Review

Reliability analysis methods can be divided into simulation methods and analytical methods. Simulation methods, such as Monte Carlo Simulation (MCS), are achieved through realizing random variables and determining whether a particular event occurs for the simulation instance (See [4]). The ratio of the number of failures to the total number of samples is regarded as the probability of failure. MCS is computationally expensive, especially when the probability of failure is low. Analytical methods are commonly used due to their efficiency compared to simulation methods. Analytical methods are generally gradient-based, including the 'worse case' analysis method, the moment matching method (See [5]) and the most probable point (MPP) based method (See [7]). The 'worse case' analysis method and the moment matching method are earlier reliability analysis methods, which are inaccurate when the random variables have large variations. The strategy of searching MPP is more efficient. Reliability index approach (RIA) (see [7] - [11]) and performance measure approach (PMA).

According to the integration strategies of reliability analysis and optimization, RBDO methods can be divided into three categories: double loop methods, single loop methods and decoupled methods (See [12] - [13]). Double-loop methods have two nested loops: the design optimization loop and reliability analysis loop. In each iteration, the design optimization loop repeatedly calls the reliability analysis loop to get the reliability information. The computational cost of double-loop method is prohibitive, especially when the performance functions are highly nonlinear.

Single-loop method (See [14] - [19]) adopts the Karush–Kuhn–Tucker (KKT) conditions to substitute the reliability analysis loop. It is very efficient for RBDO problems with linear and moderate nonlinear performance functions. Decoupled method (see [5], [20] - [24]) conducts design optimization and reliability analysis sequentially. When a new design point is obtained from deterministic optimization, reliability analysis will be performed to assess the reliability of the design and find the MPPs and shifting vectors, then limit state constraints will be moved according to these shifting vectors to convert the RBDO problem into a deterministic optimization problem. The key of the decoupled method is the shifting vector as it directly affects the efficiency and accuracy of RBDO results. Metamodels are also applied in RBDO to substitute the original performance functions. A response surface method (RSM) for RBDO was used in the literature (See [25]). An adaption of symmetric optimal Latin hypercube sampling and kriging model for design optimization under uncertainty was also discussed (See [26]). An RBDO method using the moment method and kriging model is explored (See [27]). The authors applied a constraint boundary sampling method and kriging model for RBDO. A more recent approach with a new sequential sampling method for design under uncertainty was proposed (See [28]).

The most promising structure for RBDO is the decoupled method, such as the sequential optimization and reliability analysis method (See [5]) that is also known as SORA. In SORA, limit state constraints $g_i(X) = 0; i = 1, \cdots, N$ are moved to the feasible region according to the shifting vectors. PMA (also called the inverse reliability analysis method) is usually applied to retrieve these shifting vectors in the original SORA method (See [5] and [20]). However, the shifting vector obtained from PMA is based on the specific performance function $g_i(X) = g_ik$, instead of the limit state constraint gi(X) = 0. If the performance function $g_i(X)$ is highly nonlinear, the shapes of the functions $g_i(X) = g_ik$ and $g_i(X) = 0$ may vary widely; then the shifting vector obtained from PMA will not be accurate to give the optimal moving direction for the probabilistic constraint.

Accuracy of the shifting vector in decoupled method is critical; a large error in the shifting vector may lead to more optimization iterations. In this paper, the optimal shifting vector (OSV) approach will be proposed. A new reliability analysis model will be built, in which the shifting vectors are searched utilizing the limit state functions $g_i(X) = 0$; i = 1, …., N, rather than the specific performance functions $gi(X) = g_i$; $k_i = 1$, …, N. Hence the accuracy of the shifting vectors obtained from the OSV model will not be affected by the nonlinearity of the performance functions.

The new reliability analysis model is conducted in the super sphere design space to reduce its number of constraints and design variables, and it can yield the optimal shifting vectors even for highly nonlinear constraints. These optimal shifting vectors will reduce the number of optimization iterations and constraint function calls, thus enhancing the efficiency of the proposed OSV method.

III. Methodology

There are several theory and concepts that have been used to Reliability Based Design Optimization (RBDO) problem and its solutions. The following terms and illustrations are necessary to completely understand the procedures and methods followed in this paper.

A typical RBDO problem is formulated as follows:

find: d, μ_x

$\min f(d, \mu_x, \mu_p)$

s.t. $\text{Prob}\left(g_i(d, X, P) \geq 0\right) \geq R_i$, i = 1, 2, 3,…,N

$d^{lower} \leq d \leq d^{upper}$, $\mu_x^{lower} \leq \mu_x \leq \mu_x^{upper}$

Where $f(d, \mu_x, \mu_p)$ is the objective function, Prob $(g_i\,(d, X, P) \geq 0)$ is the probability function which denotes the probability of satisfying the i^{th} performance function $g_i\,(d, X, P)$; N is the number of probabilistic constraints, d is the vector of deterministic design variables; X and P are the vectors of random design variables and random parameters; μ_x and μ_p denote the mean vectors of X and P; Ri denotes the denotes the desired design probability of satisfying the i^{th} probabilistic constraint.

Two optimization techniques will be used in this paper to solve and compare the solutions which were tested and validated by Rabiei Hosseinabad and Ahmadian in 2014 (See [6]). Those are sequential quadratic programming (SQP) and Interior point (IP) method. A brief discussion on these methods is presented in the following paragraphs.

The Sequential quadratic programming (SQP) is an iterative method for nonlinear optimization. SQP methods are used on mathematical problems for which the objective function and the constraints are twice continuously differentiable.

SQP methods solve a sequence of optimization sub-problems, each of which optimizes a quadratic model of the objective subject to a linearization of the constraints. If the problem is unconstrained, then the method reduces to Newton's method for finding a point where the gradient of the objective vanishes. If the problem has only equality constraints, then the method is equivalent to applying Newton's method to the first-order optimality conditions, or Karush–Kuhn–Tucker (KKT) conditions, of the problem. SQP methods have been implemented in many packages, including KNITRO, NPSOL, SNOPT, NLPQL, OPSYC, OPTIMA, MATLAB, GNU Octave, SQP and SciPy.

Interior point methods (also referred to as barrier methods) are a certain class of algorithms that solves linear and nonlinear convex optimization problems. John von Neumann suggested an interior point method of linear programming which was neither a polynomial time method nor an efficient method in practice. In fact, it turned out to be slower in practice compared to simplex method which is not a polynomial time method. In 1984, Narendra Karmarkar developed a method for linear programming called Karmarkar's algorithm which runs in probably polynomial time and is also very efficient in practice. It enabled solutions of linear programming problems which were beyond the capabilities of the simplex method. Contrary to the simplex method, it reaches a best solution by traversing the interior of the feasible region. The method can be generalized to convex programming based on a self-concordant barrier function used to encode the convex set.

Any convex optimization problem can be transformed into minimizing (or maximizing) a linear function over a convex set by converting to the epigraph form. The idea of encoding the feasible set using a barrier and designing barrier methods was studied by Anthony V. Fiacco, Garth P. McCormick, and others in the early 1960s. These ideas were mainly developed for general nonlinear programming, but they were later abandoned due to the presence of more competitive methods for this class of problems (e.g. sequential quadratic programming). In a recent study conducted by Rabiei Hosseinabad and Moraga in 2017, they have introduced a novel way of showing non-linearity in a system using system dynamics approach which opened a new path for future researchers looking to investigate on the non-linearity existed within a system caused by variables affecting that system (See [22] & [31]).

The class of primal-dual path-following interior point methods is considered the most successful. Mehrotra's predictor-corrector algorithm provides the basis for most implementations of this class of methods.

IV. Numerical Example

The problem of this paper is mainly concerned with the optimization of the design parameters of a speed reducer considering the reliability-based design optimization (RBDO) approach. A speed reducer shown in Fig. 1 is used to rotate the engine and propeller with efficient velocity in light plane (See [27]). This problem has seven random variables and 11 probabilistic constraints. The objective function is to minimize the weight, and the probabilistic constraints are related to physical quantities such as bending stress, contact stress, longitudinal displacement, stress of the shaft, and geometry constraints. The random design variables are gear width (X_1), gear module (X_2), the number of pinion teeth (X_3), distance between bearings (X_4, X_5), and diameter of each shaft (X_6, X_7).

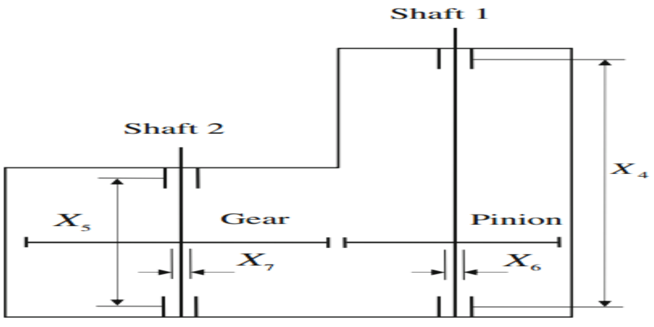

Fig 1. A speed reduced (taken from [27])

V. Mathematical Formulation

The speed reducer mass minimization problem can be formulated as below:

$$\begin{aligned}
&\text{find } d = [d_1, d_2, d_3, d_4, d_5, d_6, d_7]^T\\
&\min\ f(d) = 0.7854 d_1 d_2^2 \times (3.3333 d_3^2 + 14.9334 d_3 - 43.0934)\\
&-1.508 d_1 (d_6^2 + d_7^2) + 7.477 (d_6^3 + d_7^3)\\
&+0.7854 (d_4 d_6^2 + d_5 d_7^2)\\
&\text{subject to } \text{prob}[g_j(X) > 0] \le \varphi(-\beta_j^t), j = 1, \ldots, 11\\
&\text{where}\\
&g_1(X) = \frac{27}{X_1 X_2^2 X_3} - 1,\\
&g_2(X) = \frac{397.5}{X_1 X_2^2 X_3^2} - 1,\\
&g_3(X) = \frac{1.93 X_4^3}{X_1 X_2 X_6^4} - 1,\\
&g_4(X) = \frac{1.93 X_5^3}{X_2 X_3 X_7^4} - 1,\\
&g_5(X) = \frac{\sqrt{(745 X_4/(X_2 X_3))^2 + 16.9 \times 10^6}}{0.1 X_6^3} - 1100\\
&g_6(X) = \frac{\sqrt{(745 X_4/(X_2 X_3))^2 + 157.5 \times 10^6}}{0.1 X_7^3} - 850\\
&g_7(X) = X_2 X_3 - 40,\\
&g_8(X) = 5 - \frac{X_1}{X_2},\\
&g_9(X) = -12 + \frac{X_1}{X_2},\\
&g_{10}(X) = -1 + \frac{1.5 X_6 + 1.9}{X_4}\\
&g_{11}(X) = -1 + \frac{1.1 X_7 + 1.9}{X_5}
\end{aligned}$$

where all the variables are assumed to be non-negative.

The initial design point may be selected as the result of deterministic optimization. All random variables are statistically independent and have normal distributions. The probability values should lie in between 0 to 1 when the probability density functions are integrated to find out the value of areas under the normal curve.

VI. Result and Analysis

This section will deal with the two solution techniques and their side by side comparison. First the SQP method solution output will be analyzed and discussed followed by the interior-point algorithm solution output. The comparison table will clarify the idea on which solution methods are better in terms of the number of iterations, convergence speed and time required for solutions.

The output shown by Table 1 is generated by values obtained from MATLAB.

Table 1. Optimized values obtained by SQP and IP

Variable	SQP	IP
X_1	0.1000	0.1000
X_2	0.6964	0.6964
X_3	0.1000	0.1000
X_4	0.1000	0.1000
X_5	0.1000	0.1000
X_6	0.1000	0.1000
X_6	0.1000	0.1000

Table 2 shows the comparison between the efficiency of the two methods in terms of some metrics.

Table 2. Comparison between two methods.

Method	Iterations	Time (s)	Convergence
SQP	6	61.29	Slower
IP	4	46.61	Faster

The tolerance limit of 10^{-6} was maintained here as well to preserve consistency and feasibility. The solution from both the methods appeared to be same which proves the formulation of the RBDO to be authentic and yields a consistent result.

As we can see from the output, the sequential quadratic programming found a local minimum that satisfied the constraints. The optimized design variables are shown here as well. The values are within the specified limits. The convergence process took a total of 6 iterations and tends to be a bit slower than the interior point method (discussed above). The required time for SQP to complete the process took more than 1 minute. That is due to the 11 probabilistic constraints included in the formulation of the speed reducer RBDO problem.

For IP, the convergence process took a total of 4 iterations and tends to be a bit faster than the interior point method. The process took less than 50 seconds to complete and seemed very efficient to find out the solution swiftly.

VII. Conclusion

There are several scopes to improve the solution generated by these procedures. This study is basically an effort to successfully understand the logic behind these RBDO procedures. The reliability-based designs can be very complex and the existing standard algorithms sometimes fail to solve them effectively. So, the search for robust techniques has been essential. The author is actively looking forward to continuing this research to formulate and solve complex reliability design related problems by modifying algorithms necessary to specific design optimization.

References

[1] Sugano, S., & Endo, T. (1983). Seismic strengthening of reinforced concrete buildings in Japan. *Strengthening of building structures—diagnosis and theory*, 371-378.
[2] Higashi, Y., & Kokusho, S. (1975). The strengthening method of existing reinforced concrete buildings. *US-Japan Cooperative Research Program in Earthquake Engineering, Honolulu HI*.
[3] Kawamata, S., & Ohnuma, M. (1981). Strengthening effect of eccentric steel braces to existing reinforced concrete frames. In *7WCEE Conference, Proceedings of 2nd Seminar on Repair and Retrofit of Structures*.
[4] Sun, G., Li, G., Stone, M., & Li, Q. (2010). A two-stage multi-fidelity optimization procedure for honeycomb-type cellular materials. *Computational Materials Science*, *49*(3), 500-511.
[5] Du, X., & Chen, W. (2001). A most probable point-based method for efficient uncertainty analysis. *Journal of Design and Manufacturing*

automation, *4*(1), 47-66.

[6] Rabiei, E., & Ahmadian, P. (2014). The effects of economic sanctions on target countries over time through mathematical models and decision making. *International Journal of Resistive Economics, 2(4)*, 53-62. Retrieved from http://oajre.ir/the-effects-of-economic-sanctions-on-target-countries-over-time-through-mathematical-models-and-decision-making/.

[7] Hasofer, A.M., & Lind, N.C. (1974). Exact and invariant second-moment code format. *Journal of the Engineering Mechanics division*, *100*(1), 111-121.

[8] Nikolaidis, E., & Burdisso, R. (1988). Reliability based optimization: a safety index approach. *Computers & structures*, *28*(6), 781-788.

[9] Tu, J., Choi, K.K., & Park, Y. H. (1999). A new study on reliability-based design optimization. *Journal of mechanical design*, *121*(4), 557-564.

[10] Grandhi, R.V., & Wang, L. (1998). Reliability-based structural optimization using improved two-point adaptive nonlinear approximations. *Finite Elements in Analysis and Design*, *29*(1), 35-48.

[11] Gasser, M., & Schuëller, G. I. (1997). Reliability-based optimization of structural systems. *Mathematical Methods of Operations Research*, *46*(3), 287-307.

[12] Enevoldsen, I., & Sørensen, J. D. (1994). Reliability-based optimization in structural engineering. *Structural safety*, *15*(3), 169-196.

[13] Valdebenito, M.A., & Schuëller, G.I. (2010). A survey on approaches for reliability-based optimization. *Structural and Multidisciplinary Optimization*, *42*(5), 645-663.

[14] Aoues, Y., & Chateauneuf, A. (2010). Benchmark study of numerical methods for reliability-based design optimization. *Structural and multidisciplinary optimization*, *41*(2), 277-294.

[15] Chen, X., Hasselman, T., Neill, D., Chen, X., Hasselman, T., & Neill, D. (1997). Reliability based structural design optimization for practical applications. In *38th Structures, structural dynamics, and materials conference* (p. 1403).

[16] Kirjner-Neto, C., Polak, E., & Der Kiureghian, A. (1998). An outer approximations approach to reliability-based optimal design of structures. *Journal of optimization theory and applications*, *98*(1), 1-16.

[17] Liang, J., Mourelatos, Z. P., & Tu, J. (2008). A single-loop method for reliability-based design optimization. *International Journal of Product Development*, *5*(1-2), 76-92.

[18] Kharmanda, G., Mohamed, A., & Lemaire, M. (2002). Efficient reliability-based design optimization using a hybrid space with application to finite element analysis. *Structural and Multidisciplinary Optimization*, *24*(3), 233-245.

[19] Shan, S., & Wang, G. G. (2008). Reliable design space and complete single-loop reliability-based design optimization. *Reliability Engineering & System Safety*, *93*(8), 1218-1230.

[20] Agarwal, H., Mozumder, C. K., Renaud, J. E., & Watson, L. T. (2007). An inverse-measure-based unilevel architecture for reliability-based design optimization. *Structural and Multidisciplinary Optimization*, *33*(3), 217-227.

[21] Yin, X., & Chen, W. (2006). Enhanced sequential optimization and reliability assessment method for probabilistic optimization with varying design variance. *Structures and Infrastructure Engineering*, *2*(3-4), 261-275.

[22] Moraga, R., & Rabiei Hosseinabad, E. (2017). A System Dynamics Approach in Air Pollution Mitigation of Metropolitan Areas with Sustainable Development Perspective: A Case Study of Mexico City. *Journal of Applied Environmental and Biological Sciences*, 164-174.

[23] Royset, J. O., Der Kiureghian, A., & Polak, E. (2001). Reliability-based optimal structural design by the decoupling approach. *Reliability Engineering & System Safety*, *73*(3), 213-221.

[24] Ching, J., & Hsu, W. C. (2008). Transforming reliability limit-state constraints into deterministic limit-state constraints. *Structural Safety*, *30*(1), 11-33.

[25] Cho, Y. W., Lee, J. H., Son, H. K., Lee, S. H., Shin, C., & Johns, M. W. (2011). The reliability and validity of the Korean version of the Epworth sleepiness scale. *Sleep and Breathing*, *15*(3), 377-384.

[26] Zou, T., & Mahadevan, S. (2006). A direct decoupling approach for efficient reliability-based design optimization. *Structural and Multidisciplinary Optimization*, *31*(3), 190.

[27] Youn, B. D., Choi, K. K., Yang, R. J., & Gu, L. (2004). Reliability-based design optimization for crashworthiness of vehicle side impact. *Structural and Multidisciplinary Optimization*, *26*(3-4), 272-283.

[28] Mourelatos, Z. P., & Zhou, J. (2005). Reliability estimation and design with insufficient data based on possibility theory. *AIAA journal*, *43*(8), 1696-1705.

[29] Hyeon Ju, B., & Chai Lee, B. (2008). Reliability-based design optimization using a moment method and a kriging metamodel. *Engineering Optimization*, *40*(5), 421-438.

[30] Zhuang, X., & Pan, R. (2012). A sequential sampling strategy to improve reliability-based design optimization with implicit constraint functions. *Journal of Mechanical Design*, *134*(2), 021002.

[31] Hosseinabad E. R., Moraga R. J. (2017). Air Pollution Mitigation in Metropolitans Using System Dynamics Approach, *Institute of Industrial and Systems Engineers (IISE)*, 638-643, Retrieved from http://www.sustainableengineer.org/air-pollution-mitigation-in-metropolitans-using-system-dynamics-approach/.

AUTHOR'S PROFILE

Muhammad Adib Uz Zaman was born in Dhaka, Bangladesh back in 1992. From his childhood, he showed impressive merit and brilliance throughout his academic career. He completed his BS degree in Industrial and Production Engineering (IPE) from Bangladesh University of Engineering and Technology in 2015. He received his MS degree in Industrial and Systems Engineering (ISyE) from Northern Illinois University (NIU), De Kalb, Illinois, United States on May 2018. He is currently a doctoral student in Industrial Engineering at Texas Tech University, Lubbock, Texas, United States. He is presently working as a research assistant in the department of Industrial, Manufacturing and Systems Engineering of Texas Tech University. His research interests are optimization, data analysis, signal/ image processing and lean manufacturing. He is currently working in several projects related to optimization, image processing and additive manufacturing. His previous employment was in Northern Illinois University as a research and teaching assistant. Mr. Zaman is an active member of IEEE, IISE and Alpha Pi Mu. He is a certified (by NIU) Lean Six Sigma expert. During his Masters, Mr. Zaman received a prestigious graduate academic achievement award from ISyE department of NIU. He also received outstanding master's thesis (in STEM category) award in 2018-2019 academic year (link). For more information, please visit his personal research website: https://auzipe.wixsite.com/adib

APPENDIX

1. *MATLAB Fmincon Optimizer*

```
%.m file for the optimization assignment
% Initialize function

function [x,fval]= rbdo_speed(~)
clc;
%Define options for optimset command (mainly the specific algorithm used)
options = optimset;

%Choose the SQP algorithm for the problem
options = optimset(options,'Algorithm', 'SQP');
%options = optimset(options,'Algorithm', 'Interior-point');
%Display the number of iterations needed
options=optimset(options,'Display','iter');

%Evaluate the optimized variables and minimum value using fmincon function

[x,fval]=fmincon(@(x) rbdo_func(x),[.1 .2 .3 .4 .5 .6 .7],[],[],[],[],[0.1...
   0.1 0.1 0.1 0.1 0.1 0.1],[1 1 1 1 1 1 1],@(x)limit_state(x),options);
end
```

2. *Objective Function*

```
function [f]=rbdo_func(x)

%Define the objective function (which is to be minimized) in terms of
%Random design variables
f=0.7845*x(1)*x(2)^2*(3.3333*x(3)^2+14.9334*x(3)-43.0934)-1.508*x(1)*(x(6)^2+...
   x(7)^2)+7.477*(x(6)^3+x(7)^3)+0.7854*(x(4)*x(6)^2+x(5)*x(7)^2);
%The function consists of 7 random design variables from x(1) to x(7)
end
```

3. *Constraint Functions*

```
function [c,ceq]=limit_state(x)
%Define the value of phi(-beta) for each of the limit state constraints
b1=0.2;
b2=0.4;
b3=0.5;
b4=0.33;
b5=0.75;
b6=0.55;
b7=0.25;
b8=0.35;
b9=0.60;
b10=0.7;
b11=0.55;
%Define symbolics to perform exact integration
syms x1 x2 x3 x4 x5 x6 x7

%Constraint #1
p1=int(int(27/(x1*x2.^2*x3)-1,x1),x2),x3);
g1=matlabFunction(p1);
c1=g1(x(1),x(2),x(3))-b1;

%Constraint #2
p2=int(int(int(397.5/(x1*x2.^2*x3.^2)-1,x1),x2),x3);
g2=matlabFunction(p2);
c2=g2(x(1),x(2),x(3))-b2;
```

```
%Constraint #3
p3=int(int(int(int((1.93*x4.^3)/(x1*x2.^2*x6.^4)-1,x1),x2),x6),x4);
g3=matlabFunction(p3);
c3=g3(x(4),x(1),x(2),x(6))-b3;

%Constraint #4
p4=int(int(int(int((1.93*x5.^3)/(x2*x3*x7.^4)-1,x2),x3),x5),x7);
g4=matlabFunction(p4);
c4=g4(x(2),x(3),x(5),x(7))-b4;

%Constraint #5
p5=int(int(int(int(-1100+sqrt((745*x4/x2*x3)^2)/0.1*x6^3))));
g5=matlabFunction(p5);
c5=g5(x(2),x(3),x(4),x(6))-b5;

%Constraint #6
p6=int(int(int(int(-850+sqrt((745*x4/(x2*x3))^2)/(0.1*x7^3)))));
g6=matlabFunction(p6);
c6=g6(x(2),x(3),x(4),x(7))-b6;

%Constraint #7
p7=int(int(x2*x3-40,x2),x3);
g7=matlabFunction(p7);
c7=g7(x(2),x(3))-b7;

%Constraint #8
p8=int(int(5-x1./x2,x1),x2);
g8=matlabFunction(p8);
c8=g8(x(1),x(2))-b8;
 %Constraint #9
p9=int(int(-12+x1./x2,x1),x2);
g9=matlabFunction(p9);
c9=g9(x(1),x(2))-b9;
%Constraint #10
p10=int(int(-1+(1.5*x6+1.9)./x4,x6),x4);
g10=matlabFunction(p10);
c10=g10(x(4),x(6))-b10;
%Constraint #11
p11=int(int(-1+(1.1*x7+1.9)./x5,x7),x5);
g11=matlabFunction(p11);
c11=g11(x(5),x(7))-b11;
%Define inequality constraint
c=[c1;c2;c3;c4;c5;c6;c7;c8;c9;c10;c11];
ceq=[];
```