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One-Dimensional Radial Fin by Frobenius Method Versus Two-Dimensional Straight Radial Fin

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Abstract **– Fins or extended surfaces are extensively used in engineering applications to increase the efficiency of heat transfer of surfaces. Recent applications in compact heat exchangers increase interest in easy and applicable models for fin systems. A generalized one-dimensional radial fin model has been developed, where the modified power series expansion, the "Frobenius" method, is applied to a specific geometry. The comparison between two models, one-dimensional and two-dimensional, was presented to determine the thermal characteristics in a simple fin system. The one-dimensional model is suitable for compact fin systems, where the ratio is relatively low (K ≤ 6) and the Biot number is not very high (Bi < 0.1). The results obtained are promising and motivating, leading to the conclusion that the implementation of the generalized model should be effective.**

Keywords **– Extended Surfaces, Compact Heat Exchangers, Modified Power Series, Frobenius.**

I. Introduction

Fins or the extended surfaces are extensively used in engineering applications to increase the heat transfer efficiency of surfaces. Once the temperature distribution through the fin is known, the heat transfer rate and the efficiency can be readily determined. A large variety of fins geometries are used in heat transfer application, and the longitudinal fin of rectangular profile and the one-dimensional radial fin are the most common used [01], [23], [25], [10], [28], [17], [06], [09], [26], [14], [21].

Because the practical importance of the extend surfaces, widely works in this subject is yet developed, and the applications in compact heat exchanger increase the interest in easy and applicable models for fins systems [13], [12], [24], [27], [18].

This work is directly connected to the solution of extended surfaces, as a special topic related to heat conduction theory, which is extension of the classical heat conduction mathematical formulation [07], [22], [03], ([19; [20]), [11], [05]), [04], [15].

Was developed a generalized one-dimensional radial fin (Figure 01), where the expansion in modified power series, the Frobenius Method, is applied for a particular geometry (Figure 02). The two-dimensional straight radial fin, described for [08], was used as a reference for comparison.

II. Theoretical Analysis

A. *Frobenius Method*

Consider steady-state, one-dimensional heat conduction through a radial fin, with constant conductivity, k and subjected an ambient temperature T_{∞}.

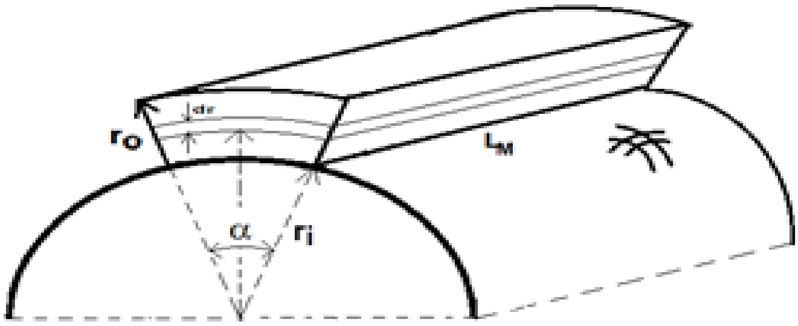

Fig. 1. Generalized Geometry for One-Dimensional Radial Fin Analysis.

$$\dot{q}_r - \dot{q}_{r+dr} = 2h_2\,(\alpha r + L_M)dr[T(r) - T_\infty] \qquad 01$$

$$\dot{q}_{r+dr} = \dot{q}_r - \frac{d\dot{q}}{dr}dr \qquad 02$$

$$\dot{q}(r) = -kA(r)\frac{dT}{dr} = -k\alpha r L_M\frac{dT}{dr} \qquad 03$$

$$\frac{d}{dr}\left[k\alpha r L_M\frac{dT(r)}{dr}\right] = 2h_2[\alpha r + L_M][T(r) - T_\infty] \qquad 04$$

$$\frac{d}{dr}\left[r\frac{dT(r)}{dr}\right] = \frac{2h_2[\alpha r + L_M]}{k\alpha L_M}[T(r) - T_\infty] \qquad 05$$

By definition:

$$\theta(r) = \frac{T(r) - T_\infty}{T(r_i) - T_\infty} \qquad 06$$

Then, we have:

$$r^2\frac{d^2\theta(r)}{dr^2} + r\frac{d\theta(r)}{dr} = \frac{2h_2[\alpha r^2 + L_M r]}{k\alpha L_M}\theta(R) \qquad 07$$

$$R = \frac{r - r_i}{r_o - r_i} \quad \rightarrow \quad r = L_o R + r_i \; with \;\; L_o = r_o - r_i \qquad 08$$

$$[R + \frac{r_i}{L_o}]^2\frac{d^2\theta(R)}{dR^2} + \left(R + \frac{r_i}{L_o}\right)\frac{d\theta(R)}{dR} = \frac{2h_2[\alpha(L_o R + r_i)^2 + L_M(L_o R + r_i)]}{k\alpha L_M}\theta(R) \qquad 09$$

$$\mathbb{P}(R)\frac{d^2\theta(R)}{dR^2} + \mathbb{Q}(R)\frac{d\theta(R)}{dR} - \mathbb{W}(R)\theta(R) = 0 \qquad 10$$

where

$$\mathbb{P}(R) = P_1 R^2 + P_2 R + P_3;\; \mathbb{Q}(R) = Q_1 R + Q_2;\; \mathbb{W}(R) = W_1 R^2 + W_2 R + W_3 \qquad 11$$

and

$$P_1 = 1.0;\; P_2 = 2K_1; P_3 = {K_1}^2;\; Q_1 = 1.0;\; Q_2 = K_1;\; W_1 = \frac{2B_{i2}K^2}{K_2};\; W_2 = B_{i2}[\frac{4K^2K_1}{K_2} + \frac{2K}{\alpha}]; W_3 = B_{i2}[\frac{2K^2K_1}{K_2} + \frac{2KK_1}{\alpha}] \qquad 12$$

with dimensionless groups defined as:

$$K = \frac{L_o}{w/2}; K_1 = \frac{r_i}{L_o}; K_2 = \frac{L_M}{w/2}; w = \alpha r_i; L_M = w; B_{i1} = \frac{h_2(\frac{w}{2})}{k}; B_{i2} = \frac{h_2(\frac{w}{2})}{k} \quad 13$$

The proposal, in this work, for one validation of the general model (Figure 01), is realized a comparison with a "Two-Dimensional Straight Radial Fin", presented by COTTA and MIKHAILOV (1997). In this case, the following simplifications were needed:

$$B_{i1} \rightarrow \infty; \ r_i = 0 \rightarrow \ K_1 = 0 \ and \ \alpha \rightarrow \infty \quad 14$$

Then,

$$P_2 = 0; \ P_3 = 0; \ Q_2 = 0; \ W_2 = 0 \ and \ W_3 = 0 \quad 15$$

and

$$R^2 \frac{d^2\theta(R)}{dR^2} + R\frac{d\theta(R)}{dR} - W_1 R^2 \theta(R) = 0 \quad 16$$

For convenience, was defined

$$K_2 = 2; \ \beta^2 = W_1 \ and \ R' = \beta R \quad 17$$

In this case,

$$R'^2 \frac{d^2\theta(R)}{dR^2} + R'\frac{d\theta(R)}{dR} - R'^2\theta(R) = 0 \quad 18$$

or

$$\frac{1}{R'}\frac{d}{dR'}\left[R'\frac{d\theta}{dR'}\right] - \theta(R') = 0 \quad 19$$

In this work the equation 18 is more convenient, because the interest is in obtaining a particular solution of equation 10, by the expansion in modified series of power, called "Frobenius method" in the specialized literature. The Equation 18 has a singular regular point in R'=0, and By Georg Frobenius (1849-1917) [04] (1986, pag.243), [15] (1969, pag.190), [03] (1966, pag.231), [11] (1962, pag.143), [21] (1955, pag.46-59), [07] (1947, pag.374-376):

$$\theta(R') = \sum_{n=0}^{\infty} a_n R'^{n+s} \quad 20$$

$$\theta'(R') = \frac{d\theta(R')}{dR'} = \sum_{n=1}^{\infty} a_{n-1}(n+s-1)R'^{n+s} \quad 21$$

$$\theta''(R') = \frac{d^2\theta(R')}{dR'^2} = \sum_{n=2}^{\infty} a_{n-2}(n+s-2)(n+s-3)R'^{n+s} \quad 22$$

Then

$$\mathbb{P}(R)\sum_{n=2}^{\infty} a_{n-2}(n+s-2)(n+s-3)R^{n+s} + \mathbb{Q}(R)\sum_{n=1}^{\infty} a_{n-1}(n+s-1)R^{n+s} - \mathbb{W}(R)\sum_{n=0}^{\infty} a_n R^{n+s} = 0 \quad 23$$

or

$$R'^2 \sum_{n=0}^{\infty} a_n(n+s)(n+s-1)R'^{n+s-2} + R' \sum_{n=0}^{\infty} a_n(n+s)R^{n+s-1} - R'^2 \sum_{n=0}^{\infty} a_n R^{n+s} = 0 \tag{24}$$

By algebraic manipulation, the following indicial equation was obtained:

$$a_0[(s^2 - s) + s]R'^s = 0 \qquad with \qquad a_0 \neq 0 \;\; and \; s = 0 \tag{25}$$

The roots of the indicial equation are equal zero and the recurrence rule is given by

$$a_n = \frac{a_{n-2}}{n^2} \tag{26}$$

or

$$a_2 = \frac{a_0}{2^2};\; a_4 = \frac{a_0}{2^2 4^2}\;;\; a_6 = \frac{a_0}{2^2 4^2 6^2} \cdots \tag{27}$$

For the situation in analysis, two equal roots, there are two linearly independent solutions, which constitute a fundamental system of solution [15]. The first is:

$$\theta_1(R) = 1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m};\quad a_{2m} = \frac{1}{2^{2m}(m!)^2} \tag{28}$$

The second linearly independent solution contains a logarithmic term and has a form:

$$\theta_2(R) = [\ln(\beta R)]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} A_m(\beta R)^m \tag{29}$$

By [07], and [04] the more convenient expression is

$$\theta_2(R) = -\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m} \tag{29.1}$$

where

$$H_m = \frac{1}{m} + \frac{1}{m-1} + \cdots + \frac{1}{2} + 1 \quad and\; \gamma \cong 0.5772 \tag{30}$$

γ is known as the Euler-Mascheroni [04] (1986, pag.247) constant.

Then

$$\theta(R) = a_0\theta_1(R) + a_1\theta_2(R) \tag{31}$$

$$\theta(R) = a_0[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}] + a_1[-\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\theta_1(R) + \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}] \tag{32}$$

or

$$\theta(R) = a_0[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}] - a_1\left\{\left[\ln\left(\frac{\beta R}{2}\right) + \gamma\right]\left[1 + \sum_{m=1,2,3..}^{\infty} a_{2m}(\beta R)^{2m}\right] - \sum_{m=1,2,3\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}\right\} \tag{32.1}$$

The first boundary condition is defined by [08]:

$$\theta(0) = 1 \quad \rightarrow \quad a_0 = 1 + a_1\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right] \qquad 33$$

Finally,

$$\theta(R) = \theta_1(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right]\theta_1(R) + \theta_2(R)\right\} \qquad 33.1$$

$$\theta'(R) = \theta_1{}'(R) + a_1\left\{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right]\theta_1{}'(R) + \theta_2{}'(R)\right\} \qquad 33.2$$

where (Figure 02)

$$R_b = R \rightarrow 0 \qquad 34$$

For the second boundary conditions:

$$\theta'(1) = -B_{i2}K\theta(1) \qquad 35$$

Then,

$$-[\theta_1(1) + B_{i2}K\theta_1'(1)] = a_1\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right][\theta_1(1) + B_{i2}K\theta_1'(1)] + [\theta_2(1) + B_{i2}K\theta_2'(1)] \qquad 35.1$$

In this case,

$$a_1 = \frac{-[\theta_1(1) + B_{i2}K\theta_1'(1)]}{\left[\ln\left(\frac{\beta R_b}{2}\right) + \gamma\right][\theta_1(1) + B_{i2}K\theta_1'(1)] + [\theta_2(1) + B_{i2}K\theta_2'(1)]} \qquad 36$$

The total exchange heat transfer is given by

$$\dot{q} = \frac{-kA_b(T_b - T_\infty)\theta'(0)}{L_0} \qquad 37$$

The dimensionless exchange heat transfer is written in the form, by definition

$$Q_b = \frac{\dot{q}}{h_2A_b(T_b - T_\infty)} \quad \rightarrow \quad Q_b = \frac{-1}{B_{i2}K}\left(\frac{d\theta}{dR}\right)_{R=0} \qquad 38$$

A_b and T_b are the base area and the base temperature respectively

Efficiency is given by

$$\eta = \frac{-1}{B_{i2}K(1+K)}\left(\frac{d\theta}{dR}\right)_{R=0} \qquad 39$$

A. *Straight Radial Fin*

The formulation, in dimensionless form, is written as:

$$\frac{1}{R}\frac{\partial}{\partial R}\left[R\frac{\partial\theta(R,Y)}{\partial R}\right] + K^2\frac{\partial^2\theta(R,Y)}{\partial Y^2} = 0 \qquad 40$$

with boundary conditions

$$\theta(R_b, Y) = 1; \qquad \frac{\partial\theta(1,Y)}{\partial R} + B_{i2}K\theta(1,Y) = 0, \qquad 0 \le Y \le 1 \qquad 41$$

$$\frac{\partial\theta(R,0)}{\partial Y}=0; \qquad \frac{\partial\theta(R,1)}{\partial Y}+B_{i2}K\theta(R,1)=0, \qquad R_b\le R\le 1 \tag{42}$$

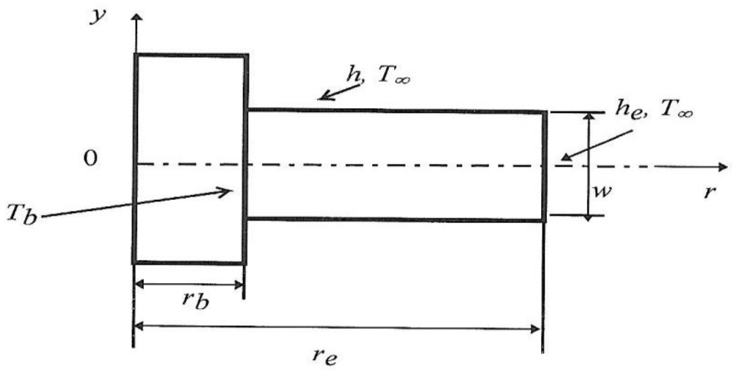

Fig. 2. Geometry System for Straight Radial Fin Analysis by [08].

Dimensionless groups are defined as:

$$K=\frac{r_e}{w/2}; \qquad R_b=\frac{r_b}{r_e}; \qquad B_{i2}=\frac{h_e(\frac{w}{2})}{k} \tag{43}$$

The exact solution of the Two-Dimensional Straight Radial Fin is obtainable by separation of variables, and the dimensionless average temperature at each circumferential section is given by [08]:

$$\theta_{Av}(R)=2\sum_{n=1}^{\infty}\frac{sin^2\lambda_n}{\lambda_n[\lambda_n+sin\lambda_n cos\lambda_n]}F(\lambda_n,R) \tag{44}$$

where,

$$F(\lambda_n,R)=\frac{\{\mathbb{K}_0(\lambda_nKR)[B_{i2}\mathbb{I}_0(\lambda_nK)+\lambda_n\mathbb{I}_1(\lambda_nK)]-\mathbb{I}_0(\lambda_nKR)[B_{i2}\mathbb{K}_0(\lambda_nK)-\lambda_n\mathbb{K}_1(\lambda_nK)]\}}{\{\mathbb{K}_0(\lambda_nKR_b)[B_{i2}\mathbb{I}_0(\lambda_nK)+\lambda_n\mathbb{I}_1(\lambda_nK)]-\mathbb{I}_0(\lambda_nKR_b)[B_{i2}\mathbb{K}_0(\lambda_nK)-\lambda_n\mathbb{K}_1(\lambda_nK)]\}} \tag{45}$$

$\mathbb{I}_\nu, \mathbb{K}_\nu$ are modified Bessel functions and the $\lambda_n's$ are obtained from the solution of the transcendental equation:

$$\lambda_n tan\lambda_n = B_{i2} \tag{46}$$

For small R [19] (1989, pag.493):

$$\mathbb{I}_n(R)\cong\frac{1.0}{2^n n!}R^n \tag{47}$$

$$\mathbb{K}_n(R)\cong -l_nR \;\; for\; n=0 \;\; and\; \mathbb{K}_n(R)\cong\frac{2^{n-1}(n-1)!}{R^n} \;\; for\;\; n\neq 0 \tag{48}$$

For $R\ge 10$:

$$\mathbb{I}_0(R)\cong\frac{0.3989e^R}{R^{\frac{1}{2}}}\left\{1+\frac{1}{8R}+\frac{9}{128R^2}+\frac{75}{1024R^3}\right\} \tag{49}$$

$$\mathbb{I}_1(R)\cong\frac{0.3989e^{-R}}{R^{\frac{1}{2}}}\left\{1+\frac{3}{8R}-\frac{15}{128R^2}+\frac{105}{1024R^3}\right\} \tag{50}$$

$$\mathbb{K}_0(R)\cong\frac{1.2533e^{-R}}{R^{\frac{1}{2}}}\left\{1-\frac{1}{8R}+\frac{9}{128R^2}-\frac{75}{1024R^3}\right\} \tag{51}$$

$$\mathbb{K}_1(R) \cong \frac{1.2533e^{-R}}{R^{\frac{1}{2}}}\left\{1+\frac{3}{8R}-\frac{15}{128R^2}+\frac{105}{1024R^3}\right\} \qquad 52$$

For large R:

$$\mathbb{I}_n(R) \cong \frac{e^R}{\sqrt{2\pi R}} \qquad 53$$

$$\mathbb{K}_n(R) \cong \sqrt{\frac{\pi}{2R}}e^{-R} \qquad 54$$

The total heat exchange through the fin's base, in dimensionless form, is given by:

$$Q_b = \sum_{n=1}^{\infty} \frac{sin^2\lambda_n}{[\lambda_n + sin\lambda_n cos\lambda_n]} G(\lambda_n) \qquad 55$$

where,

$$G(\lambda_n) = \frac{\{\mathbb{K}_1(\lambda_n K R_b)[B_{i2}\mathbb{I}_0(\lambda_n K) + \lambda_n \mathbb{I}_1(\lambda_n K)] + \mathbb{I}_1(\lambda_n K R_b)[B_{i2}\mathbb{K}_0(\lambda_n K) - \lambda_n \mathbb{K}_1(\lambda_n K)]\}}{\{\mathbb{K}_0(\lambda_n K R_b)[B_{i2}\mathbb{I}_0(\lambda_n K) + \lambda_n \mathbb{I}_1(\lambda_n K)] - \mathbb{I}_0(\lambda_n K R_b)[B_{i2}\mathbb{K}_0(\lambda_n K) - \lambda_n \mathbb{K}_1(\lambda_n K)]\}} \qquad 56$$

$$\eta = \frac{Q_b}{B_{i2}K^2} \qquad 57$$

Table 1. Obtained Results for Modified Bessel Functions.

X	**K0(X)**	**K0(X)**	**I0(X)**	**I0(X)**	**K1(X)**	**K1(X)**	**I1(X)**	**I1(X)**
1.00E - 01	2.427E + 00	2.426E + 00	1.003E + 00	1.003E + 00	9.854E + 00	1.000E + 01	5.010E - 02	5.000E - 02
1.00E + 00	4.210E - 01	4.210E - 01	1.266E + 00	1.376E + 00	6.019E - 01	6.272E - 01	5.906E - 01	4.394E - 01
2.00E + 00	1.139E - 01	1.135E - 01	2.280E + 00	2.270E + 00	1.399E - 01	1.404E - 01	1.591E + 00	1.606E + 00
3,00E + 00	3.474E - 02	3.471E - 02	4.881E + 00	4.867E + 00	4.016E - 01	4.020E - 01	3.953E + 00	3.970E + 00
4.00E + 00	1.116E - 02	1.116E - 02	1.130E + 01	1.129E + 01	1.248E - 02	1.249E - 02	9.760E + 00	9.771E + 00
5.00E + 00	3.691E - 03	3.691E - 03	2.724E + 01	2.723E + 01	4.045E - 03	4.045E - 03	2.434E + 01	2.434E + 01
6.00E + 00	1.244E - 03	1.244E - 03	6.723E + 01	6.722E + 01	1.344E - 03	1.344E - 03	6.134E + 01	6.135E + 01
7.00E + 00	4.248E - 04	4.248E - 04	1.686E + 02	1.686E + 02	4,542E - 03	4,542E - 03	1.560E + 02	1.560E + 02
8.00E + 00	1.465E - 04	1.465E - 04	4.276E + 02	4.275E + 02	1.554E - 04	1.554E - 04	3.999E + 02	3.999E + 02
9.00E + 00	5.088E - 05	5.088E - 05	1.094E + 03	1.093E + 03	5.364E - 05	5.364E - 05	1.030E + 03	1.031E + 03
9.09E + 00	1,975E - 05	1,975E - 05	2.561E + 03	2.561E + 03	2.072E - 05	2.072E - 05	2.428E + 03	2.428E + 03

Bold: M. NECATTI ÖZISIK (1980; 1989)

Table 2. Six First Eigenvalue of the Equation 46

B_{i2}	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
0.101	0.3125	3.1734	6.2992	9.4354	12.5744	15.7143
0.201	0.4338	3.2042	6.315	9.446	12.5823	15.7207
0.301	0.5226	3.2344	6.3307	9.4566	12.5903	15.7271

0.401	0.5939	3.2638	6.3463	9.4671	12.5982	15.7334
0.501	0.6538	3.2926	6.3618	9.4776	12.6061	15.7398
0.601	0.7055	3.3206	6.3771	9.488	12.614	15.7461
0.701	0.751	3.348	6.3924	9.4984	12.6218	15.7524
0.801	0.7914	3.3746	6.4075	9.5088	12.6297	15.7587
0.901	0.8277	3.4006	6.4226	9.5191	12.6375	15.7650

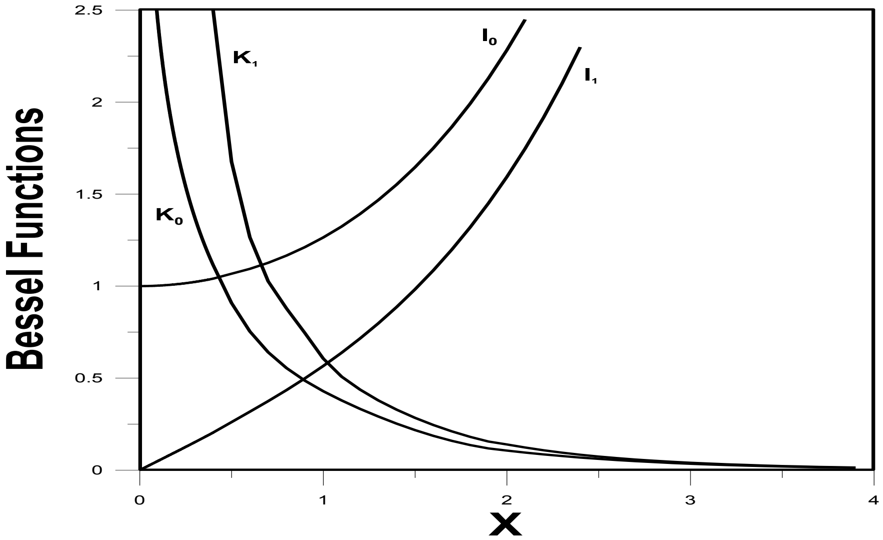

Fig. 3. Implemented Modified Bessel Functions

III. Results and Discussion

In Figure 04, below, the temperature is measured as a function of the radial position for aspect ratio K = 2. The comparison between the models for this aspect ratio value demonstrates that the models, one-dimensional and two-dimensional, present equivalent results for relatively low Biot number. The difference, for average temperature, is only noticeable for Biot number near and above 10.

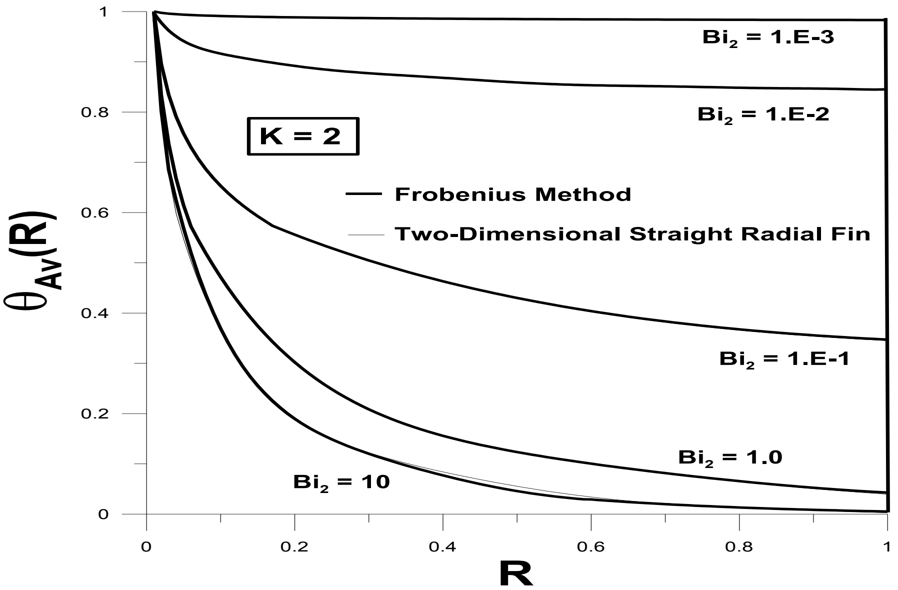

Fig. 4. Average Temperature for Aspect Ratio K = 2 versus Biot Number.

Through Figure 05, with a K = 6 aspect ratio, it can be observed that the difference between the models, one-dimensional and two-dimensional, already occurs for Biot number values above 1.0. The same occurs for aspect ratio K = 10 (Figure 06).

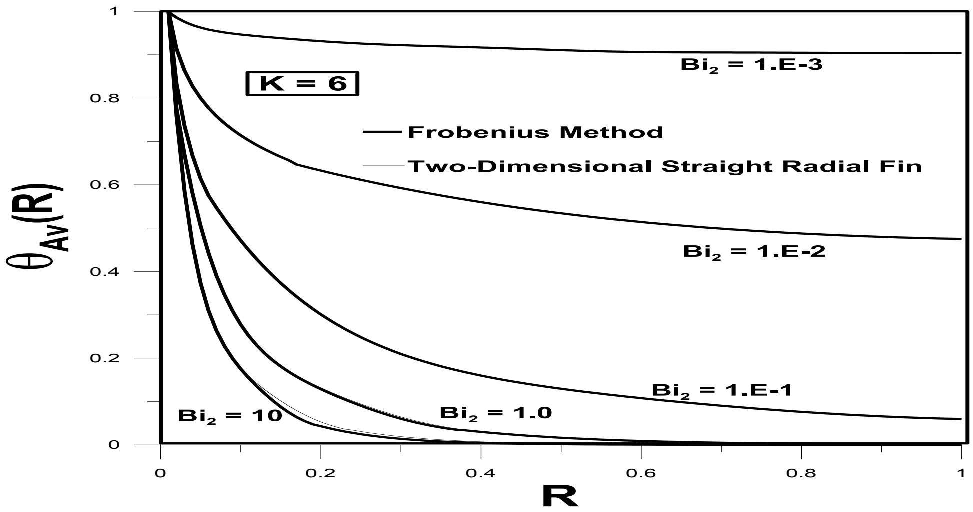

Fig. 5. Average Temperature for Aspect Ratio K = 6 versus Biot Number.

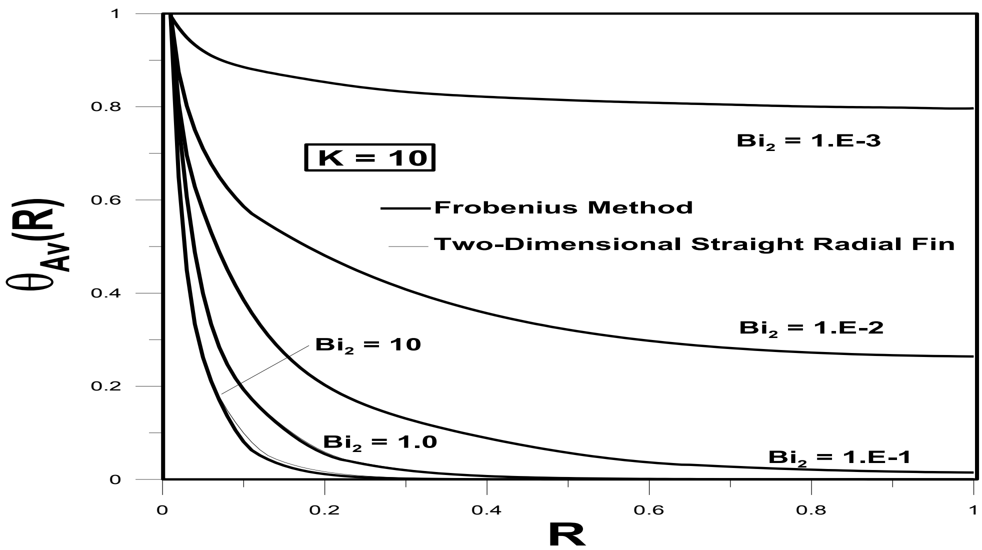

Fig. 6. Average Temperature for Aspect Ratio K = 10 versus Biot Number.

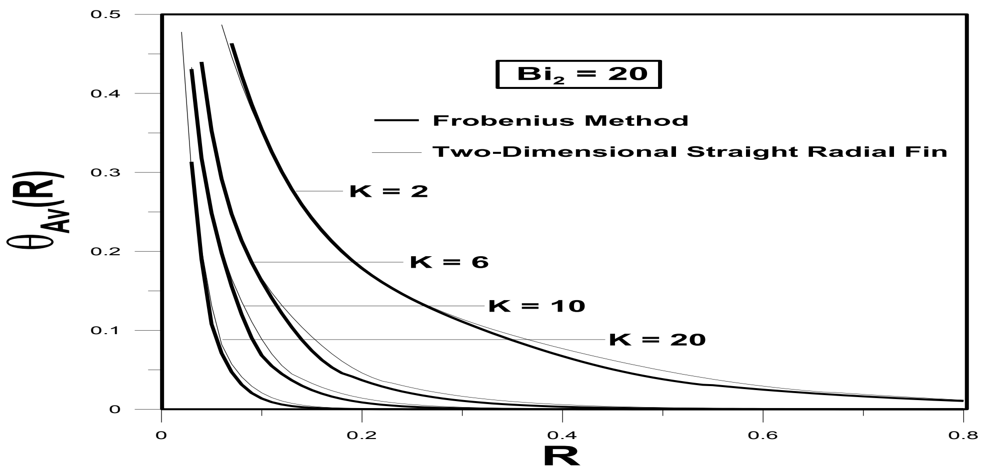

Fig. 7. Average Temperature for Biot Number $B_{i2} = 20$ versus Aspect Ratio.

Figure 07, above, demonstrates that for high Biot number value, Biot equal to 20, even for low aspect ratio, the difference between models is noticeable and meaningful. The difference between the models, one-dimensional and two-dimensional, becomes more evident through the rate of heat transfer dimensionless, Figure 08.

The one-dimensional model works properly for low aspect ratio value, in a wide range of Biot number. In fact, the results obtained show that the one-dimensional model presented is suitable for compact systems, where the aspect ratio of the fin is low and the Biot number is not very high.

In Figure 09 there are values for efficiency according to the heat transfer coefficient, where the length of the base and the conductivity of the fin were obtained from an electric motor finned, with K close to 6. It is observed that there is a maximum efficiency for the exchange of heat in all cases. For K = 6 The maximum efficiency corresponds to an approximate value of 80 W/ (m^2. K), for the heat transfer coefficient, in Biot number less than 7.3 10-3.

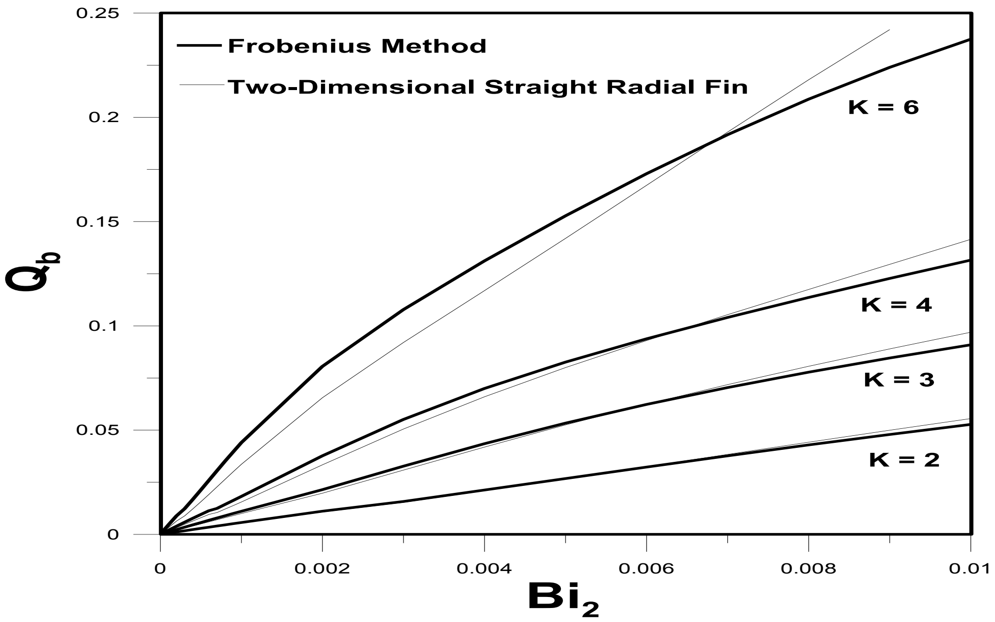

Fig. 8. Total Dimensionless Heat Exchange versus Biot Number.

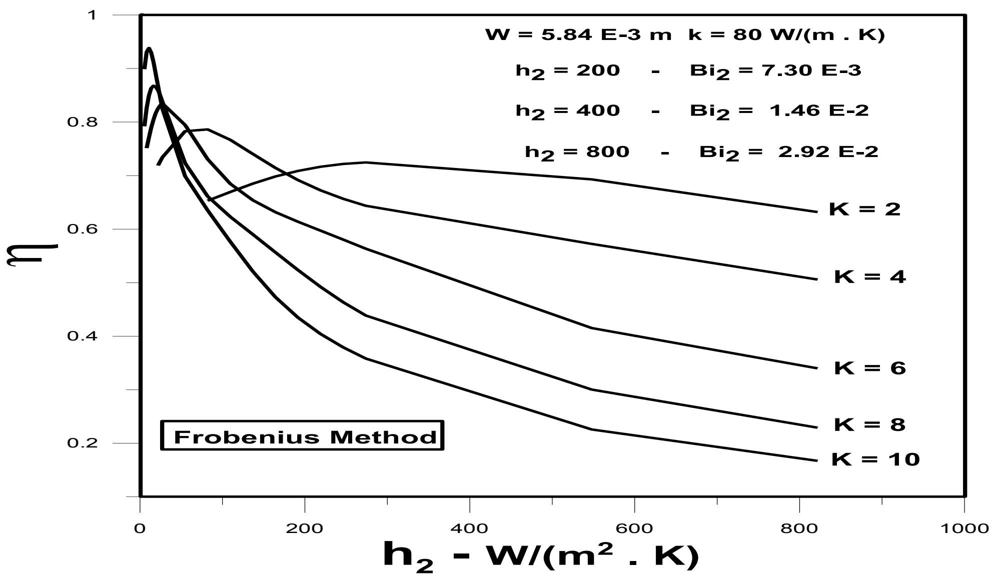

Fig. 9. Effectiveness versus Convection Heat Transfer Coefficient.

IV. Conclusions

The comparison between two models, one-dimensional and two-dimensional, was presented to determine the thermal characteristics in a simple radial fin system.

The one-dimensional model is simpler and easier to be deployed than the two-dimensional model, as demonstrated by the formulation of Frobenius Method, implemented to a particular geometric situation, used in electrical motors.

The one-dimensional model is suitable for compact fins systems, where the aspect ratio is relatively low ($K \leq 6$) and the number of Biot is not very high ($Bi<0.1$). The smaller the value of the aspect ratio, the greater the range of Biot number in which the one-dimensional model works properly.

It is a first approximation to the generalized one-dimensional model using the Frobenius method, and it can to be used, for example, in finned compact heat exchangers, because of the low aspect ratio.

The results obtained are promising and motivating, leading to the conclusion that the implementation of the generalized model should be effective.

References

[1] APARECIDO, J. B.; COTTA, R. M. "IMPROVED ONE-DIMENSIONAL FIN SOLUTIONS". Heat Transf. Eng., V. 11, no. 1, 49-59, 1988.
[2] APARECIDO, J. B.; COTTA, R. M. "MODIFIED ONE-DIMENSIONAL ANALYSIS OF RADIAL FINS". Proceedings of the Second National Meeting of Thermal Sciences, ENCIT, pp. 225 ± 228, 1988.
[3] ARPACI, V.S. "CONDUCTION HEAT TRANSFER". Addison-Wesley Publishing Company, Inc., Printed in United State of America, Library of Congress Catalog, No. 66-25602, 1966.
[4] BOYCE, W. E.; DIPRIMA, R. C. "ELEMENTARY DIFFERENTIAL EQUATIONS AND BOUNDARY VALUE PROBLEMS". John Wiley & Sons, New York, 1986.
[5] BUTKOV, E. "MATHEMATICAL PHYSICS". Guanabara Dois, Rio de Janeiro, 1983.
[6] CAMPO, A.; KUNDU, B. "EXACT ANALYTIC HEAT TRANSFER FROM AN ANNULAR FIN WITH STEPPED RECTANGULAR PROFILE". American Journal of Heat and Mass Transfer, Vol. 4 No. 4, pp. 146-155 doi:10.7726/ajhmt.2017.1013, 2017.
[7] CARSLAW, H. S., JAEGER, J. C. "CONDUCTION OF HEAT IN SOLIDS". Oxford of Clarendon Press, Glasgow, 1948.
[8] COTTA, R. M., MIKHAILOV, M. D. "HEAT CONDUTION – LUMPED ANALYSIS, INTEGRAL TRANSFORM, SYMBOLIC COMPUTATION". John Wiley & Sons, New York, 1997.
[9] COTTA, R.M.; RAMOS, R. "ERROR ANALYSIS AND IMPROVED FORMULATIONS FOR EXTENDED SURFACES". Proceedings of the NATO - Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258, pp. 753±787, 1993.
[10] DENISE; NOVAIS, A; NOGUEIRA, E. "RECTANGULAR PROFILE ANALYTIC SOLUTION: COMPARISON OF THERMAL PERFORMANCE BETWEEN ALUMINUM AND CAST IRON". Cadernos UniFOA (Printed), v. 20, p. 43, 2012.
[11] HILDEBRAND, F. B. "ADVANCED CALCULUS FOR APPLICATIONS". Prentice-Hall, INC; Englewood Cliffs, New Jersey, 1962.
[12] KAKAÇ, S. "BOYLERS, EVAPORATORS, AND CONDENSERS". John Wiley & Sons, INC., New York, 1991.
[13] KAYS, W.M., LONDON, A.L. "COMPACT HEAT EXCHANGERS". Krieger Publishing Company, Florida, 1984.
[14] KRAUS, A.D. "ANALYSIS OF EXTENDED SURFACE ARRAYS FOR AIR-COOLED ELECTRONIC EQUIPMENT". Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258, 1993.
[15] KREYSZIG, E. "ADVANCED ENGINEERING MATHEMATICS". Technical and Scientific Books Publisher, Rio de Janeiro, 1969.
[16] MIKHAILOV, M. D.; ÖZISIK, M. N. "UNIFIED ANALYSIS AND SOLUTIONS OF HEAT AND MASS TRANSFER". John Wiley & Sons, New York, 1994.
[17] NOVAIS, A ; CHAGAS, R. D. F .; NOGUEIRA, Élcio "THEORETICAL ANALYSIS OF THERMAL PERFORMANCE OF FINNED ELECTRIC INDUCTION MOTORS". Cadernos UniFOA (Online), v. IX, p. 19-34, 2014.
[18] OKTAY, S. "BEYOND THERMAL LIMITS in COMPUTER SYSTEMS COOLING". Advanced Study Institute on Cooling of Electronic Systems, NATO ASI Series E: Applied Sciences, vol.258, 1993.
[19] OZISIK, M. N. "BOUNDARY VALUE PROBLEMS OF HEAT CONDUCTION". Dover Publications, INC., New York, 1989.
[20] OZISIK, "HEAT CONDUCTION". John Wiley and Sons, New York, 1980.
[21] R.ENGR. KARINATE VALENTINE OKIY "AN ASSESSMENT OF EXTENDED SURFACES TWO-DIMENSIONAL EFFECTS". International Journal of Engineering Research in Africa Vol 15 pp 71-85, 2015.
[22] SCHNEIDER, P. J. "CONDUCTION HEAT TRANSFER". Addison-Wesley Publishing Company, Inc., Printed in United State of America, Library of Congress Catalog, No. 55-5025, 1957.
[23] SOARES, M. V. F. "ANALYSIS OF THE INFLUENCE OF THE CONVECTION HEAT TRANSFER COEFFICIENT ON THE TEMPERATURE OF THE ELECTRIC MOTOR FLOW WITH FLOWED PRESCRIPTED HEAT FLOW". Completion of course work. (Degree in Mechanical Engineering) - Volta Redonda University Center, Oswaldo Aranha Foundation. Advisor: Élcio Nogueira, 2015.
[24] STONE, K. M. "REVIEW OF LITERATURE ON HEAT TRANSFER ENHANCEMENT IN COMPACT HEAT EXCHANGERS". ACRC Project 65, Investigation of Wavy Fins for Heat Transfer Augmentation in Refrigeration/Air Conditioning Systems, University

of Illinois, 1996.

[25] SANTOS, T.A.M. "THERMAL PERFORMANCE ANALYSIS OF FINNED ELECTRIC MOTORS: BIDIMENSIONAL SOLUTION IN RECTANGULAR FIN WITH PRESCRIPTION BASED TEMPERATURE". Completion of course work. (Degree in Mechanical Engineering) - Oswaldo Aranha University Center - Volta Redonda. Advisor: Élcio Nogueira, 2017.

[26] SOMMERS, A. D.; JACOBI, A. M. "AN EXACT SOLUTION TO STEADY HEAT CONDUCTION IN A TWO-DIMENSIONAL ANNULUS ON A ONE-DIMENSIONAL FIN: APPLICATION TO FROSTED HEAT EXCHANGERS WITH ROUND TUBES". Journal of Heat Transfer APRIL 2006, Vol. 128, 2006.

[27] WEBB, R. L. "PRINCIPLES OF ENHANCED HEAT TRANSFER". John Wiley & Sons, INC., New York, 1994.

[28] YURI; DENISE; NOGUEIRA E. "ALUMINUM AND CAST IRON IN CASE PRODUCTION OF FIXED ELECTRIC MOTORS: EFFICIENCY, COSTS, OPERATIONAL AND ENVIRONMENTAL ASPECTS". Cadernos UniFOA (Printed), v. IX, p. 11-19, 2014.

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