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From Pythagorean Mathematical Music Theory to the Density of the Dark Energy Sector of the Cosmos and Unification of Art with Science

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Abstract **– We present briefly the philosophical ideas of the Pythagorean school of thoughts then proceed from there to link these ideas to the golden mean number system of E-Infinity Cantorian space time. This way, it may come as a mild surprise to realize that one can explain and compute accurately the cosmic dark energy density of the universe as well as conclude the basic unity of art and science. As grandiose as all that may be, the rationale behind it all is too simple to be noticed let alone believed at once and the time it took to see the golden mean in the work of art and music as well as fundamental quantum physics of Hardy's quantum entanglement and Alain Connes' noncommutative Penrose fractal tiling universe is really due to this utter unexpected simplicity and beauty of our finding as will be clear from the present work.**

Keywords **– Von Neumann Continuous Geometry, A Connes Noncommutative Geometry, Deductive Dimensional Theory, Nikolay Umov Mass Energy Equivalence, Cantorian Spacetime, Golden Mean Number System, E-Infinity Theory, Super Strings, Exceptional Lie Symmetry Groups, Fractal Witten M-Theory, Hidden Extra Dimensions, Fractal Vafa F-Theory, Dvoretzky's Theorem, Heterotic String Theory, Dark Energy, Zero Set, Empty Set.**

I. INTRODUCTION - PYTHAGOREAN PHILOSOPHY

The present work may be extensive [1-45] but it is really quite condensed and is relatively a rather short account of the long story of the impact of utilizing the golden mean number system [1-22], [38-41] in quantum cosmology in particular [14-22] and in unifying art with science in general [6, 8], [10-12]. To make a long story short which started in Egypt with Pythagoras and maybe even before, we presuppose a fair familiarity with the Pythagorean philosophy and their theory of cosmic music as spanned by number theory [8-15] as well as the tremendous work which sprung from it [11-12] and influenced many modern research directions which are visible in our very own present time [6, 8, 12]. Suffice for us to see that the present work brings some of the most advanced, cutting edge research in string theory, noncommutative geometry, fractal spacetime and dark energy research together into a single coherent theory [20-49].

II. BACKGROUND INFORMATION

Concurrent to what we just said above in the introduction, we draw attention to parallel developments coming from different directions of logical and historical motivation converging on the same critical point, namely number theory in general [13, 23] and the golden mean number system in particular [19, 21]. To put the preceding point in explicit terms, we just need to recall the fundamental fact that the spacetime manifold of our E-Infinity Cantorian theory is virtually made of an infinite number of union and intersection of random Cantor sets with the golden mean $\phi = (\sqrt{5} - 1)/2$ to the power of n as Hausdorff dimension where n takes the value from one to infinity as well as n = o which gives us unity (i.e. $\phi^0 = 1$) as shown in numerous previous publications [1-9], [13-22] [40-49].

III. Cantorian Spacetime-Bijection Formula - Non - Commutative Geometry and the Dimensional Function

Let us stress that due to the well known theorem of Maudlin and William, a random Cantor set has the golden mean as its Hausdorff dimension and that is why we are using ϕ^n as statistical weight for the hierarchical E-Infinity Cantorian spacetime [1-3], [19] [24]. In this way the deductive dimensional theory developed by K. Menger and P. Urysohn entered into E-infinity and was integrated into its mathematics via the bijection formula [1, 2, 3, 19, 24].

$$d_c^{(n)} = \left(1/d_c^{(0)}\right)^{n-1},\ d_c^{(0)} = \phi \tag{1}$$

which in turn serves the same purpose and has the same meaning as Von-Neumann-Connes' dimensional function of Penrose fractal tiling universe [25-28]

$$D = a + b\phi,\ \ a, b \in z \ \text{ and } \ \phi = \left(\sqrt{5}-1\right)/2 \tag{2}$$

Let us now show how equations one and two stated above deliver the same Hausdorff dimension of a spacetime which is topologically four dimensional.

In case of equation (2) all what we need is to set n =4 and $d_c^{(0)} = \phi$ to find immediately the famous Hausdorff dimensionality. Consequently for Einstein's spacetime with $D(4) = n = 4$ one finds [1-4], [25, 26]

$$\begin{aligned} d_c^{(4)} &= \left(1/\phi\right)^{4-1} \\ &= \left(1/\phi\right)^3 \\ &= 4.2360679... \\ &= 4 + \phi^3 \\ &= 4 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + ...}}} \end{aligned} \tag{3}$$

The result we just found goes far beyond Einstein D = 4 because every single internal region is also four dimensional and so is everything which appears in this space to be a mere point turns out on careful observation to be an entire Cantor set as obvious from the continued fraction representation of $4 + \phi^3$ given in equation (3). Such a space is clearly scale invariant [6, 15, 19, 21]. This scale invariance is in general an important and essential property of a truly consistent theory as discussed in many earlier publications [1, 8, 15, 17, 19, 21, 24].

IV. Extension to Fractal M-Theory and Dark Energy

In a similar manner to the above we have shown how Wittens' M-Theory D = 11 combined with dark energy can be converted to a general fractal and scale invariant M-Theory with a dimensionality equal to eleven plus the exact Hardy quantum entanglement $P(H) = \phi^5$ [2, 3, 29]. Consequently we see that we have [15]

$$\begin{aligned} D &= 11 + \phi^5 \\ &= 11 + \cfrac{1}{11 + \cfrac{1}{11 + ...}} \end{aligned} \tag{4}$$

i.e. an eleven dimensional space time inside a larger one and so on indefinitely [15, 30].

In fact in the last 25 years or so we have deduced numerous similar relations which the reader may find mathematically derived and physically explained in the relevant voluminous literature on E-Infinity Cantorian spacetime theory [1-8], [13-31].

In the present work however we will concentrate on and recall only on those theoretical results which are of immediate relevance and direct use in the aspect we are concerned with here and stated at the very beginning, namely the connection between science and the artistic golden mean [5, 6] as testified by our derivation of the energy density of the dark section of the cosmos i.e. pure dark energy which amounts to almost 73.5 % of the total energy of the universe as well as the dark matter energy density found using accurate cosmic measurement to be 22% [3, 6, 16, 17, 18, 32]. Consequently the total dark energy sector is about 95.5%, which harmonizes well with the result of the cosmic energy density which can be measured in a direct way rather than indirectly inferred, namely the ordinary cosmic energy of about 4.5%. In other words, the total energy density adds to the maximal energy density given by that most famous of all equations [33] namely the coming equation (5) as will be discussed next.

V. UMOV-POINCARE-LORENZ-EINSTEIN'S FORMULA AND COSMIC DARK ENERGY

Let us start with Einstein's mass-energy equivalence equation:

$$\begin{aligned} E &= \gamma mc^2 \\ &= mc^2 \end{aligned} \tag{5}$$

so that we may conclude that the energy E given by equation (5), where c is the velocity of light and m is the mass is the maximal energy density possible [17] while $\gamma = 1$ marks the corresponding 100% cosmic energy density [17, 31, 33]. We note on passing the remarkable form of equation (5) which shows that only the value of γ changes but nothing else except the velocity $v \to c$ so that:

(a) $\gamma = \frac{1}{2}$ gives the kinetic energy for v<<c

$$E = \frac{1}{2}mv^2, \tag{6}$$

(b) $\gamma = 1$ gives the maximal energy density possible [17]

$$E = mc^2, \tag{7}$$

(c) $\gamma \cong 0.73$ gives pure dark energy density [3, 18]

$$E = 0.73mc^2 \tag{8}$$

(d) $\gamma = 0.22$ gives dark matter energy density [3, 18, 29]

$$E = 0.22mc^2 \tag{9}$$

Finally for ordinary energy we have [33-36]

(e) $\gamma \cong 0.045$ and one finds [33-36]

$$E = 0045mc^2 \tag{10}$$

so that the total adds up to maximal cosmic energy destiny [29-30], [32-36]

$$\begin{aligned} E &= (0.735 + 0.22 + 0.045)\, mc^2 \\ &= (0.955 + 0.045)\, mc^2 \\ &= (1)\, mc^2 \\ &= E(\text{Einstein}) \end{aligned} \tag{11}$$

as should be [28-36]. It is relevant at this point that all the preceding results were anticipated indirectly by N. Umov's E = kmc2 as discussed in Reference [34]. Remarkably the work of Umov was long before the advent of the theory of relativity and quantum mechanics. Thus more or less in a free-spirited research somewhat similar to E-Infinity theory [34-45].

VI. Zero Set Quantum Particle and Empty Set Quantum Wave - Set Theoretical Foundation of Quantum Mechanics

Returning now to our results given by equation 3 namely $d_c^{(4)} = 4 + \phi^3$ we may ask two relevant questions. First how to get the same results using equation two rather than equation one. That is easily answered by setting a = 3 and b = 2 in equation [2] and finding that [16-19], [34-36].

$$D = 3 + (2)(\phi) = 3 + 1 + \phi^3 = 4 + \phi^3 \tag{12}$$

Which is what we were expecting due to the equivalence of equation (1) and equation (2), Second we would like to decide which is the real dimension of this space? Is it the topological 4 or the Hausdorff $4 + \phi^3$? The answer is actually clear from our very mathematical derivation procedure and a little contemplation teaches us that both dimensions together is an optimal way of fixing this space. That way we are naturally led to the notion of the bi-dimensionality [2, 19, 35].

$$D \equiv \left(4, 4 + \phi^3\right) \tag{13}$$

Which is vital in our computation of the dark energy density of the cosmos [3, 19] as we will see momentarily. We mean by this that the two fundamental notions of the set theoretical quantum mechanics of E-Infinity theory namely the zero set and the empty set can be of use only when defined using E-Infinity bi-dimension [2, 19, 31]. To show this point in a systematic way we recall one of the most important starting points of E-Infinity set theoretical formulation enshrined in the basic tenant of real life and potent philosophy of being and nothingness [35]. In short the point is that all what we need is to assert what every pure mathematician working on set theory has long known, namely that zero, empty and nothing are entirely different [19]. Zero in set theory is a set full with zeros i.e. it is not empty so it is a very respectable something to reckon with as it is full and even when all the things which we call zero disappear, leaving the set functioning, we still refer to it as the empty set. It was the fundamental contribution of a great Australian mathematician K. Menger and probably a little earlier the achievement of a very young Russian mathematical genius P. Urysohn that they discovered the deductive topological dimensional system and gave the zero set the obvious dimension zero and the empty set the somewhat

not as obvious topological dimension minus one [19]. It took then some time and the combined ingenuity of many mathematicians particularly J. Von Neumann, R. Penrose and the inventor of non-commutative geometry A. Connes to come up with equation (2) the present work. Subsequently after some focused effort by the pioneers of E-Infinity, including the present author it was found that the zero set is uniquely and pragmatically fixed by the bi-dimension [15-22].

$$\mathrm{D}(0) \equiv (0, \phi) \tag{14}$$

and was immediately recognized by the present author to be the best way to model the pre-quantum particle while the empty set is given by [2,19]

$$\mathrm{D}(-1) \equiv \left(-1, \phi^2\right) \tag{15}$$

and the best model possible for the illusive pre-quantum wave. As a consequence of the above, it follows then that there are infinitely many empty sets with increasing degree of emptiness i.e. we have [2,19]

$$\mathrm{D}(-2) \equiv \left(-2, \phi^3\right) \tag{16}$$

which is what we call fractal scale invariant Einstein spacetime because $1/\phi^3 = 4 + \phi^3$, and [2,19]

$$\begin{aligned} \mathrm{D}(-3) &\equiv \left(-3, \phi^4\right) \\ \mathrm{D}(-4) &\equiv \left(-4, \phi^5\right) \\ \mathrm{D}(-5) &\equiv \left(-5, \phi^6\right) \end{aligned} \tag{17}$$

and so on till we find [2, 19]

$$\begin{aligned} \mathrm{D}(-\infty) &\equiv \left(-\infty, \phi^{\infty}\right) \\ &\equiv (-\infty, 0) \end{aligned} \tag{18}$$

which is the true mathematical definition of nothingness. This nothingness has kept philosophers busy for as long ago as there were ancient philosophies such as Socrates and Plato up to our time and modern philosophers who shaped modern time such as Heidelberg, Hussler, Sartre and Camus [36] to mention a few names of modern philosophers who impacted our lives even on the artistic and political levels [37].

VII. Zero, Infinity, Hardy's Quantum Entanglement, Unruh Temperature and the Immirzi ϕ^6

In the scheme of things, it follows also that we can define infinity using equation (2) to be [2, 19, 35, 36].

$$\begin{aligned} \mathrm{D}(+\infty) &\equiv \left(+\infty, \phi^{-\infty}\right) \\ &\equiv \left(+\infty, (1/\phi)^{\infty}\right) \\ &\equiv (+\infty, +\infty) \end{aligned} \tag{19}$$

which is in fact not only infinite but incommensurately infinite i.e. it is an infinity which cannot be even counted making it an infinity which is larger than ordinary infinity [19].

Building on the above we have shown on many previous occasions that there are at least three fundamental physical phenomena of predominant theoretical notions, which may be topologized using the E-Infinity golden mean system [19, 35, 38].

Here we are referring to the topological Unruh temperature given by $\left(-3,\phi^4\right)$, the topological Hardy quantum entanglement given by $\left(-4,\phi^5\right)$ and the topological Barbero-Immirzi parameter given by $\left(-5,\phi^6\right)$ and which links loop quantum gravity to super string theory in order to produce the correct black hole results [14].

VIII. The Degree of Emptiness and an Empty Set

In what follows, the following empty sets with increasing degree of emptiness [2, 19, 35, 38]:

$$\begin{aligned}&\mathrm{D}(-2)=\left(-2,\phi^3\right),\ \mathrm{D}(-3)=\left(-3,\phi^4\right),\\&\mathrm{D}(-4)=\left(-4,\phi^5\right)\text{ and }\mathrm{D}(-5)=\left(-5,\phi^6\right)\end{aligned}\tag{20}$$

will play a pivotal role and it is relatively easy to show that they are a sequence of cobordism meaning that the empty set is the surface of the zero set and that [2, 19, 35, 38]

$$\mathrm{D}(-2)=\left(-2,\phi^3\right)\tag{21}$$

spacetime set is thus the surface of the empty set. Consequently, the topological Unruh temperature set [2, 19]

$$\mathrm{D}(-3)=\left(-3,\phi^4\right)\tag{22}$$

is the surface of the spacetime set and [2, 19, 35, 38]

$$\mathrm{D}(-4)=\left(-4,\phi^5\right)\tag{23}$$

of the Hardy quantum entanglement set is the surface of the previous set D (-3) and so on indefinitely [19]. This particular feature is behind the existence of the hidden dark energy sector of the cosmos as we will reason in this work which in turn means that the non classical character of the quantum universe is nothing more or less that the role which an infinite dimensional hierarchal topology of our cosmos display to three dimensional time bound creatures which are known as humans.

IX. Ordinary Energy Density and Dark Energy Density of the Cosmos

Now our next task is to draw on the zero set and the empty set as well as the fact known to E-Infinity researchers since quite a long time that Kaluza-Klein's five dimensionality is the exact mean dimension of a Cantorian space time consistent with the phenomenon of the hidden dark energy of the cosmos [2, 19, 35, 38]. This is all what we need to do the elementary calculation apart of realizing that while the ordinary energy of the cosmos is correlated, the dark energy is uncorrelated as evident from the Rindler wedge space theory [43]. Consequently, applying this to the ordinary energy found from a five dimensional intersection of the zero set will lead us to the value ϕ^5 which is the magnitude of Hardy's quantum entangled probability so that letting V go to C in our classical formula of the kinetic energy would capture the exact solution with deceptive ease leading to [3, 16, 19, 38]

$$\begin{aligned}\mathrm{E}(0)&=\frac{1}{2}\phi^5\mathrm{mc}^2\\&=\left(\phi^5/2\right)\left(\mathrm{mc}^2\right)\end{aligned}\tag{24}$$

Within approximate rational terms this may be written as [2, 3, 19]

$$E(0) = mc^2 / 22 \tag{25}$$

In other words the ordinary energy density of the universe is only $\phi^5 / 2$, 1/22 percent of the maximum energy i.e. 4.5% and that is in complete agreement with measurements and observations. In analogy with the above however using the empty five dimensional set, we find the total dark energy density of the cosmos as the uncorrelated union rather than intersection of five empty sets with the topological volume $5\varphi^2$ so that one finds the total uncorrelated dark energy density of the universe to be [2, 3, 19, 38]

$$\begin{aligned} E(D) &= \frac{1}{2}\gamma(D)(m)(v \to c)^2 \\ &= \frac{1}{2}\left(5\phi^2\right)mc^2 \\ &= \left(5\phi^2 / 2\right)mc^2 \end{aligned} \tag{26}$$

Ignoring irrationality and writing the above in a rational term, we find our accurate approximation for E(D) to be given by [2,3,19]:

$$E(D) = \left(\frac{21}{22}\right)mc^2 \tag{27}$$

which is about 95.5% of the total maximum energy of $E = mc^2$ and this is again consistent with all accurate measurements and observations [3, 17, 29].

X. Pure Dark Energy and Dark Matter Energy of the Cosmos

Now we are reasonably equipped to tackle the new main points of the present work, which is to accurately discriminate between pure dark energy and dark matter energy and give it a mathematical and physical meaning. First and to avoid considerable misunderstanding, we repeat what we know for sure by now namely that ordinary energy is directly proportionate to a five dimensional zero set pre quantum particle of a topological value ϕ^5 where ϕ^5 is Hardy's quantum probability of two entangled quantum particles and the total dark energy density is similarly proportionate to uncorrelated 5 dimensional topological volume of an empty set pre quantum wave i.e. $5\phi^2$. Thus we are truly content now to have derived the astonishing fact that the pure dark energy is proportionate to a ten dimensional Unruh temperature uncorrelated set given by (10) (ϕ^4) and leading to

$$(PD) = \frac{1}{2}\left(10\phi^4\right) \tag{28}$$

This is thus an energy density equal to

$$\begin{aligned} E &= \left(5\phi^4\right)mc^2 \\ &= 0.7294901699mc^2 \end{aligned} \tag{29}$$

On the other hand and quite similarly the dark matter energy is proportionate to the topological Barbero-Immirzi parameter

$$\gamma(DM) = (8)\left(\phi^6\right) \tag{30}$$

where ϕ^6 is the Barbero-Immirzi parameter [14] leading to the corresponding energy density in 8 dimensional super space [1-3], [18]

$$\begin{aligned} E(DM) &= \left(\frac{1}{2}\right)\left(8\phi^6\right)mc^2 \\ &= \left(4\phi^6\right)\left(mc^2\right) \\ &= (0.2229123605)\,mc^2 \end{aligned} \tag{31}$$

This means the dark matter energy of the universe is about 22% of the maximal energy $E = mc^2$ in very reasonable agreement with the most accurate measurement [3, 32].

Alas the preceding result, although qualitatively very accurate, it is quantitatively not more than a very good approximation. This becomes obvious when we add the dark energy sector as given by equation 23 and 30 and find that

$$mc^2\left(5\phi^4 + 4\phi^6\right) = (0.9524025)\,mc^2 \tag{32}$$

while the exact sum should be

$$\left(mc^2\right)\left(\frac{5\phi^2}{2}\right) = (0.954915)\,mc^2 \tag{33}$$

As we said this result of $0.9524mc^2$ is a very good approximation to $(0.954915)\ mc^2$ but is inexact. The reason is that both pure dark energy and dark matter energy are weakly coupled as shown in previous publications using a different method which we will discuss next [18].

XI. Cosmic Energy Density from Heterotic String Theory

As we know from earlier derivations using the Heterotic string theory, this theory is defined dimensionally by [18]:

$$26\text{-}10 = 16 \tag{34}$$

where 26 is the dimension of the old bosonic string theory and 10 are the super strings dimensionality while 16 is the David Grosse et al extra bosonic dimensions. Utilizing this theory by subtracting Einstein's D = 4 from D 26 we are left with a reference dimension equal to $D = 26 - 4 = 22$ and find in the first crude approximation the following equation [18]:

$$\begin{aligned} E &= \left(\frac{22}{22}\right)mc^2 \\ &= \left(\frac{1+5+16}{22}\right)mc^2 \\ &= \left(\frac{1}{22}+\frac{5}{22}+\frac{16}{22}\right)mc^2 \\ &= \left(\gamma_0 + \gamma_{DM} + \gamma_{PE}\right)mc^2 \end{aligned} \tag{35}$$

where $\gamma_0, \gamma_{DM}, \gamma_{PD}$ are the cosmic energy destiny of the Ordinary energy, γ_{DM} the cosmic energy destiny of dark matter and γ_{PE} is the cosmic energy destiny of pure dark energy [18]. The corresponding equations are thus: [2, 3, 18, 19]

$$E(0) = mc^2 / 22 = (0.04545)\,mc^2 \tag{36}$$

for ordinary energy

$$E(DM) = (5/22)\,mc^2 = (0.22727)\,mc^2 \tag{37}$$

for dark matter energy and

$$E(PD) = (16/22)\,mc^2 = (0.7272)\,mc^2 \tag{38}$$

for pure dark energy [18].

The above result can be refined by using the exact transfinitely corrected Heterotic dimension [18] 26 + k, 10, 16 + k and consequently 22 + k where k is 'tHooft's renormalization equal to [16]

$$K = \phi^3\left(1-\phi^3\right) = 0.18033989 \tag{39}$$

Consequently, we have [3, 16]

$$\begin{aligned} E(0) &= \frac{mc^2}{22+k} \\ &= (0.0450849)\,mc^2 \end{aligned} \tag{40}$$

which is the exact value. As for the dark section, we obtain in this case [18]

$$\begin{aligned} E(DM) &= (5/22+k)\,mc^2 \\ &= (0.225424)\,mc^2 \end{aligned} \tag{41}$$

and [18]

$$\begin{aligned} E(PD) &= \left[(16+k)/(22+k)\right]mc^2 \\ &= 0.7294901688mc^2 \end{aligned} \tag{42}$$

This last value is exactly equal to the approximation given by equation (28) which is an improvement compared to (16/22). However the first result (5/22+k) = 0.225424 is an improvement on 5/22 = 0.227 but is not equal to $4\phi^6 = 0.2229123$. As we said earlier, the reason for all these small discrepancies is that E(DM) and E(PD) are weakly couples and this coupling can be easily determined and is equal to $\Delta = 0.080325$. That way the exact expression is given by [18]

$$\gamma(DM) = (5-\Delta)/(22+k) = 22.1803\% \tag{43}$$

which is the exact value and give dark matter a convincing picture namely being due to the compactified 22 + k dimension of the old superstring theory [2, 3, 19]. Similarly the exact value for the pure dark energy density is found to be

$$\begin{aligned} E(PD) &= mc^c\left(\frac{16+k+\Delta}{22+k}\right) \\ &= \frac{16.26066429}{22+k}\,mc^2 \\ &= (0.733111619)\,mc^2 \end{aligned} \tag{44}$$

Again this is the exact value as discussed in previous publications [2, 3, 18].

One maybe interested in seeing how the Heterotic theory could give the same approximation found using equation (30). This is the case indeed as shown below

$$\begin{aligned} E(PD) &= \frac{5-\phi^6}{22+k} mc^2 \\ &= \frac{4.94427191}{22.18033989} mc^2 \\ &= 0.22291236 mc^2 \end{aligned} \tag{45}$$

which is exactly equal to the result of the said equation 30 namely

$$\begin{aligned} E(PD) &= 4\phi^6 mc^2 \\ &= 0.22291236 mc^2 \end{aligned} \tag{46}$$

XII. Hidden Extra Dimension, Measure Concentration of Dvoretzky's Theory and the Holographic Boundary

Last but not least, it is not irrelevant to contemplate the geometrical topological picture which relates the total energy to ten dimensions that surround an eight dimension super space which in turn surrounds five dimensional space while the entire structure is embedded and paradoxically so in 5 dimensional space. So how could this be consistent? To understand this we have to understand the nature of infinite dimensional topology in general and E-Infinity as well as hierarchal topology in particular. E-infinity has formally infinite dimensions but its mean dimension is equal to 4.23606797 as an average Hausdorff dimension as well as an equal average topological dimension $\langle d_c \rangle$ and $\langle n \rangle$ respectively as discussed in length in the relevant literature [3, 16, 20, 39]. That is how we come to dimensions like $5+\phi^3$ fractal Kaluza-Klein space $11+\phi^5$ Fractal Witten space [15] and fractal Vafa's 12-dimensional space [6]. Then and even more importantly we have the measure concentration phenomena as per Dvoretzky's theorems which apply where dimensionality exceeds four when it is basically fractal as in 'tHooft's-Weltman-Wilson space 4-k compared to Einstein's 4 from which we could deduct the total cosmic dark energy density by simply finding [16]:

$$\begin{aligned} \gamma(D) &= \frac{4-k}{4} \\ &= 95.49150275\% \end{aligned} \tag{47}$$

while ordinary energy is given by:

$$\begin{aligned} \gamma(0) &= \frac{k}{4} \\ &= 4.50849725\% \end{aligned} \tag{48}$$

And this is not all or the end of the story as we have the possibility of dividing space into bulk and holographic boundary implied by the fundamental equation relating the dimension of E8E8 Lie symmetry group of super string and Heterotic super string 496 with the dimensionality of the boundary governed by the symmetry group SL(2, 7) i.e. 336 dimensions as given by

$$
\begin{aligned}
|\,E_8 E_8\,| &= |\,SL(2.7)\,| + \bar{\alpha}_0 + G + SU(2) \\
&= 336 + 137 + 20 + 3 \\
&= 496
\end{aligned}
\tag{49}
$$

where $\bar{\alpha}_0$ is the inverse electro-magnetic fine structure constant, G is the number of the components of the Riemannian tensor in four dimensions and SU(2) the weak force compactification of SL(2, 7) [1, 3, 44, 45].

XIII. THE UNITY OF ART AND SCIENCE

There remains one simple point which this paper wishes to illustrate and which is immensely important for understanding the enormous power of unification dormant inside number theory as applied to physics in general and the golden mean number system of E-Infinity in particular which does not unify all the fundamental interactions only but upon deep consideration unifies art and science in the same spirit of Pythagorean mathematical cosmic music of numbers from which the universe emerges [6], [8-13]. Thus these esoteric ideas become far less esoteric and far more real when we look deeply and realize how the golden mean is present in works of art as a harmonic principle of composition of painting and music as well as the experimentally tested Hardy quantum entanglement ϕ^5, to mention only one example of many which we have discussed in various old and recent publications [3, 6, 7, 8, 15, 18, 19, 21, 22, 30], [38-45].

XIV. CONCLUSION

The present work gives a truly grand unification theory which explains in essence not only why quantum groups leads to the topological dimension D = 4 [48, 49] of Einstein while E-infinity leads to the scale invariant E-infinity Hausdorff dimension $D = 4 + \phi^3$ [48, 49] and that for the same golden mean backbone value ϕ. But even more than that, it explains why Hopf algebra is beautiful [49] and why beauty is a scientific truth. In any accurate experiment, testing a theory entails in general finding certain numbers to find out if the theory is right or wrong. Hence on this account alone when some call numbers numerology then at a minimum this would be immensely misguided and short-sighted beyond measure. It was the invention of the zero by the ancient Indian civilization and its sweeping implementation by the Arabs in the decimal number system that gave us a solid basis for the subsequent modern sciences. This simple step must be thus considered as one of the most important intellectual steps which man took towards modern civilization. The same could be said and much more about modern set theory and the deep meaning of countable infinity and incommensurate infinity. Even far more advanced than all of that and far more efficient than the binary number system employed in modern digital computers, is the almost miraculous golden mean based number system which surpasses anything we could think of and deserves to be labeled a transfinite Alan Turing machine or golden computer for short [41-42]. How Pythagoreans surmised that music, harmony, number theory and the truth are all linked and are the basic blueprints of nature's language, is and will remain a mystery at least for the present author.

The paper at hand starts from this ancient time and takes the ancient theory of Pythagoras seriously and uses its basic premises to solve the mystery of dark energy of the cosmos and the fundamental question about the unity of art and science [6].

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AUTHOR'S PROFILE

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Professor M.S. El Naschie was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree in Engineering plus being a professional chartered engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt. Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work is cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems. Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 80 and his i-10 index is 784 and total citations are 35432 according to Google Scholar Citation.