An Approach for Engineering Tuning of Smith Predictor with Dynamic Object from High Order

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Abstract – An approach is proposed for engineering tuning of Smith predictor with a dynamic object from high order. There is a proposal to solve the problem by solving the characteristic equation (controller-compensator). As a result of the high order dynamic system analysis, the tuning parameters of the Smith predictor are calculated. The transitional processes of the system (controller-compensator) are dealt with by assignment. For the transitional process by assignment, overshoot $\sigma = 16$, $2\%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, $0, 1\%$ inaccuracy is observed in theory. Therefore, the proposed approach for engineering tuning of Smith predictor with a high order dynamic object is suitable for use in high order dynamic systems analysis.

Keywords – Smith Predictor, Tuning, Dynamic System, High Order Object, Transfer Function.

I. INTRODUCTION

There is a time-delay (pure or transient) in each automatic control system, which can be small or significant and it leads to a deterioration of the control system. This is a major cause of poor quality control, which can lead to major overshoot and, in some cases, to unstable control system. PID-controllers work successfully when the relative delay of the object does not exceed one. In many cases, however, it exceeds a unit, so special delay compensators (Smith Predictor) have been developed. The basic idea behind this development is how to set up a controller operating with a delayed object can be reduced to a controller setting for an object without delay. This is possible if a delayed object model is incorporated into the master controller structure so the controller can be "misled" to operate as if the object was not delayed.

II. PROBLEMS WITH THE TUNING OF SMITH PREDICTOR IN HIGH ORDER SYSTEMS

Since the implementation of a time-delay unit with analogue means is very difficult, Smith's controller has not found a practical application right after publishing the idea. With the introduction of microprocessor controllers, its wide use is also possible. This example shows the limitations of PI-control for processes with long dead time and illustrates the benefits of a control strategy called "Smith Predictor". Note that the delay is more than twice the time constant. This model is representative of many chemical processes. The performance of the PI-controller is severely limited by the long dead time. This is because the PI-controller has no knowledge of the dead time and reacts too "impatiently" when the actual output $y$ does not match the desired set point $y_{sp}$. Everyone has experienced a similar phenomenon in showers where the water temperature takes a long time to adjust. There, impatience typically leads to alternate scalding by burning hot and freezing cold water. A better strategy consists of waiting for a change in temperature setting to take effect before making further adjustments. And once we have learned what knob setting delivers our favorite temperature, we can get the right temperature in just the time it takes the shower to react. This “optimal” control strategy is the basic idea behind the Smith Predictor scheme. With the help of the Smith Predictor control structure we are able to increase the open-loop bandwidth to achieve faster response and increase the phase margin to reduce the overshoot. The Smith Predictor provides much faster response with no overshoot. The difference is also visible in the frequency domain by plotting the closed loop Bode response from $y_{sp}$ to $y$. Note the higher bandwidth for the Smith Predictor [14].

III. POSSIBLE OPTIONS FOR SOLUTION OF THE ASSIGNED TASK

Many papers [1]-[14] have proposed a number of options to improve the control system with a Smith controller or to solve specific problems. The principal controller typically implements a proportional-integral (PI) control law, including an control object model. Here, a differential component is not necessary because predictions are made by the new time-delay compensation structure.

One of the possible ways to facilitate the setting-up of the Smith controller is known as a predictive PI-controller (PPI) and consists of the following. The controller in the main loop realizes a PI-algorithm, and the control object model in the controller is a aperiodic unit and a time-delay unit [15].

The purpose of this paper is to offer an approach for engineering tuning of Smith predictor with a dynamic object from high order by solving the characteristic equation of the system (controller - compensator).

IV. PROPOSAL FOR SOLVING THE PROBLEM BY SOLVING THE CHARACTERISTIC EQUATION

Figure 1 shows the structural diagram of a ACS comprising a high order control object and a Smith predictor.

In the structure under consideration, the controller is enclosed by a negative feedback in which the compensator K is also included. The transfer function of the closed system with respect to the reference is:

$$W_{sp}(s) = \frac{Y_{sp}(s)}{Y_{sp}(s)} = \frac{W_c(s)W_{ac}(s)e^{-\alpha}}{1 + W_c(s)W_{ac}(s) + W_c(s)W_{ac}(s)e^{-\alpha}}.$$
where \( W_c(s) \) - is the transfer function of the controller; 
\( W_o(s) \) - is the transfer function of the control object; 
\( W_k(s) \) - is the transfer function of the compensator.

The transfer function of the compensator has the kind:

\[
W_k(s) = W_o(s)(1 - e^{-\tau s}).
\]

For a more precise model of the control object in the controller a second order control object is chosen (with two aperiodic link), i.e.

\[
W_o(s) = \frac{k_o}{(T_{o1}s + 1)(T_{o2}s + 1)}.
\]

It is known that the expression in the denominator \( W_o(s) \) defines the characteristic equation of the system and the presence of time-delay deteriorates the stability. For this purpose additional feedback and the presence of time is introduced so as to eliminate the impact of time-delay on the stability of the system, i.e.

\[
1 + W_c(s)W_o(s) + W_c(s)W_o(s)e^{-\tau s} = 1 + W_c(s)W_o(s)
\]

Therefore, it is necessary to have a model of the object in order to put into practice Smith's controller. The transient function of the Smith’s controller will be the type

\[
W_{Smith Predictor}(s) = \frac{W_c(s)}{1 + W_c(s)W_o(s)(1 - e^{-\tau s})}
\]

![Fig. 1. A system with a high order control object and a Smith predictor](image)

The transfer function of the loop system (controller - compensator) without time-delay (fig. 1), when working under set point is:

\[
W_p(s) = \frac{W_c(s)W_o(s)}{1 + W_c(s)W_o(s)} = \frac{k_o}{(T_{o1}s + 1)(T_{o2}s + 1)} \frac{T_s + 1}{T_s} = \frac{\frac{k_o}{T_s}}{(T_{o1}s + 1)(T_{o2}s + 1)} \frac{T_s + 1}{T_s} = \frac{\frac{k_o}{k_o}}{T_s + 1}
\]

The transfer function of the loop system (controller - compensator) without time-delay (fig. 1), in case of disturbance is:

\[
W_p(s) = \frac{W_c(s)W_o(s)}{1 + W_c(s)W_o(s)} = \frac{k_o}{(T_{o1}s + 1)(T_{o2}s + 1)} \frac{T_s + 1}{T_s} = \frac{\frac{k_o}{T_s}}{(T_{o1}s + 1)(T_{o2}s + 1)} \frac{T_s + 1}{T_s} = \frac{\frac{k_o}{k_o}}{T_s + 1}
\]

We propose that the analysis of the system (controller - compensator) be carried out with a successively connected oscillating and aperiodic link, i.

\[
W(s) = \frac{1}{T_o s^2 + 2\xi T_o s + 1}\cdot T_s + 1
\]

Assuming that the time constant of the aperiodic link (first order low pass filter) is equal to the time constant of the oscillating link, i. \( T = T_o \) is obtained

\[
W(s) = \frac{1}{T_o s^2 + 2\xi T_o s + 1}\cdot T_o s + 1
\]

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

\[
(T_o s^2 + 2\xi T_o s + 1)(T_o s + 1) = T_o s^3 + (2\xi + 1)T_o s^2 + (2\xi + 1)T_o s + 1.
\]

If we equal the corresponding coefficients in front of \( s^3 \), \( s^2 \) etc. from the characteristic equation (5) to the coefficients of \( s^3 \), \( s^2 \) etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

\[
W_p(s) = \frac{\frac{k_o}{k_o}}{T_o s^3 + (2\xi + 1)T_o s^2 + (2\xi + 1)T_o s + 1}
\]

where \( k_{sp} = 1 \) is called a coefficient of the system assignment.

The transmission function of the closed disturbance system will have the final appearance

\[
W_p(s) = \frac{\frac{k_o}{k_o}}{T_o s^3 + (2\xi + 1)T_o s^2 + (2\xi + 1)T_o s + 1}
\]
where \( k_s = \frac{T_i}{k_p} \cdot \frac{1}{T_0} = \frac{T_i}{k_p} \sqrt{\frac{k_s k_p}{T_{1a} T_{2a}}} = \frac{T_i^2 k_a}{k_p T_{1a} T_{2a}} \) is called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of \( s \) in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

\[
T_\alpha = \sqrt{\frac{T_{1a} T_{2a} T_i}{k_s k_p}}.
\]  

(8)

Similarly, the attenuation coefficient \( \xi \) is determined. For it two expressions of \( s^2 \) and \( s \) of (6) are obtained, i.e.

The first expression that can be determined \( \xi \) is

\[
(2\xi + 1)T_0^2 = \frac{T_i(T_{1a} + T_{2a})}{k_s k_p}.
\]  

(9)

If we only express \( \xi \) we obtained

\[
\xi = \frac{1}{2} \left[ \frac{T_i(T_{1a} + T_{2a})}{T_0 k_s k_p} - 1 \right].
\]  

(10)

The second expression from which can be determined \( \xi \) is

\[
(2\xi + 1)T_0 = \frac{T_i(1 + k_s k_p)}{k_s k_p}.
\]  

(11)

If we express only \( \xi \) it is obtained

\[
\xi = \frac{1}{2} \left[ \frac{T_i(1 + k_s k_p)}{T_0 k_s k_p} - 1 \right].
\]  

(12)

If the expressions (9) and (11) are divided into one another, it is obtained

\[
T_\alpha = \frac{T_{1a} + T_{2a}}{1 + k_s k_p}.
\]  

(13)

If the expressions (10) and (12) are equal to one another, i.e.

\[
\frac{1}{2} \left[ \frac{T_i(T_{1a} + T_{2a})}{T_0 k_s k_p} - 1 \right] = \frac{1}{2} \left[ \frac{T_i(1 + k_s k_p)}{T_0 k_s k_p} - 1 \right]
\]  

(14)

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal, i.e.

\[
T_\alpha = \sqrt{\frac{T_{1a} T_{2a} T_i}{k_s k_p}} = \frac{T_{1a} + T_{2a}}{1 + k_s k_p}.
\]  

(15)

If an expression (15) is solved regarding the time const-

-tant of integration \( T_i \), it is obtained

\[
T_i = \frac{(T_{1a} + T_{2a})^3 k_s k_p}{T_{1a} T_{2a} (1 + k_s k_p)^3}.
\]  

(16)

The proportionality coefficient of the controller \( k_p \) can be determined by an expression (11), i.e.

\[
k_p = \frac{T_i}{(2\xi + 1)T_0 - T_i k_s k_p}.
\]  

(17)

Example: The transitional process of the control object is of high order and is described with the following transfer function [17].

\[
W_{co}(s) = \frac{1}{(10s + 1)^3} e^{-60s}.
\]

The following algorithm performs the following:

1. Take the transitional process of the control object that is smooth and normalizing.
2. Since the control object model is of second order – fig. 2 (two consecutively connected aperiodic links with equal time constants) - the transitional characteristic is monotone with a transient delay, it is chosen to approximate the method of Ormans [16]. After the approximation, it is determined: \( k_o = 1 \), \( T_{1a} = T_{2a} = 13,125 \sec \) and \( \tau_a = 64.5 \sec \).

3. By the expressions (16) and (17), the tuning parameters of the PI-controller are calculated using the iteration procedure.

First, calculate the value of \( k_p \), at a set value of \( T_i \approx 3\tau_a \), where \( \tau_a \) is determined after a Cupfmler approximation

\[
k_p = \frac{T_i}{(2\xi + 1)T_0 - T_i k_s k_p} = 1.
\]

Finally \( T_i \) is calculated, if its value is close to the one above, the calculation procedure is terminated.

\[
T_i = \frac{(T_{1a} + T_{2a})^3 k_s k_p}{T_{1a} T_{2a} (1 + k_s k_p)^3} = 13,125 \sec.
\]
If the value of $T_i$ differs greatly from the set above, the calculation procedure starts from the beginning by selecting a value $T_i$, of so to minimize the difference between the set value and the value obtained.

4. By the expression (12) the damping factor $\zeta$ is calculated and approximately what is the value of the overshoot $\sigma$ from [15]

$$\zeta = \frac{1}{T_i k_p} \left[ \frac{1}{2} \left( 1 + k_p k_a \right) T_i \right]^{-1} = 0,5$$

$$\sigma^2 = \exp \left( -\frac{\zeta}{\sqrt{1-\zeta^2}} \right) = 0,0266 \quad \text{or} \quad \sigma = 16,3 \%.$$  

5. Determine the maximum dynamic deviation $y_1$ in the expression given in [15]

$$y_1 = \exp \left( -\frac{\zeta}{\sqrt{1-\zeta^2}} \right) = 0,561.$$  

6. If any of the above two parameters does not meet the prerequisites for quality, adjust the controller.

It is compared how an ordinary PI controller and two Smiths’ controllers would handle with this object [17].

The PI controller (PI) setting is made using the Astrom-Hagglung formulas [17] (at model parameter values.

$$k_a = 1, \quad T_a = 19,6 \text{ sec and } \tau_a = 72 \text{ sec};$$

$$k_p = 0,15 * k_a = 0,15$$

$$T_i = 0,4 * \tau_a = 28,8 \text{ sec}.$$  

The Smith’s Controller setting (Smith 2) in the model of the control object in the controller is a aperiodic link with time-delay [17] (at model parameter values $k_a = 1$, $T_a = 19,6 \text{ sec and } \tau_a = 72 \text{ sec};$)

$$k_p = \frac{1}{k_a} = 1$$

$$T_i = T_a = 19,6 \text{ sec}.$$  

Setting the Smith’s controller (Smith 1) in the model of the control object in the controller from second order with time-delay (at model parameter values $k_a = 1$, $T_{ta} = T_{2a} = 13,125 \text{ sec and } \tau_a = 64,5 \text{ sec};$)

$$k_p = \frac{T_i}{\left( \frac{2\tau_i}{T_i} + 1 \right) T_{ta} - T_i} k_a = 1$$

$$T_i = \frac{T_{ta} k_p}{(1 + k_a k_p) T_{2a}} = 13,125 \text{ sec}.$$  

For the closed loop system with PI-controller it is seen that the process is quite slow with an adjustment time of about 800 seconds (with a precision of ± 2%). This is due to the low value of the controller coefficient $k_p$. For the Smith’s controller system, it appears that the transitional process of the (Smith 1) is faster and better than that indicated by (Smith 2). Both transitional processes have a low adjustment time compared to the regular PI-controller. The small fluctuations in the process (Smith 2) are due to the inaccurate model of the control object. It can be seen that in the exact model of the control object (Smith 1) these fluctuations are missing. The simulation shown in fig. 3 shows the advantages of the Smith controller in front of the conventional PI-controller in the case of system with a long timedelay. It should be noted that the use of a Smith controller makes sense only when the relative delay in the control object is greater than one. Otherwise, a well-tuned PID-controller would do just as successful, while being considerably evident and simple in design.

![Fig. 3. Transitional processes by assignment](image)

The transitional processes of the closed system (fig. 1) by assignment and by disturbance are shown in fig. 3. For the transitional process by assignment, overshoot $\sigma = 16, 2\%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0, 1% inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering tuning of a Smith predictor with a dynamic object from high order is suitable for use in the analysis of high order dynamic systems.

**V. CONCLUSIONS**

An approach is proposed for engineering tuning of Smith predictor with a dynamic object from high order. There is a proposal to solve the problem by solving the characteristic equation (controller-compensator). As a result of the analysis of high order dynamic system, the tuning parameters of the Smith predictor are calculated.

**REFERENCES**


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