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Hall Effect on Oscillatory Hydromagnetic Free Convective Flow of a Visco-elastic Conducting Fluid Past an Infinite Vertical Porous Plate in the Presence of Dissipation

P.K. Mishra[1], S.Biswal[2], G.S.Roy[3]
[1]Department of Physics. Bhadrak Autonomous College Bhadrak, India
[2]Department of Physics. SFRC, Bhubaneswar. India
[3]PG Departmentof appliedPhysics. College of Engg.& Tech. CET, BBSR.
e-mail: pkmishra1367@gmail.com

***Abstract* – Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic conducting fluid past an infinite vertical porous plate in the presence of dissipation has been analysed. The problem is formulated with the development of constitutive equations of the flow and the solutions are obtained by applying small parameter regular perturbation technique. The velocity and temperature profiles are plotted. Numerical values of shear stresses and the rate of heat transfer are entered in the tables. It is observed that the speed of flow is minimum for Newtonian flow and increases for non-Newtonian flow. The resulting temperature is fluctuating. Similar nature is noticed in case of primary and secondary velocity. Increase in non-Newtonian parameter (R_C) and external magnetic field strength(M) reduce the shear stresses.**

***Keywords* – Hall effect, Unsteady, Hydromagnetic, Free convective flow, Visco-elastic, Conducting fluid, Porous plate, Dissipation.**

I. Introduction

Magneto hydrodynamic (MHD)unsteady free convective flow of both viscous and visco elastic conducting fluids have a wide range of application such as relevancy in a global general circulation with convection in equatorial zones. Further, the study of effects of Hall currents on fluid flow and heat transfer has a lot of application in cooling of nuclear reactor. MHD power generation and in several astrophysical situations.

The problem of steady and unsteady free convection flow along a vertical plate in the presence of an applied transverse magnetic filed is of general interest from the view point of many applications, such as, in space flight and nuclear fusion research. The literature is replete with many examples of those studies. A few of them are the works of Gupta (1960), Singh and Singh (1983), Mishra and Mahapatra (1975), Sahoo, Data and Biswal (2003), Hossain (1986) and Hossain and Mohammed (1988). Hall effects on hydromagnetic free convection flow along a porous flat plate with mass transfer has been analysed by Hossain and Rashid (1987). Hall effect on oscillatory hydromagnetic free convective flow past an infinite vertical porous flat plate has been investigated by Mohapatra and Tripathy (1988).

When the external transverse magnetic field strength is very strong, one can not neglect the effect of Hall current on the flow field. In the MHD flow, the Hall effect rotates the current vector away from the direction of the electric field and generally reduces the effect of the force that the magnetic field exerts on the flow. All the works mentioned above are based upon the studies of Hall current effects on MHD free convective viscous flow without heat sources mass transport and Dissipation etc. Biswal and Pradhan (2002) have studied the Hall effects on oscillatory hydromagtic free convective flow past an infinite vertical porous flat plate with mass transfer. Muduli, Biswal and Jena (2000) have analysed the Hall effects on oscillatory MHD free convective flow past an infinite vertical porous flat plate with mass transfer and internal heat generating sources/heat absorbing sinks. Inboth these papers, viscous conducting fluids have been taken into account. But, the Hall current effects on the unsteady MHD visco-elastic flow has wide applications in the problems of MHD power generation and the Hall accelerators as with the case of Newtonian fluid. Consequently, recent studies on the MHD unsteady visco-elastic flow with the Hall current have attracted many research schloars to investigate various such problems with different physical situations. Dash and Ojha (1889) have analysed the hydromagnetic flow and heat transfer of an elastic-viscous fluid over a porous plate in the slip flow regime. Biswal and Pattanaik (1999) have studied the Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic fluid past an infinite vertical porous flat plate. They have not taken into account the viscous and Joulean dissipation in the heat transfer. Our aim here is to investigate the problem of Hall current effects on oscillatory hydromagnetic free convective flow of a visco-elastic conducting fluid past an infinite vertical porous plate in the presence of both viscous, visco-elastic dissipation.

II. Formulation of the Problem

The vertical porous flat plate is considered to be perpendicular to the plane of this paper. The X-axis is chosen along the vertical plate while the Y-axis is normal to it. The Z-axis lies on the plate, perpendicular to both X and Y axis.

Under such a physical situation, we write the equation of continuity as

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$\frac{\partial V}{\partial y}=0,$ (2.1)

Which yields V=-V$_0$ (constant), V$_0$>0, where v is the fluid velocity along y-axis. Since the electrically conducting fluid taken here for consideration is assumed to be incompressible and an external uniform transverse magnetic field is applied to it, we write

div $\vec{H}=0$ and div $\vec{J}=0$, (2.2)

Which implies $\frac{\partial H_y}{\partial y}=0$ and $\frac{\partial J_y}{\partial y}=0$, respectively.

Neglecting the polarization effect and the induced magnetic field, we have

$\vec{E}=0$, $H_x=H_z=0$ and H_y=Const. = H_0

Hence

$$\left.\begin{aligned}\vec{J}&=(J_X,0,J_Z)\\ \vec{H}&=(0,H_0,0)\\ \vec{V}&=(u,-V_0,w)\end{aligned}\right\} \quad (2.3)$$

As the effect of Hall current is taken into the purview of discussion , the Ohm's law is modified and is represented as

$$\vec{J}+\frac{\omega_e r_e}{B_0}\left(\vec{J}\times\vec{B}\right)=\sigma\left(\vec{E}+\vec{V}\times\vec{B}+\frac{1}{en_e}\nabla P_e\right) \quad (2.4)$$

Where, ω_e is the cyclotron frequency, τ_e is the electron collision time, σ is the electrical conductivity of the fluid, e is the charge of an electron, n_e is the number density of electrons. P_e electron pressure, $B_0(=\mu_0H_0)$ is the magnetic induction, $\vec{J}$ is the current density by magnetic field. Eqns. (2.3) and (2.4) yield

$$J_x=\frac{\sigma B_0}{1+m^2}(mu-w),\ J_z=\frac{\sigma B_0}{1+m^2}(u-mw), \quad (2.5)$$

Where $m=\omega_e\tau_e$ is the Hall parameter.

The equations of motion and energy along with equation (2.5) yield eqn. of motion along X-axis.

$$\frac{\partial u}{\partial t}-V_0\frac{\partial u}{\partial y}=v\frac{\partial^2u}{\partial y^2}-\frac{K_0}{\rho}\frac{\partial^2u}{\partial y^2\partial t}-\frac{\sigma B_0^2(u+mw)}{\rho(1+m^2)}+g\beta\theta \quad (2.6)$$

And eqn. of motion along Z-axis as:

$$\frac{\partial w}{\partial t}-V_0\frac{\partial w}{\partial y}=v\frac{\partial^2w}{\partial y^2}-\frac{K_0}{\rho}\frac{\partial^2w}{\partial y^2\partial t}+\frac{\sigma B_0^2(mu-w)}{\rho(1+m^2)} \quad (2.7)$$

Eqn. of energy with viscous and Joulean dissipation is

$$\frac{\partial\theta}{\partial t}-V_0\frac{\partial\theta}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2\theta}{\partial y^2}+\frac{v}{\varepsilon c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho C_p}u^2-\frac{K_0V_0}{\rho C_p}\left(\frac{\partial\theta}{\partial y}\right)\left(\frac{\partial^2u}{\partial y^2}\right) \quad (2.8)$$

Where,

β is the co-efficient of volume expansion,

v is the kinematic viscosity,

K is that thermal conductivity,

$\theta(y,t)=T(y,t)-T_\infty$

T_∞ is the temperature of the fluid far from the plate,

T is the temperature of the fluid,

The viscous and Joulean dissipation have been taken into account here since viscous dissipation occurs in the free convection flow of a visco-elastic fluid and Joulean dissipation comes into play in the flow of a conducting fluid.

Introducing the following non-dimensional parameters,

$$\eta'=\frac{V_0y}{v},t'=\frac{V_0^2t}{4v},u'=\frac{u}{V_0},w'=\frac{w}{V_0},\theta'=\frac{\theta}{a}$$

, (where a is to be defined later)

$$G'=\frac{4g\beta va}{V_0^3},M'=\frac{4vB_0^2\sigma}{\rho V_0^3},P_r'=\frac{vPC_p}{\varepsilon K},E'=\frac{V_0^4K}{\rho v^2C_p^2},R_c'=\frac{K_0V_0^2}{\rho v^2} \quad (2.9)$$

in the eqns. (2.6) to (2.8), we obtain (dropping the dashes.)

$$\frac{\partial u}{\partial t}-4\frac{\partial u}{\partial\eta}=4\frac{\partial^2u}{\partial\eta^2}-R_c\frac{\partial^3u}{\partial\eta^2\partial t}-\frac{M}{1+m^2}(mw+u)+G\theta \quad (2.10)$$

$$\frac{\partial w}{\partial t}-4\frac{\partial w}{\partial\eta}=4\frac{\partial^2w}{\partial\eta^2}-R_c\frac{\partial^3w}{\partial\eta^2\partial t}+\frac{M}{1+m^2}(mu-w) \quad (2.11)$$

$$\frac{\partial\theta}{\partial t}-4\frac{\partial\theta}{\partial\eta}=\frac{4}{\varepsilon P_r}\frac{\partial^2\theta}{\partial\eta^2}+P_rE\left(\frac{\partial u}{\partial\eta}\right)^2+\varepsilon P_rMEu^2-R_cEP_r\left(\frac{\partial u}{\partial\eta}\right)\left(\frac{\partial^2u}{\partial\eta^2}\right) \quad (2.12)$$

Where R_c is the non-Newtonian elastic parameter.

M is the Hartmann number,

m is the Hall parameter,

P_r is the Prandtlnumber,

E is the Eckert number,

G is the Grashof number,

ε is the porosity of the porous medium.

Eqns. (2.10)-(2.12) are subjected to the boundary condition, at any time *t*.

$$\left.\begin{aligned}&u=w=0 && at\ \ y=0\\ &u=w=0 && at\ \ y=\to\infty\\ &\theta=ae^{i\omega t} && at\ \ y=0\\ &\theta=0 && at\ \ y\to\infty\end{aligned}\right\} \quad (2.13)$$

II. Method of Solution

Taking $\psi=u+iw$ (3.1)

and combining eqns. (2.10) and (2.11) we obtain

$$\frac{\partial\psi}{\partial t}-4\frac{\partial\psi}{\partial\eta}=4\frac{\partial^2\psi}{\partial\eta^2}-R_c\frac{\partial^3\psi}{\partial\eta^2\partial t}-\frac{M}{1+m^2}(1-im)\psi+G\theta \quad (3.2)$$

Introducing the non-dimensional parameter $\Omega=\frac{4vw}{V_0^2}$, the frequency parameter, and

Using eqns. (2.9) and (3.1) the boundary condition (2.13) are transformed to

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$$\left.\begin{array}{l}\psi(0,t)=\psi(\infty,t)=0\\ \theta(0,t)=e^{i\Omega t}\\ \theta(\infty,t)=0\end{array}\right\} \quad (3.3)$$

Using small parameter regular perturbation technique, we can solve equations (2.12) and (2.15). Let us take the Ecket number (E) as the perturbation parameter. Then, we can write

$$\left.\begin{array}{l}\psi=\psi_0+E\psi_1\\ \theta=\theta_0+E\theta_1\\ u=u_0+Eu_1 \text{ and}\\ w=w_0+Ew_1\end{array}\right\} \quad (3.4)$$

Neglecting the terms containing higher powers of E (≥ 2) as these terms contribute very little to the values of ψ and θ. Substituting (3.4) in the equations (2.12) and (3.2) and equating the co-efficient of E^0 and E^1 in each case, we obtain

$$\frac{\partial\psi_0}{\partial t}-4\frac{\partial\psi_0}{\partial\eta}=4\frac{\partial^2\psi_0}{\partial\eta^2}-R_c\frac{\partial^3\psi_0}{\partial\eta^2\partial t}-\frac{M}{1+m^2}(1-im)\psi_0+G\theta_0 \quad (3.5)$$

$$\frac{\partial\psi_1}{\partial t}-4\frac{\partial\psi_1}{\partial\eta}=4\frac{\partial^2\psi_1}{\partial\eta^2}-R_c\frac{\partial^3\psi_1}{\partial\eta^2\partial t}-\frac{M}{1+m^2}(1-im)\psi_1+G\theta_1 \quad (3.6)$$

$$\frac{\partial\theta_0}{\partial t}-4\frac{\partial\theta_0}{\partial\eta}=\frac{4}{\varepsilon P_r}\frac{\partial^2\theta_0}{\partial\eta^2} \quad (3.7)$$

and

$$\frac{\partial\theta_1}{\partial t}-4\frac{\partial\theta_1}{\partial\eta}=\frac{4}{\varepsilon P_r}\frac{\partial^2\theta_1}{\partial\eta^2}+P_r\left(\frac{\partial u_0}{\partial\eta}\right)^2+\varepsilon P_r M-R_cP_r\left(\frac{\partial u_0}{\partial\eta}\right)\left(\frac{\partial^2u_0}{\partial\eta^2}\right) \quad (3.8)$$

With the boundary conditions,

$$\left.\begin{array}{l}\psi_0(0,t)=0,\psi_1(0,t)=0,u_0(0,t)=0\\ \psi_0(\infty,t)=0,\psi_1(\infty,t)=0,\\ \theta_0(0,t)=e^{i\Omega t},\theta_1(0,t)=0,\\ \theta_0(\infty,t)=\theta_1(\infty,t)=0,\\ w_0(0,t)=0,u_1(0,t)=0,w_1(0,t)=0\\ u_0(\infty,t)=u_1(\infty,t)=0,w_0(\infty,t)=w_1(\infty,t)=0\end{array}\right. \quad (3.9)$$

Taking $\theta_0=e^{i\Omega t}f_0(\eta)$, eqn. (3.7) can be transformed to

$$\frac{\partial\theta_0}{\partial t}-4\frac{\partial\theta_0}{\partial\eta}=i\Omega e^{i\Omega t}f_0(\eta)-4e^{i\Omega t}f_0'(\eta)=\frac{4}{\varepsilon P_r}e^{i\Omega t}f_0''(\eta)$$

or $\frac{4}{\varepsilon P_r}f_0''(\eta)+4f_0'(n)-i\Omega f_0(\eta)=0$

or $f_0''(\eta)+\varepsilon P_r f_0'(n)-i\varepsilon\frac{P_r}{4}\Omega f_0(\eta)=0$

(3.10)

The modified boundary conditions become

$$\left.\begin{array}{l}f_0(\eta)=1 \text{ at } \eta=0\\ f_0(\eta)=1 \text{ at } \eta\to\infty\end{array}\right\}$$

(3.11)

Then, the solution of eqn. (3.10) is

$$f_0(\eta)=e^{\frac{1}{2}\left\{\varepsilon P_r+\sqrt{i\varepsilon\Omega P_r+P_r^2\varepsilon^2}\right\}\eta}$$

(3.12)

Substituting θ_0 in eqn. (3.7), we have

$$\frac{\partial\psi_0}{\partial t}-4\frac{\partial\psi_0}{\partial\eta}=4\frac{\partial^2\psi_0}{\partial\eta^2}-R\frac{\partial^3\psi_0}{\partial\eta^2\partial t}-\frac{M}{1+m^2}(1-im)\psi_0+Ge^{i\Omega}e^{\frac{1}{2}\left\{\varepsilon P_r\sqrt{i\varepsilon P_r+\varepsilon^2P_r^2}\right\}\eta} \quad (3.13)$$

Taking $\psi_0=e^{i\Omega t}F_0(\eta)$

In the above eqn. (3.13), we obtain

$$F_0''(\eta)+P_1F_0'(\eta)-P_2F_0(\eta)=P_3F_0(n) \quad (3.14)$$

Where, $P_1=\frac{4}{4-i\Omega R_c}$, $P_2=\frac{-\left[i\Omega+\frac{M}{1+m^2}(1-im)\right]}{4-i\Omega R_c}$

And $P_3=\frac{-G}{4-i\Omega R_c}$

The modified boundary conditions are

$$\left.\begin{array}{l}\eta=0,\psi_0=0,F_0(0)=0\\ \eta=\infty,\psi_\infty=0,F_0(\infty)=0\end{array}\right\}$$

(3.15)

Then, the solution of eqn. (3.14) is

$$F_0(\eta)=-P_7e^{-P_5\eta}+P_7e^{-P_4\eta}, \quad (3.16)$$

Where $P_4=\frac{1}{2}\left\{\varepsilon P_r+\sqrt{i\varepsilon\Omega P_r+\varepsilon^2P_r^2}\right\}$

$$P_5=\frac{1}{2}\left(P_1-\sqrt{P_1^2-4P_2}\right)$$

$$P_6=\frac{1}{2}\left(P_1-\sqrt{P_1^2-4P_2}\right)$$

$$P_7=P_3P_4^2-P_1P_3P_4-P_2P_3$$

Hence, $\psi_0(\eta)=e^{i\Omega t}F_0(\eta)$

$$=e^{i\Omega t}\left(-P_7e^{-P_5\eta+}P_7e^{-P_4\eta}\right) \quad (3.17)$$

As $\psi_0(\eta)=u_0(\eta)+iw_0(\eta)$

We have $u_0(\eta)=\cos\Omega t\left(-P_7e^{-P_5\eta}+P_7e^{-P_4\eta}\right)$, (3.18)

and $w_0(\eta)=\sin\Omega t\left(-P_7e^{-P_5\eta}+P_7e^{-P_4\eta}\right)$, (3.19)

Substituting (3.18) in the equn. (3.8), we have

$$\frac{\partial\theta_1}{\partial t}-4\frac{\partial\theta_1}{\partial\eta}-\frac{4}{\varepsilon P_r}\frac{\partial^2\theta_1}{\partial\eta^2}=\cos^2\Omega t\left[P_8e^{-2P_4\eta}+P_9e^{-2P_5\eta}+P_{10}e^{-(P_4+P_5)\eta}\right] \quad (3.20)$$

Taking $\theta_1=e^{i\Omega t}f_1(\eta)$

Equation (3.20) takes the form

$$f''(\eta)+\varepsilon P_r f_1'(\eta)-\frac{1}{4}i\varepsilon P_r\Omega f_{1(\eta)}$$
$$-\frac{1}{4}\varepsilon P_re^{-i\Omega}\cos^2(\Omega)e^{-i\Omega}\left[P_8e^{-2P_4\eta}+P_9e^{-2P_5\eta}-P_{10}e^{-(P_4+P_5)\eta}\right] \quad (3.21)$$

With the modified boundary conditions

$$\left.\begin{array}{l}\eta=0,\theta_1=0\Rightarrow f_1(\eta)=0\\ \eta=\infty,\theta_1=0\Rightarrow f_1(\eta)=0\end{array}\right\} \quad (3.22)$$

Solving equn. (3.21) , we obtain

$$f_1(\eta)=\frac{1}{4}\varepsilon P_re^{-i\Omega t}\cos^2\Omega t\left[\frac{P_8\left(e^{-P_4\eta}-e^{-2P_4\eta}\right)}{4P_4^2-2\varepsilon P_rP_4-\frac{1}{4}i\varepsilon P_r\Omega}+\frac{P_9\left(e^{-P_4\eta}-e^{-2P_5\eta}\right)}{4P_5^2-2\varepsilon P_5P_r-\frac{1}{4}i\varepsilon P_r}-\frac{P_{10}\left\{e^{-P_4\mu}-e^{-(P_4+P_5)\eta}\right\}}{(P_4+P_5)^2-2\varepsilon P_r(P_4+P_5)-\frac{1}{4}i\varepsilon P_r\Omega}\right] \quad (3.22)$$

$$\theta_1 = \tfrac{1}{4}\varepsilon P_r \cos^2 \Omega t + \left[\frac{P_8\left(e^{-P_4\eta} - e^{-2P_4\eta}\right)}{4P_4^2 - 2\varepsilon P_r P_4 - \frac{1}{4} i\varepsilon\Omega P_r} + \frac{P_9\left(e^{-P_4\eta} - e^{-2P_5\eta}\right)}{4P_5^2 - 2\varepsilon P_5 P_r - \frac{1}{4} i\varepsilon P_r} - \frac{P_{10}\left\{e^{-P_4\eta} - e^{-(P_4+P_5)\eta}\right\}}{(P_4+P_5)^2 - 2\varepsilon P_r (P_4+P_5) - \frac{1}{4} i\varepsilon P_r \Omega} \right] \quad (3.23)$$

With the help of eqn. (3.28), the equation (3.5) is solved taking

$$\psi_1 = e^{i\Omega t} F_1(\eta)$$

Thus $F_1(\eta) = (P_{16} + P_{17} + P_{18} - P_{15})e^{-P_6\eta} + P_{15}e^{-P_4\eta} - P_{16}e^{-2P_4\eta} - P_{17}e^{-2P_5\eta} - P_{18}e^{-2(P_4+P_5)\eta}$ (3.24)

Thus

$$u_1(\eta) = \cos\Omega t\left[(P_{16} + P_{17} + P_{18} - P_{15})e^{-P_6\eta} + P_{15}e^{-P_4\eta} - P_{16}e^{-2P_4\eta} - P_{17}e^{-2P_5\eta} - P_{18}e^{-2(P_4+P_5)\eta} \right] \quad (3.25)$$

And $w_1(\eta) = \sin\Omega t\left[(P_{16} + P_{17} + P_{18} - P_{15})e^{-P_6\eta} + P_{15}e^{-P_4\eta} - P_{16}e^{-2P_4\eta} - P_{17}e^{-2P_5\eta} - P_{18}e^{-2(P_4+P_5)\eta} \right]$ (3.26)

Obtaining u_0, u_1, w_0 and w_1, we find $u(\eta)$ and $w(\eta)$ as

$$u(\eta) = u_0(\eta) + Eu_1(\eta)$$
$$= \cos\Omega t\Big[P_7\left(e^{-P_4\eta} - e^{-P_5\eta}\right) + E\Big[P_{15}e^{-P_4\eta} + (P_{16} + P_{17} + P_{18} - P_{15})e^{-P_6\eta} - P_{16}e^{-2P_4\eta} - P_{17}e^{-2P_5\eta} - P_{18}e^{-(2P_4+P_5)\eta}\Big]\Big] \quad (3.27)$$

$$w(\eta) = w_0(\eta) + Ew_1(\eta)$$
$$= \sin\Omega t\Big[P_7\left(e^{-P_4\eta} - e^{-P_5\eta}\right) + E\Big[P_{15}e^{-P_4\eta} + (P_{16} + P_{17} + P_{18} - P_{15})e^{-P_6\eta} - P_{16}e^{-2P_4\eta} - P_{17}e^{-2P_5\eta} - P_{18}e^{-(2P_4+P_5)\eta}\Big]\Big] \quad (3.28)$$

Likewise,

$$\theta(\eta) = \theta_0(\eta) + E\theta_1(\eta)$$
$$= e^{i\Omega t}\left[f_0(\eta) + Ef_1(\eta)\right]$$
$$= e^{i\Omega t}e^{-P_4\eta} + P_{19}\left(e^{-P_4\eta} - e^{-2P_4\eta}\right) + P_{20}\left(e^{-P_4\eta} - e^{-2P_5\eta}\right) - P_{21}e^{-P_4\eta}\left(1 - e^{-P_5\eta}\right)$$
$$= (\cos\Omega t + i\sin\Omega t)e^{-P_4\eta} + P_{19}\left(e^{-P_4\eta} - e^{-2P_4\eta}\right) + P_{20}\left(e^{-P_4\eta} - e^{-2P_5\eta}\right) - P_{21}e^{-P_4\eta}\left(1 - e^{-P_5\eta}\right) \quad (3.29)$$

Hence

$$\theta_r(\eta) = \cos\Omega t e^{-P_4\eta} + P_{19}\left(e^{-P_4\eta} - e^{-2P_4\eta}\right) + P_{20}\left(e^{-P_4\eta} - e^{-2P_5\eta}\right) - P_{21}e^{-P_4\eta}\left(1 - e^{-P_5\eta}\right) \quad (3.30)$$

and

$$\theta_i(\eta) = \sin\Omega t e^{-P_4 n} + P_{19}\left(e^{-P_4\eta} - e^{-2P_4\eta}\right) + P_{20}\left(e^{-P_4\eta} - e^{-2P_5\eta}\right) - P_{21}e^{-P_4\eta}\left(1 - e^{-P_5\eta}\right) \quad (3.31)$$

Shearing Stress:

$$\tau_1 = \left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} + R_c \left.\frac{\partial^2 u}{\partial \eta^2}\right|_{\eta=0}, \quad (3.32)$$

and $\tau_2 = \left.\frac{\partial u}{\partial \eta}\right|_{\eta=0} + R_c \left.\frac{\partial^2 u}{\partial \eta^2}\right|_{\eta=0}$ (3.33)

Substituting the values of u and w in (3.32) and 3.33) respectively, we have

$$\tau_1 = \cos\Omega t\big[P_5P_7(1 - R_cP_5) - EP_6(P_{16} + P_{17} + P_{18} - P_{15})(1 - R_cP_6) - P_4(P_7 + EP_{15})(1 - R_cP_4) + 2P_4P_{16}E(1 - 2R_cP_4) + 2P_5P_{17}E(1 - 2R_cP_5) + 2(P_4 + P_5)P_{18}E\{1 - (P_4 + P_5)R_C\}\big]. \quad (3.34)$$

And

$$\tau_2 = \sin\Omega t\big[P_5P_7(1 - R_cP_5) - EP_6(P_{16} + P_{17} + P_{18} - P_{15})(1 - R_cP_6) - P_4(P_7 + EP_{15})(1 - R_cP_4) + 2P_4P_{16}E(1 - 2R_cP_4) + 2P_5P_{17}E(1 - 2R_cP_5) + 2(P_4 + P_5)P_{18}E\{1 - (P_4 + P_5)R_C\}\big]. \quad (3.35)$$

Rate of heat transfer:

The rate of heat transfer is given by

$$Nu = -\left.\frac{\partial\theta}{\partial\eta}\right|_{\eta=0}$$
$$= P_4\left(e^{i\Omega t} - P_{19} + P_{20}\right) + P_5\left(P_{21} - 2P_{20}\right), \quad (3.36)$$

Separating real and imaginary parts, we have

$$(Nu)_r = P_4(\cos\Omega t - P_{19} + P_{20}) + P_5(P_{21} - 2P_{20}) \quad (3.37)$$

and

$$(Nu)_i = P_4(\sin\Omega t - P_{19} + P_{20}) + P_5(P_{21} - 2P_{20}) \quad (3.38)$$

Where the constants involved are

$$P_8 = P_rP_7^2\left(P_4^2 + \varepsilon M + R_cP_4^3\right),$$
$$P_9 = P_rP_7^2\left(P_5^2 + \varepsilon M + R_cP_5^3\right),$$
$$P_{10} = P_rP_7^2\left(2P_4P_5 + 2\varepsilon M + R_cP_4^2P_5 + R_cP_4P_5^2\right),$$
$$P_{11} = -\tfrac{1}{4} i\Omega,$$
$$P_{12} = \left(\tfrac{1}{4}P_3P_r\varepsilon e^{-i\Omega t}\cos^2\Omega t\right) \times \frac{P_8}{4P_4^2 - 2\varepsilon P_rP_4 + \varepsilon P_rP_{11}},$$
$$P_{13} = \left(\tfrac{1}{4}P_3P_r\varepsilon e^{-i\Omega t}\cos^2\Omega t\right) \times \frac{P_9}{4P_5^2 - 2\varepsilon P_rP_4 + \varepsilon P_rP_{11}}$$
$$P_{14} = \left(-\tfrac{1}{4}P_3P_r\varepsilon e^{-i\Omega t}\cos^2\Omega t\right) \times \frac{P_{10}}{\left(P_4^2 + P_5^2\right) - 2\varepsilon P_r(P_4 + P_5) + \varepsilon P_rP_{11}},$$
$$P_{15} = \frac{P_{12} + P_{13} + P_{14}}{P_4^2 - P_1P_4 + P_2}, \quad P_{16} = \frac{P_{12}}{4P_4^2 - P_1P_4 + P_2},$$

$$P_{17} = \frac{P_{13}}{4P_5^2 - 2P_1P_5 + P_2},$$

$$P_{18} = \frac{P_{14}}{(P_4+P_5)^2 - P_1(P_4+P_5) + P_2}$$

$$P_{19} = \left(\frac{P_{12}}{P_3}\right)Ee^{i\Omega t},\ P_{20} = \left(\frac{P_{13}}{P2}\right)Ee^{i\Omega t},\ P_{21} = \left(-\frac{P_{14}}{P_3}\right)Ee^{i\Omega t}$$

III. Result and Discussion

Hall effect on oscillatory hydromagnetic free convective flow of a visco-elastic (Walters' $B^{'}$ liquid) conducting fluid past an infinite vertical porous plate in the presence of dissipation has been studied with the help of graphs and tables. The velocity and temperature profiles have been shown by graphs and the skin-friction and rate of heat transfer have been presented by tables. The effects of fluid parameters like the non-Newtonian parameter $R_{c,}$ magnetic parameter *M*, Hall parameter *m*, Grashof number *G*, Prandtl number P_r and Eckert number *E* on the velocity and temperature field have been unveiled. The numerical values of shear-stress and Nusselt number entered in the tables illustrate the effects of $R_{c,}$ *M*,*m*, P_r and *E* on the shear-stresses and the rate of heat transfer.

Fig. 1 shows the effects of $R_{c,}$ *G*, *M* and *m* on the primary velocity *U* keeping other parameters like P_r, *E*, ε and Ω fixed. It is observed that the increase in $R_{c,}$ increases the speed of flow. Same effect is marked in case of Grashof number *G*. Rise in Hall parameter *m*decelerates the flows. The rise in Hartmann number *M* reduces the speed of flow further. In each case, the speed of flow first rises and then falls with the distance from the plate. The speed of flow is minimum for Newtonian flow ($R_{c,}$ = 0.0) and increases for non-Newtonian flow (curves 11 and III)

The profiles of secondary velocity *W* have been exhibited in Fig.2 which contains curves drawn between *W* and η varying the values of fluid parameters $R_{c,}$ *G*, *M* and *m*. The natures of the curves are similar to those of primary velocity. Generally, the speed of flow first rises, attains peak value and then falls with the rise of distance from the plate. The strength of the external transverse magnetic field reduces the speed of non-Newtonian flow maximum.

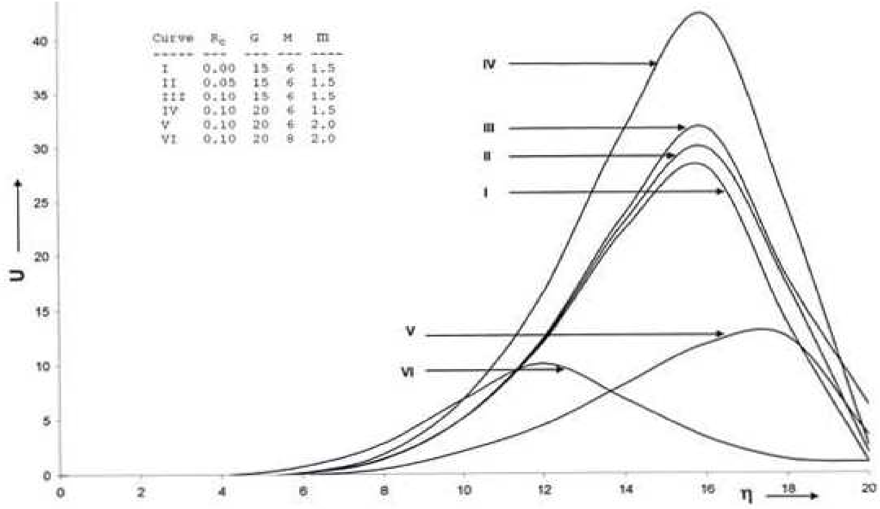

Fig. 1. Profile of primary velocity *U* for P_r=1.0, ε =0.05, E=0.001, Ω =1.0, Ωt= $\pi/6$

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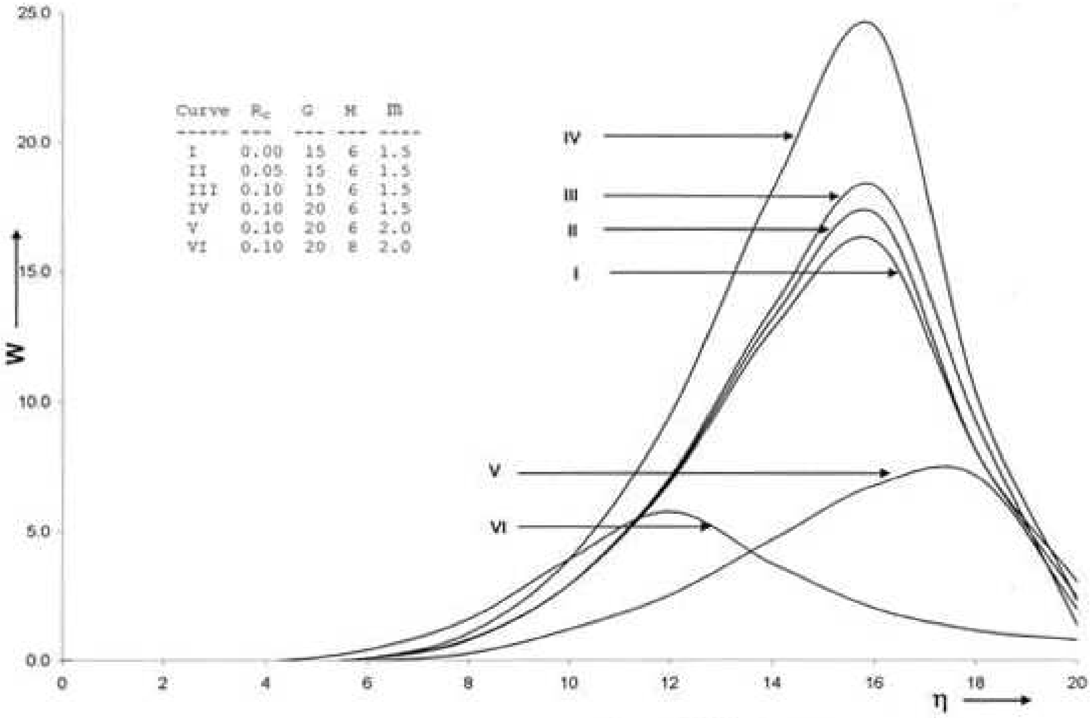

Fig. 2. Profiles of Secondary Velocity W for $P_r = 1.0$, ε =0.05, E=0.001, Ω =1.0, Ω t= $\pi/6$

Figure 3 explains the characteristics of transient temperature with the variation of Prandtl number P_r. it is observed that the transient temperature is fluctuating. The amplitude of oscillatory temperature first decreases and then rises for P_r=0.025, 1.0, 2.3, maintaining the sequence throughout with the distance from the plate. Reverse is gleaned form curves II and V drawn for P_r=0.71 (air) and P_r=7.0 (water). The amplitude of fluctuating temperature is the minimum in case of water (curve V).

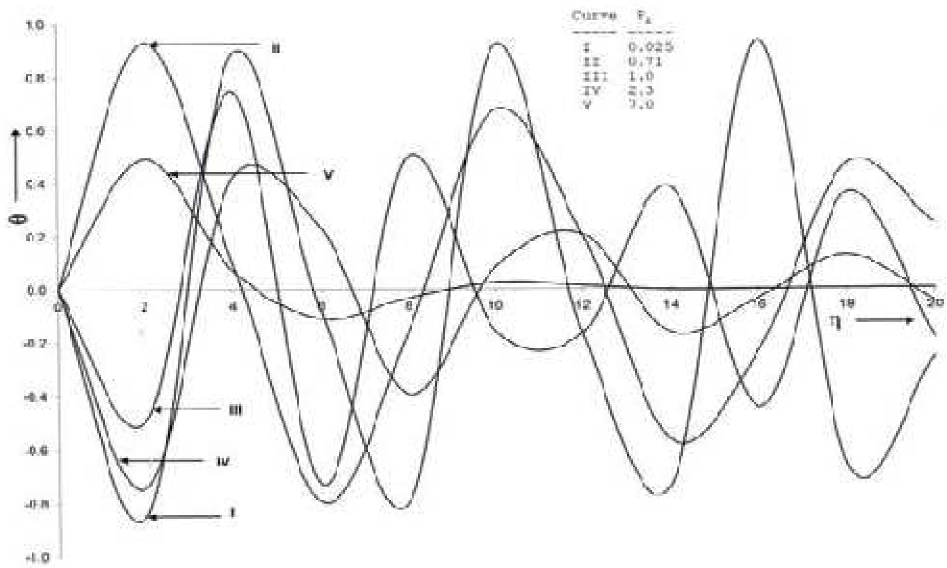

Fig. 3. Transient temperature profile for M=2.0, m=0.5, ε =0.05, E=0.002, G=5.0, Ω t= $\pi/6$

The effects of angular velocity Ω on the transient temperature θ_1 have been presented in the Fig.4. it is observed that the increase in Ω reduces the amplitude of transient temperature (curves I and II)while the high value of Ω decreases the amplitude as well as reverses its nature its nature. Generally, the amplitude falls with the distance (η) from the plate.

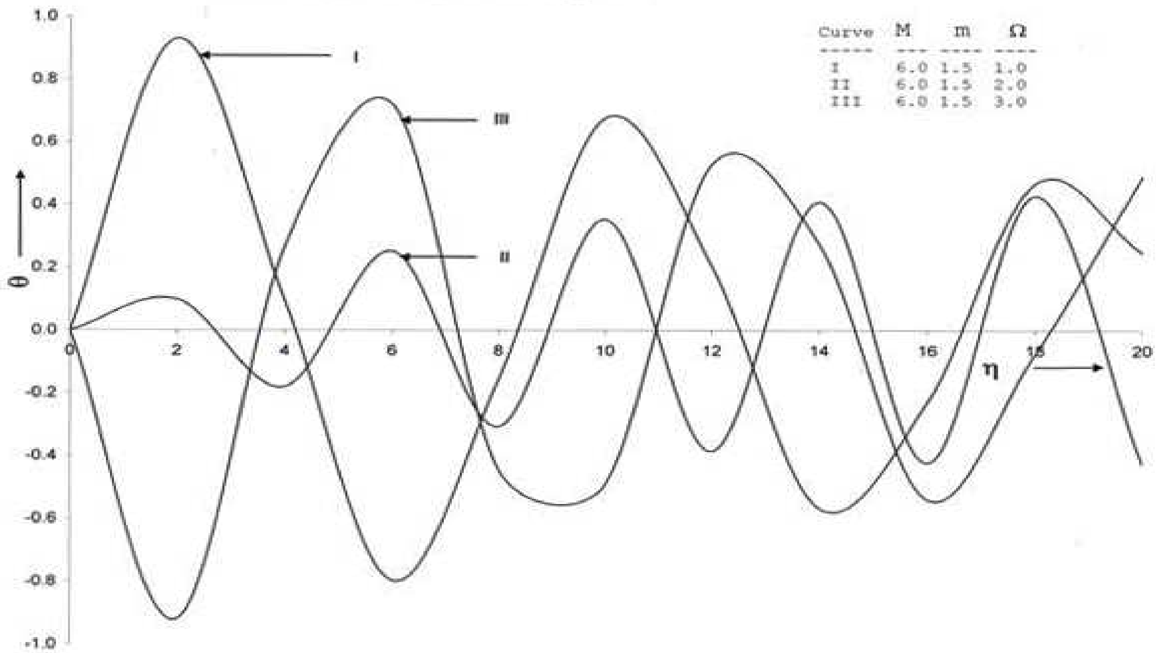

Fig. 4. Transient Temperature profile for R_C =0.05, ε =0.05, E=0.002, G=5.0, Ω t= $\pi/2$, P_r=1.0.

Shear Stresses: The values of the shear stresses τ_1 and τ_2 due to primary and secondary velocity of flow have been presented in the table 1 for different values of R_C *M and E.*

Table 1. Values of the shear stresses τ_1 and τ_2 for P_r=1.0, m=1.0, G=5, ε =0.05 and Ω t= $\pi/6$

R_c E		τ_1		τ_2	
M		0.001	0.002	0.001	0.002
0.00	2.0	-0.047551	-0.047551	-0.027467	-0.027467
0.05	2.0	-0.097178	-0.097178	-0.056133	-0.056133
0.10	2.0	-0.147221	-0.147221	-0.085039	-0.085039
0.10	4.0	-0.150510	-0.150510	-0.086939	-0.085939
0.10	6.0	-0.153398	-0.153398	-0.088608	0.088608

It is noticed that the increase in the elastic parameter (R_c) decreases the shear stresses τ_1 and τ_2 . The increase in Hartmann number (*M*) further reduces the shear stresses. But the Eckert number (*E*) produces no effect on the values τ_1 and τ_2 .

Rate of heat transfer

The rate heat transfer is characterized by the Nusselt number (*Nu*). The values of the Nusselt number for various values of Prandtl number P_r . Hall parameter *m*, Hartmann number *M* and Eckert number *E* have been entered in table 2 . It is observed that the rise in the values of P_r enhances the rate of heat transfer. The variation in *m* ans *M* produces no effect on *Nu*. Also the rate of heat radiation does not vary with the Eckert number. The negative values of *Nu* implies that there is loss of heat due to radiation.

Table 2. Values of the Nusselt number for R_c =0.05, G=5, ε =0.05 and Ω t= $\pi/2$

P_r	m E		Nu	
	M		**0.001**	**0.002**
0.025	0.5	2.0	-200.0000	-200.0000
0.710	0.5	2.0	-7042274	-7042274
1.000	0.5	2.0	-5.000031	-5.000031
7.000	0.5	2.0	-0.714507	-0.714507
7.000	1.0	2.0	-0.714507	-0.714507
7.000	1.0	4.0	-0.714507	-0.714507
7.000	1.5	6.0	-0.714507	-0.714507

Conclusions

Following conclusions are drawn from the above investigation.

1) The speed of flow (both primary and secondary) is minimum for Newtonian flow (R_c = 0.0) and increases for non-Newtonian flow ($R_c \neq$ 0.0).
2) The transient temperature is fluctuating.
3) The amplitude of transient temperature first decreases and then rises for P_r= 0.025, 1.0 and 2.3.
4) Reverse effect is marked in case of the transient temperature for Newtonian fluids like air (P_r= 0.71) and water (P_r = 7.0)
5) The increase in the angular velocity (Ω) reduces the amplitude of the transient temperature (θ_1).
6) It is interesting to note that the high value of the angular velocity reduces the amplitude of θ_1 as well as reverses its nature.
7) The increase in the elastic parameter (R_c) decreases the shear stresses τ_1 and τ_2 developed due to primary and secondary flow.
8) The increase in external magnetic field strength decreases the values of τ_1 and τ_2 further.
9) The rise in the values of the Prandtl number enhances the rate of heat radiation.
10) The negative values of the Nusselt number show that there is loss of heat due to radiation.

References

[1] A.S. Gupta. "Steady and transient free convection of an electrically conducting fluid from a vertical plate in the presence of a magnetic field", Appl. Sci. Res. 9A. pp. 319-333, 1960.

[2] A. K. Singh and J. Singh, "MHD flow through a porous medium bounded by an impulsively started porous plate". Ind. J. Theo, Phys., 51. No.4. pp. 8-12. 1983.

[3] S.P. Mishra and P. Mahapatra. "Unsteady free convection flow from a vertical plate in the presence of a uniform magnetic field". ZAMM. 55. pp. 759-762. 1975.

[4] P.K. Sahoo, N. Datta and S. Biswal."Magneto hydrodynamic unsteady free corvection flow past an infinite vertical plate with constant suction and heat sink".Indian J. Pure appl. Math., 34 (1).pp, 145-155, 2003.

[5] M.A. Hossain. "Effect of Hall current on unsteady hydromagnetic free convection flow near an infinite vertical porous plate". J. Phys. Soc. Japan, 55, No. 7, pp. 2183-2190, 1986.

[6] MA. Hossain and K. Mohammad."Effect of Hall current on hydromagnetic free convection flow near an accelerated porous plate". Japanese J. appl. Phys., 27, No. 8, pp. 1531-1535.

[7] M.A. Hossain and R.I.M.A. Rashid, "Hall effects on hydromagnetic free convection flow along a porous flat plate with mass transfer", J. Phys. Soc. Japan, 56, No.1, pp. 97-104, 1987.

[8] P. Mohapara and A. Tripathy, "Hall effect on oscillatory hydromagnetic free convective flow past an infinite vertical porous flat plate", J. Orissa Math.Soc, 7, No.1, pp. 49-63, 1988.

[9] S. Biswal and M. Pradhan,, "Hall effect on oscillatory hydromagnetic free convective flow past an infinite vertical porous plate with mass transfer", Acta.Ciencia Indica, 28 p, No.l. pp. 41-52, 2002.

[10] J.C. Muduli, S.Biswal and B.Jena, "Hall effect on oscillatory hydromagnetic free convective flow past an infinite vertical porous flat plate with mass transfer and internal heat generating sources/heat absorbing sinks", Bull. Orissa Physical Society , 12, pp. 130-152, 2000.

[11] G.C. Dash and B.K. Ojha, "Hydromagnetic flow and heat transfer of an elastic-viscous fluid over a porous plate in the slip flow regime", J.AMSE, Modelling , Simulation and Control, B.20, No.21, pp. 39-53, 1989.

[12] S. Biswal and B.K. pattnaik, "Hall effect on oscillatory hydromagnetic free convective flow of a visco –elastic fluid past an infinite vertical porous flat plate", Orissa Physical Society, 7, pp. 47-65, 1999.