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An Improvement of the Non-Local Filter in Image Denoising by a Rotation-Invariant Similarity Measure

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Abstract **– An improved image denoising technique based on the non-local means algorithm is investigated. The proposed method defines the similarity-distance between two patches by rotating either one and taking the minimal Weighed Euclidean distance. The comparative experimental results show that the improved NL-means filter achieves better denoising performances.**

Keywords **– Image Denoising, Non-local Filter, Rotation-Invariant Similarity Measure.**

I. Introduction

Consider the following image model:

$$v(i)=u(i)+n(i) \tag{1.1}$$

where $v(i)$ is the observed value of image, $u(i)$ would be the "true" value, and $n(i)$ is the noise perturbation at a pixel i.

Given a discrete noisy image $v=\{v(i)|i\in I\}$, the Non-Local Means (*NLM*) algorithm estimates the grey value of pixel i as a weighed average of all the pixels in a learning window centered at pixel i,

$$NL(v)(i)=\frac{\sum_{j\in N_i(D)} w(i,j)v(j)}{\sum_{j\in N_i(D)} w(i,j)} \tag{1.2}$$

where $N_i(D)$ denotes the learning window of size D centered at pixel i, the family of weights $\{w(i,j)\}$, dependent on the similarity between the comparing windows N_i and N_j, is usually defined as

$$w(i,j)=\exp\left\{-\left\|v(N_i)-v(N_j)\right\|_a^2/2\sigma_r^2\right\} \tag{1.3}$$

Here $\sigma_r>0$ is a control parameter, and $v(N_j)=(v(k),k\in N_j)$ is the vector composed of the grey values $v(k)$ in the window N_j with center *j*, and $\|v(N_i)-v(N_j)\|_a^2$ is a kind of weighted norm:

$$\|v(N_i)-v(N_j)\|_a^2 = \sum_{\|h\|\le(d-1)/2} a(h)|v(i+h)-v(j+h)|^2, \tag{1.4}$$

where $a(h)>0$ is usually chosen to be a decreasing function of the Euclidean norm $\|h\|$, e.g.,

$$a(h)=\frac{1}{(d-1)^2}\sum_{l=|h|}^{(d-1)/2}\frac{1}{(2l+1)^2}, \tag{1.5}$$

where d denotes the size of the comparing windows. One may see Refs. [1, 2, 3, 4, 5] for more details. The *NLM* algorithm achieves better denoising performances than some classical methods, e.g., *DFT* or *DWT,* one may compare their performance by denoising experiments or see related comparison in literature.

In this paper we aim to improve the *NLM* by a Rotation-Invariant Similarity Measure. Let us illustrate our main idea and motivation by some images with tree leaves. In Fig. 1.1, many leaves have the same shape, but different pose. They are really similar to each other, so their center have similar color and brightness. However, they may not be similar according to the definition of similarity above.

Fig. 1.1. An image with leaves on the ground

Let us make it clear by Fig.1.2, where a leaf rotates anti-clockwise 90, 180 and 270 degrees respectively. The four leaves are the same, but they are not similar according to the definition of the similarity above. So it is reasonable to update the definition of similar-patches by a rotation invariant similarity measure, i.e., patch $v(N_i)$ is similar to patch $v(N_j)$ if any rotation of $v(N_i)$ is similar to $v(N_j)$.

Fig. 1.2. Rotation of a leaf

This paper is organized as follows: In section 2, we give our definition of similarity by rotation and formulate our algorithm. In Section 3, we report our experiment results.

II. Rotation Algorithm

Let N_i and N_j be two comparing windows, a local patch $v_\theta(N_i)$ is obtained by rotating the image around pixel i of angle θ, then we define the similarity-distance between them as follows:

$$Dist_r(i,j)=\min_\theta \left\| v_\theta(N_i)-v(N_j)\right\|_a, \qquad (2.1)$$

and the corresponding similarity-weight is as

$$w^r(i,j)=\exp\left(-Dist_r(i,j)/2\sigma_r^2\right) \qquad (2.2)$$

Therefore the counterpart to (1.2) is

$$NL(v)(i)=\frac{\sum_{j\in N_i(D)} w^r(i,j)v(j)}{\sum_{j\in N_i(D)} w^r(i,j)}. \qquad (2.3)$$

The rotated patch $v_\theta(N_i)$ can be described as

$$v_\theta(N_i)(i+k)=v(N_i)(i+k_\theta)$$

where

$$k_\theta=([k_1\cos\theta-k_2\sin\theta],[[k_1\cos\theta+k_2\sin\theta]]),$$

$k=(k_1,k_2)$, and $[\cdot]$ denotes the integer part of $\cdot$.

Of course, it is impossible to calculate the exact similarity distance between two patches by (2.1), so we do by its approximate version as follows:

$$Dist_r(i,j)$$
$$=\min\left\{\left\|v_\theta(N_i)-v(N_j)\right\|_a : \theta=k\Delta\theta, 0\le k\le\frac{2\pi}{\Delta\theta}\right\}, \qquad (2.4)$$

where $\Delta\theta\in[0,2\pi]$ is the step length of the discretization.

III. Simulation and Choice Of Parameters

In this section, we present some experimental results. We take $\Delta\theta=\pi/2$, and use *PSNR* to measure the quality of a restored image. In our experiments, we use the 512×512 image of Peppers, the image boundaries are handled by assuming symmetric boundary conditions.

Denote the deviation of the Gaussian noise by □, the size of the learning window by *D*, and the size of the comparing window by *d*. For □, d and D given, we choose the weight factor $□_r$ which maximize the *PSNR* of the restored images.

The experiment results are illustrated by Table 3.1 and Figs. 3.1-3.4. Table 3.1 shows the performance of *PSNR* for removing Gaussian noise by the *NLM* and *RISM* with $\Delta\theta=\pi/2$ respectively. Fig. 3.1 is the original images, Figs. 3.2-3.4 are the restored images by the *RISM*.

Table 3.1: PSNR values for removing Gaussian noise by *NLM* and *RISM*

σ	10	20	30
d	7	9	13
D	7	9	11
NlM	34.4465	31.9308	29.8438
RISM	34.5143	32.0424	30.0198

Fig. 3.1. Original image and noisy images of Peppers. First row from left to right are the original image and a noisy image with the standard deviation 10; second row (from left to right) are two noisy images with the standard deviation 20 and 30 respectively.

Fig. 3.2. Restored images by *RISM.* First row from left to right are the restored images from noisy images with the standard deviation 10 and 20 respectively; second row is the restored image from noisy image with the standard deviation 30.

Comparing the *PSNR* values of the restored images listed in Table 3.1, one can see that the *RISM* filter gives better images than the *NLM* filter.

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