

# A Group Decision-Making Method Based on EOWA and ILOWGA Operators for Coal Mine Safety Evaluation

Chunfu Wei

School of Mathematics & Information Science, Henan Polytechnic University, Jiaozuo, 454003-China.  
Corresponding author email id: mathwcf@163.com

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**Abstract** – Coal mine safety is the most important issue in global coal mine enterprises. In order to effectively identify and control hazard sources, it's critical to establish a scientific and rational evaluation method of coal mine safety which not only consider the weight of the factors affecting coal mine safety, but also take the ordered position of the factors in aggregation process and the weight of the experts. To this end, first, EOWA and ILOWGA operators will be elaborated to help the decision-makers who may have vague knowledge about the decision information, and can't estimate their decision information with exact numerical values. Second, a group decision-making method based on EOWA and ILOWGA operators for coal mine safety evaluation is proposed. By the method, coal companies can enhance the efficiency, improve the level and reduce the cost of security management.

**Keywords** – OWA Operator, ILOWGA Operator, Group Decision-Making, Aggregation Operator, Weight.

## I. INTRODUCTION

In the coal mine safety evaluation, the decision-makers usually have vague knowledge about the decision information, and can't estimate their decision information with exact numerical values. It is more suitable to provide their preferences by means of linguistic variables rather than numerical ones. For example, the experts usually evaluate geological condition of coal mine with a linguistic term set {very active, active, fair, steady, very steady}. However, how to aggregate the decision information described by linguistic variables. There are some scholars who have studied the linguistic terms [1-4] in early years. In many cases, the preference values are human natural language. Frequently, computing with words can be used to solve decision making problems. Decision-making problems generally consist of finding the most desirable alternative(s) from a given alternative set. Later, some new decision-making methods based on induced OWA operators were introduced [5-12]. Based on EOWA and ILOWGA Operators we develop a group decision-making method for coal mine safety evaluation.

## II. BASIC DEFINITIONS AND RELATED THEOREMS

### A. OWA operator

OWA operator can be expressed as following[13]: if

$$OWA_w(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j, \text{ where } w = (w_1,$$

$w_2, \dots, w_n)$  is the associated weighting vector, with  $w_j \in [0, 1]$  such that  $\sum_{j=1}^n w_j = 1$ , and  $b_j$  is the  $j$  th largest element in the set  $\{a_1, a_2, \dots, a_n\}$ , then the function OWA is called the ordered OWA operator of dimension  $n$ .

The OWA operator has the following properties[13].

1) (Commutativity) Let  $(a_1, a_2, \dots, a_n)$  be a collection of arguments, and  $(a'_1, a'_2, \dots, a'_n)$  be any permutation of  $(a_1, a_2, \dots, a_n)$ . Then  $OWA_w(a_1, a_2, \dots, a_n) = OWA_w(a'_1, a'_2, \dots, a'_n)$ .

2) (Idempotency) Let  $(a_1, a_2, \dots, a_n)$  be a collection of arguments if  $a_i = a$ , for any  $i$ . Then  $OWA_w(a_1, a_2, \dots, a_n) = a$ .

3) (Monotonicity) Let  $(a_1, a_2, \dots, a_n)$  and  $(b_1, b_2, \dots, b_n)$  be two collections arguments, if  $a_i \leq b_i$ , for any  $i$ . Then  $OWA_w(a_1, a_2, \dots, a_n) \leq OWA_w(b_1, b_2, \dots, b_n)$ .

4) (Bounded) The OWA operator lies between the max and min operators:  $\min(a_i) \leq OWA_w(a_1, a_2, \dots, a_n) \leq \max(a_i)$ .

5) If  $w = (\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n})$ , the OWA operator is reduced to the arithmetic operator:  $OWA_w(a_1, a_2, \dots, a_n) = \frac{1}{n} \sum_{i=1}^n a_i$ .

6) If  $w = (1, 0, \dots, 0)$ , the OWA operator is reduced to the max operator:  $OWA_w(a_1, a_2, \dots, a_n) = \max(a_i)$ .

7) If  $w = (0, 0, \dots, 1)$ , the OWA operator is reduced to the min operator:  $OWA_w(a_1, a_2, \dots, a_n) = \min(a_i)$ .

### B. EOWA operator

An EOWA operator[14] is a mapping EOWA:  $\bar{S}^n \rightarrow \bar{S}$ , in which  $w = (w_1, w_2, \dots, w_n)$  is an weighting vector

with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ , such that

$$EOWA_w(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = s_{\beta}$$

Where  $\beta = \sum_{j=1}^n w_j \beta_j$ ,  $s_{\beta_j}$  is the  $j$ th largest element in the set  $\{s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}\}$ , then the function EOWA is called the ordered EOWA operator of dimension  $n$ .

### C. ILOWGA Operator

An ILOWGA operator[15] is a mapping  $ILOWGA: \bar{S}^n \rightarrow \bar{S}$ , in which  $w = (w_1, w_2, \dots, w_n)$  is an exponential weighting vector with  $w_j \in [0, 1]$  and  $\sum_{j=1}^n w_j = 1$ ,  $\bar{\omega} = (\bar{\omega}_1, \bar{\omega}_2, \dots, \bar{\omega}_n)$  is the weighting of the  $s_{\alpha_i}$  with  $\bar{\omega}_j \in [0, 1]$  and  $\sum_{j=1}^n \bar{\omega}_j = 1$  such that  $ILOWGA_{w, \bar{\omega}}(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \dots \otimes (s_{\beta_n})^{w_n} = s_{\beta}$ .

Where  $\beta = \prod_{j=1}^n \beta_j^{w_j}$ ,  $s_{\beta_j}$  is the  $j$ th largest element in the set  $\{\bar{s}_{\alpha_1}, \bar{s}_{\alpha_2}, \dots, \bar{s}_{\alpha_n}\}$ , i.e.,  $\bar{s}_{\alpha_i} = r \bar{\omega}_i s_{\alpha_i} = s_{r \bar{\omega}_i \alpha_i}$ , in which,  $i = 1, 2, \dots, n$ ,  $r$  is the balancing coefficient, the value of  $r$  is usually equal to  $n$ .

Let  $S = \{s_i\} (i = 1, \dots, n)$  be a finite and totally ordered discrete term set. Any label,  $s_i$ , represents a possible value for a linguistic variable, and it must have the following characteristics[16]:

- (1) The set is ordered:  $s_i \geq s_j$  if  $i \geq j$ ;
- (2) There is the negation operator:  $neg(s_i) = s_j$  such that  $j = t + 1 - i$ ;
- (3) Max operator:  $\max(s_i, s_j) = s_i$  if  $s_i \geq s_j$ ;
- (4) Min operator:  $\min(s_i, s_j) = s_i$  if  $s_i \leq s_j$ ;

To preserve all the given information, Xu[5] extended the discrete term set  $S$  to a continuous linguistic term set  $\bar{S} = \{s_{\alpha} | s_1 < s_{\alpha} < s_t, \alpha \in [1, t]\}$ , where, if  $s_{\alpha} \in S$ , then  $s_{\alpha}$  is called the original linguistic term, otherwise, is called the virtual linguistic term. Consider any two linguistic terms  $s_{\alpha}, s_{\beta} \in \bar{S}$ , and  $\mu, \mu_1, \mu_2 \in [0, 1]$ , some operational laws are defined as follows[6]:

- (1)  $\mu s_{\alpha} = s_{\mu \alpha}$ ;
- (2)  $(s_{\alpha})^{\mu} = s_{\alpha^{\mu}}$ ;
- (3)  $(s_{\alpha})^{\mu_1} \otimes (s_{\alpha})^{\mu_2} = (s_{\alpha})^{\mu_1 + \mu_2}$ ;
- (4)  $(s_{\alpha} \otimes s_{\beta})^{\mu} = (s_{\alpha})^{\mu} \otimes (s_{\beta})^{\mu}$ ;
- (5)  $s_{\alpha} \otimes s_{\beta} = s_{\beta} \otimes s_{\alpha} = s_{\alpha \beta}$ ;

## III. THE GROUP DECISION-MAKING METHOD FOR COAL MINE SAFETY EVALUATION

The coal mining industry is a special high-risk industry, and the underground work environment is complex and changeable, safety accidents usually arised in coal mines, the number of coal mining deaths up to hundreds of people each year, and cause serious loss of life and property to the country, enterprises and people. Therefore, it is critical to implement a rational and targeted method of safety evaluation in the process of production for the coal mines which have specific situation, so that potential risks to personnel may be predicted and safety accidents may be prevented with a minimum investment.

### A. Establishing a safety evaluation indicator system

For this paper, we combined with research literatures and the advices of the specialists in the study of the coal mine safety, then we establish a safety evaluation indicator system with six main factors[15]. Where six main factors of coal mine safety evaluation are usually evaluated by the experts with six linguistic term sets in Table I.

Table I. The linguistic term sets of the factors

Factors	Linguistic term sets $s_i (i=1,2,\dots,6)$
Geological condition	{very active, active, fair, steady, very steady}
Technological equipment	{fvery behindhand, behindhand, fair, advanced, very advanced}
Human diathesis security education	{very low, low, fair, high, very high}
Environment security management level	{very poor, poor, fair, good, very good}
	{fvery formidable, formidable, fair, fine, very fine}
	{ very low, low, fair, high, very hig}

In this section, a decision-making problem involves the evaluation of five coal mines safety  $x_i (i = 1, 2, 3, 4, 5)$  of an area. There are four decision-makers  $d_k (k = 1, 2, 3, 4)$ , whose weight vector  $\bar{\omega}' = (0.3, 0.2, 0.2, 0.3)^T$ . When making a decision, the attributes considered include:  $u_1$ : geological condition;  $u_2$ : technological equipment;  $u_3$ : human diathesis;  $u_4$ : security education;  $u_5$ : environment security;  $u_6$ : management level. The decision-makers use the linguistic terms in the set  $S = \{s_1, s_2, s_3, s_4, s_5\}$  and the decision information[15] about coal mines safety is presented in Table II-V.

Table II. The decision information by D1

	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$
$x_1$	$s_3$	$s_2$	$s_2$	$s_5$	$s_3$	$s_3$
$x_2$	$s_4$	$s_3$	$s_3$	$s_2$	$s_4$	$s_3$
$x_3$	$s_5$	$s_3$	$s_4$	$s_4$	$s_4$	$s_3$
$x_4$	$s_5$	$s_3$	$s_5$	$s_4$	$s_4$	$s_5$
$x_5$	$s_5$	$s_5$	$s_4$	$s_4$	$s_5$	$s_3$

Table III. The decision information by d2

	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$
$x_1$	$s_4$	$s_4$	$s_1$	$s_4$	$s_4$	$s_4$
$x_2$	$s_5$	$s_4$	$s_5$	$s_4$	$s_5$	$s_4$
$x_3$	$s_3$	$s_4$	$s_4$	$s_5$	$s_3$	$s_2$
$x_4$	$s_4$	$s_5$	$s_4$	$s_5$	$s_3$	$s_4$
$x_5$	$s_3$	$s_4$	$s_4$	$s_5$	$s_4$	$s_2$

Table IV. The decision information by d3

	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$
$x_1$	$s_2$	$s_4$	$s_4$	$s_5$	$s_3$	$s_3$
$x_2$	$s_4$	$s_3$	$s_4$	$s_2$	$s_4$	$s_4$
$x_3$	$s_5$	$s_3$	$s_4$	$s_4$	$s_4$	$s_3$
$x_4$	$s_5$	$s_3$	$s_5$	$s_3$	$s_4$	$s_5$
$x_5$	$s_4$	$s_5$	$s_4$	$s_4$	$s_4$	$s_4$

Table V. The decision information by d4

	$u_1$	$u_2$	$u_3$	$u_4$	$u_5$	$u_6$
$x_1$	$s_3$	$s_2$	$s_2$	$s_5$	$s_3$	$s_4$
$x_2$	$s_4$	$s_3$	$s_3$	$s_2$	$s_4$	$s_3$
$x_3$	$s_5$	$s_3$	$s_4$	$s_4$	$s_4$	$s_3$
$x_4$	$s_4$	$s_3$	$s_5$	$s_3$	$s_4$	$s_4$
$x_5$	$s_5$	$s_4$	$s_4$	$s_4$	$s_3$	$s_5$

To get the safest mine(s), the following steps are involved:

Step 1 According to Table II-V, the linguistic decision matrices by the four decision-makers for the decision-making for coal mine safety are  $A_1, A_2, A_3$  and  $A_4$ .

Step 2 Utilize the fuzzy linguistic quantifier “most” with the pair (0.3,0.8), and by (1) and (2), we obtain the weighting vector (associated with the EOWA operator),

$$w = (0, 0.067, 0.333, 0.333, 0.267, 0)^T$$

Step 3 Utilize the decision information given in matrix  $A_{1-4}$  and the EOWA operator

$$EOWA_w(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = s_{\alpha_1} \cdot w_1 + s_{\alpha_2} \cdot w_2 + \dots + s_{\alpha_n} \cdot w_n$$

$$A_1 = \begin{bmatrix} s_3 & s_2 & s_2 & s_5 & s_3 & s_3 \\ s_4 & s_3 & s_3 & s_2 & s_4 & s_3 \\ s_5 & s_3 & s_4 & s_4 & s_4 & s_3 \\ s_5 & s_3 & s_5 & s_4 & s_4 & s_5 \\ s_5 & s_5 & s_4 & s_4 & s_5 & s_3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} s_4 & s_4 & s_1 & s_4 & s_4 & s_4 \\ s_5 & s_4 & s_5 & s_4 & s_5 & s_4 \\ s_3 & s_4 & s_4 & s_5 & s_3 & s_2 \\ s_4 & s_5 & s_4 & s_5 & s_3 & s_4 \\ s_3 & s_4 & s_4 & s_5 & s_4 & s_2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} s_2 & s_4 & s_4 & s_5 & s_3 & s_3 \\ s_4 & s_3 & s_4 & s_2 & s_4 & s_4 \\ s_5 & s_3 & s_4 & s_4 & s_4 & s_3 \\ s_5 & s_3 & s_5 & s_3 & s_4 & s_5 \\ s_4 & s_5 & s_4 & s_4 & s_4 & s_4 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} s_3 & s_2 & s_2 & s_5 & s_3 & s_4 \\ s_4 & s_3 & s_3 & s_2 & s_4 & s_3 \\ s_5 & s_3 & s_4 & s_4 & s_4 & s_3 \\ s_4 & s_3 & s_5 & s_3 & s_4 & s_4 \\ s_5 & s_4 & s_4 & s_4 & s_3 & s_5 \end{bmatrix}$$

We have

$$Z_1^{(1)} = 0.067s_3 \oplus 0.333s_3 \oplus 0.333s_3 \oplus 0.267s_2$$

$$= s_{3*0.067} \oplus s_{3*0.333} \oplus s_{3*0.333} \oplus s_{2*0.267}$$

$$= s_{3*0.067+3*0.333+3*0.333+2*0.267} = s_{2.733}$$

We obtain the other values through the same method.

$$Z_2^{(1)} = s_{3.067}, Z_3^{(1)} = s_{3.733}, Z_4^{(1)} = s_{4.4}, Z_5^{(1)} = s_{4.4};$$

$$Z_1^{(2)} = s_{4.0}, Z_2^{(2)} = s_{4.4}, Z_3^{(2)} = s_{4.4}, Z_4^{(2)} = s_{4.067}, Z_5^{(2)} = s_{3.733};$$

$$Z_1^{(3)} = s_{3.4}, Z_2^{(3)} = s_{3.733}, Z_3^{(3)} = s_{3.733}, Z_4^{(3)} = s_{4.133}, Z_5^{(3)} = s_4;$$

$$Z_1^{(4)} = s_{2.8}, Z_2^{(4)} = s_{3.067}, Z_3^{(4)} = s_{3.733}, Z_4^{(4)} = s_{3.733}, Z_5^{(4)} = s_{4.067}.$$

Step 4 We obtain the decision information given in matrix A by Step 3.

$$A = \begin{bmatrix} s_{2.733} & s_{4.0} & s_{3.4} & s_{2.8} \\ s_{3.067} & s_{4.4} & s_{3.733} & s_{3.067} \\ s_{3.733} & s_{4.4} & s_{3.733} & s_{3.733} \\ s_{4.4} & s_{4.067} & s_{4.133} & s_{3.733} \\ s_{4.4} & s_{3.733} & s_4 & s_{4.067} \end{bmatrix}$$

Step 5 Utilize the fuzzy linguistic quantifier “most” with the pair (0.3,0.8), and by (1) and (2), we obtain the weighting vector (associated with the ILOWGA operator),  $w = (0, 0.4, 0.5, 0.1)^T$ .

Step 6 Utilize the decision information given in matrix A and the ILOWGA operator According to (3), we have

$$\bar{s}_{\alpha_1} = 4 * 0.3 * s_{2.733} = s_{4*0.3*2.733} = s_{3.2796},$$

$$\bar{s}_{\alpha_2} = 4 * 0.2 * s_{4.0} = s_{4*0.2*4.0} = s_{3.2},$$

$$\bar{s}_{\alpha_3} = 4 * 0.2 * s_{3.4} = s_{4*0.2*3.4} = s_{2.72},$$

$$\bar{s}_{\alpha_4} = 4 * 0.3 * s_{2.8} = s_{4*0.3*2.8} = s_{3.36},$$

Thus, we have

$$s_{\beta_1} = s_{3.36}, s_{\beta_2} = s_{3.2796}, s_{\beta_3} = s_{3.2}, s_{\beta_4} = s_{2.72}.$$

and the overall value  $Z_i$  of alternative  $x_i$ , We have

$$Z_1 = (s_{3.36})^0 \otimes (s_{3.2796})^{0.4} \otimes (s_{3.2})^{0.5} \otimes (s_{2.72})^{0.1}$$

$$= s_{3.36^0 * 3.2796^{0.4} * 3.2^{0.5} * 2.72^{0.1}} = s_{3.1795}.$$

We obtain the other values through the same method

$$Z_2 = S_{3.5249}, Z_3 = S_{3.8093}, Z_4 = S_{3.7275}, Z_5 = S_{3.7625}.$$

Step 7 Utilize  $Z_i$  to rank the alternative as

$$Z_3 \succ Z_2 \succ Z_4 \succ Z_5.$$

And thus the safest coal mine is  $x_3$ .

#### IV. CONCLUSIONS

In this paper, we not only consider the weight of the factors affecting coal mine safety, but also take the ordered position of the factors in aggregation process and the weight of the experts. Then, a new multi-expert decision-making method based on EOWA and ILOWGA operators for coal mine safety evaluation is proposed to ensure integration and objectivity. We obtain the safest coal mine is  $x_3$ , the evaluation result is different from [15]. Analysis of its causes, ILOWGA operators increased penalties for deviation.

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#### AUTHOR'S PROFILE



**Chunfu Wei**, School of Mathematics & Information Science, Henan Polytechnic University, Jiaozuo, China. Associate professor, Doctor of Engineering, His research interests include intelligent information processing, Mine informatization. Email: mathwcf@163.com