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Developed Regression Models of Mining Rate Fit on Resource and Reserve Variations in Gold Mining

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***Abstract* – Mining rate determines profitability of a mine and when set at an optimal rate generates maximum NPV. Although mining rate depends on many parameters, ore tonnage has been used as the main determinant of mining rate in the development of empirical models for predicting mining rate. However, variable results have been obtained depending on the characteristics of ore-body and mine and also the number of mines used. The main aim in this paper is to develop regression models that can be used in predicting mining rate in any gold mining in the pre-production stage depending on resource and reserve variations. The data used in this study include mining rate, resource-tonnage and resource-grade as well as reserve-tonnage and reserve-grade of 160 gold mines/deposits obtained from Raw Material Group database. Multiple Linear Regression Analysis using stepwise method was used to develop regression models. Results indicated that mining rate in the gold mining can be estimated based on the resource-tonnage and reserve-tonnage. Resource-tonnage accounted for 68.6 per cent of the determinant variables of mining rate while reserve-tonnage accounted for 77 per cent of the total variation.**

***Keywords* – Gold Mining, Mining Rate, Reserve-Tonnage, Resource-Tonnage.**

I. Introduction

Mining rate determines profitability of a mine and when set correctly at an optimal level it maximizes the Net Present Value (NPV) [1]. Understanding the importance of mining rate in the maximization of NPV has increased a body of literature on the determinants of mining rate. On one side, a body of literature indicates that mining rate should be determined based on physical, economic and financial factors. However, the obtained mining rate is criticized since it seems to be biased toward high rate, unrealistic, unachievable and undesired [2], [3]. On the other side, reference [4] established a model where by mining rate is determined by one parameter- ore tonnage only. Although the model is very practical and useful, it is associated with many limitations as viewed by other researchers such as [5] and [6]. In their views, the model encompassed a wide range of mines of different ore-body characteristics as a result, the obtained mining rates are not always correct. Taylor rule was therefore tested, re-estimated, and fitted using data obtained from narrow ranges of ore-body and mining conditions. Although the ore-bodies of gold ores slightly vary from mine to mine, gold mining may be considered as a mining constituting more or less similar ore-body and mining conditions in relation to other minerals including coal and building minerals. And also behaves more or less similar to the variable mining environment where technology, price and public policies are continuously changing to ensure the growing global demand is met [7]. The main aim in this paper is to develop separate regression models of mining rate fits on ore resource and also fits on ore reserve variations that can be used in the estimation of mining rate in the gold mining.

II. Determination of Mining Rate

Mining rate alternatively known as production rate expressed in million tonnes per year (Mt/y) is the annual tonnage of ore material as obtained from a mine/mines going to the mill for subsequent mineral and metallurgical processing aimed at recovering gold and determines capacity of a mine. Selection of the level of mining rate justifies the economy of the mine.

A. Variables Used

The variables used in selecting mining rate are the resource and/or reserve variables [8] and [9].

Resource Variable

Resource may be defined as a concentration or occurrence of valuable mineral of intrinsic economic interest in or on the earth's crust in such a form and quantity that there are reasonable prospects for eventual economic extraction. The location, quantity, grade, geological characteristics and continuity of mineral resource are known, estimated or interpreted from specific geological evidence and knowledge. The total mineral resource is sub-divided in order of increasing geological confidence, into inferred, indicated and measured categories [10] – [13]. Resource variable consists of two parameters namely resource-tonnage and resource-grade. Resource-tonnage of a resource measured in million tonnes (Mt) is referred to as a total tonnage of resource consisting of indicated and measured classes of gold resources. Resource-grade is an amount of gold in a tonne of resource expressed as a percentage, ounce, troy ounces or grams per tonne (gpt or g/t). Resource-tonnage and resource-grade are highly interrelated [14]. High resource-tonnage is associated with low grade and vice versa.

Reserve Variable

Reserve is the economically mineable part of a measured or indicated mineral resource. It includes diluting materials and allowances for losses which may occur when mineral is mined. It includes consideration and modification by realistically assumed mining, metallurgical, economic, marketing, legal, environmental, social and government factors. Total mineral reserves are sub-divided in order of increasing confidence into probable mineral reserves and proved mineral reserve [14]. Reserve variable consists of two parameters namely reserve-tonnage and reserve-grade. Reserve-tonnage measured in million tonnes (Mt) is referred to as total tonnage of reserve consisting of all classes of gold reserve probable and proved. Reserve-grade of reserve is an amount of gold in a tonne of reserve expressed as a percentage, ounce, troy ounces or grams per tonne (gpt or g/t). Reserve tonnage and grade are highly interrelated (*ibid*). High reserve-tonnage is associated with low grade and vice versa.

B. Existing Models used in the Estimation of Mining Rate

The models used to determine mining rate can be grouped into two: NPV- based models and tonnage-based models.

NPV-based Models

The NPV-based models consider a wide range of basic economic parameters and methods in the establishment of optimal mining rates that seek to maximize NPV. Reference [8] used present value ratio (PVR) as a criterion to determine optimal mining rate. PVR is the ratio of the present value of positive cash flows (PVIN) to the present value of negative cash flows (PVOUT). Any PVR greater than 1 represents a profitable mining rate while PVR less than represents an unprofitable mining rate. The optimum mining rate is the rate that causes the PVR to be at its maximum values. Further, reference [15] looked at cash flow, stochastic risk modeling and option pricing techniques to determine mine life that maximizes NPV while [16] used open ended dynamic programming to research on optimal mining rate. [17] Proposed a range of mining rates with an upper limit as the rate that resulted in the highest NPV. The lower limit of this range was found to be the rate that best repaid capital costs. [18] Used mathematical model that considered various physical, economic and financial factors for determination of the optimal mining rate. The factors considered by [18] were tonnage, grade of deposit, gold price, expected growth rate of gold prices, capital cost (discount rate). However, the main disadvantage of NPV-based models is that the mining rate obtained is biased toward high rates, unrealistic, unachievable and undesired.

Tonnage-based Models

In these models, mining rate is considered as dependent variable of the ore tonnage as compared to the NPV-based models where mining rate was considered as independent variable that explains NPV. The general form of the models is shown in (1) though other forms may exist.

$$Mining\ Rate\left(\frac{mt}{day}\right) = a\ x\ Tonnage^{(b)} \qquad (1)$$

Other forms of the tonnage-based models as proposed by [19] and the Half Vertical Tonnage are shown in (2) and (3).

$$Mining\ Rate\left(\frac{tonnes}{year}\right) = rate\ factor\ x\ rate\ multiplier \qquad (2)$$

Where: rate factor – tonnes/vertical meter; rate multiplier - an empirical value based on the deposit thickness and the risk the mine designer willing to take.

$$Mining\ Rate = \frac{1}{2} x \frac{\text{Tonnage}}{\text{Depth (ft)}} \qquad (3)$$

Tatman's model is used in the steeply dipping deposit while Half Vertical Tonnage is used in the steeply dipping and vertical deposits in the mechanized mines. Table 1 provides details of the tonnage-based models in the form of formula (1).

Table1: Summary of the Chronological Development of Tonnage-based Models

S/N	Source or scholar	Mine Type	No. of Mines	a	b	Model
1	[4] and [20]	Mining projects	30	0.0143	0.75	$Production\left(\frac{mt}{day}\right) = 0.0143\ x\ Tonnage^{(0.75)}$
2	[21]	Open Pit gold/ silver	41	0.416	0.5874	$Production\left(\frac{mt}{day}\right) = 0.416\ x\ Tonnage^{(0.5874)}$
3	[22]	Underground – massive sulfide	28	0.0248	0.704	$Production\left(\frac{mt}{day}\right) = 0.0248\ x\ Tonnage^{(0.704)}$
4	[23]	Open pit copper mine	45	0.0236	0.74	$Production\left(\frac{mt}{day}\right) = 0.0236\ x\ Tonnage^{(0.74)}$
5	[24]	Open pit/block caving-other	342	0.123	0.649	$Production\left(\frac{mt}{day}\right) = 0.123\ x\ Tonnage^{(0.649)}$
6	[24]	Underground –other	197	0.297	0.562	$Production\left(\frac{mt}{day}\right) = 0.0297\ x\ Tonnage^{(0.562)}$

Source: Derived by author

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III. Development of Regression Models of Mining Rate

A. Formulation

The general formulation of the mining rate is in the form of a curve shown in (1) or may be written as a linear equation shown in (4) using common logarithm:

$$\log Mining\ Rate\ \left(\frac{Mt}{year}\right) = a_{ij} + b_{ij} logX_{ij} \quad (4)$$

Where: $a\ and\ b - constants$; $x - tonnage\ or\ grade$; $i - resource$; $and\ j - reserve$.

B. Database and Regression Analysis

The database, nature of data and the regression analysis used in developing models of mining rate are explained in this section.

Database

Raw Materials Group database constitutes the main source of the information in this study since data related to tonnage and grade of resource and reserve as well as mining rate of global gold mines and deposits are collected and kept in this database. Such huge information justifies performance of observational study. The database was accessed by the author in 2013 and year 2012 was selected for the study. Gold deposits in the stages of pre-feasibility and feasibility studies, and mines in the construction and development stages were the main targets. The main reason was that the size of resource-tonnage and reserve- tonnage in these deposits/mines are unaffected by mining operations since the mining operations have not yet started. Based on the availability of data, 160 gold mines/deposits were selected. Some of the selected mines/deposit whose data were not available in the Raw Materials Group Database, their data were obtained from annual and technical reports compiled by the controlling companies.

Normality of Data

Normality of data for resource-tonnage, reserve-tonnage and mining rate parameters was tested using Kolmogorov-Smirnov test of normality. The results indicated the non-normality of data with p = .000. The data were then normalized by transforming them into common logarithm. Fig. 1 shows the histogram and overlaid normal distribution for the resource-tonnage, reserve-tonnage and mining rate data. The skewness and kurtosis for resource-tonnage were 0.061 and 0.07, respectively while for reserve-tonnage were 0.18 and 0.083, respectively. The skewness and kurtosis for mining rate were 0.062 and - 0.112. All values of skewness and kurtosis for these parameters approached zero indicating that they were fairly normally distributed.

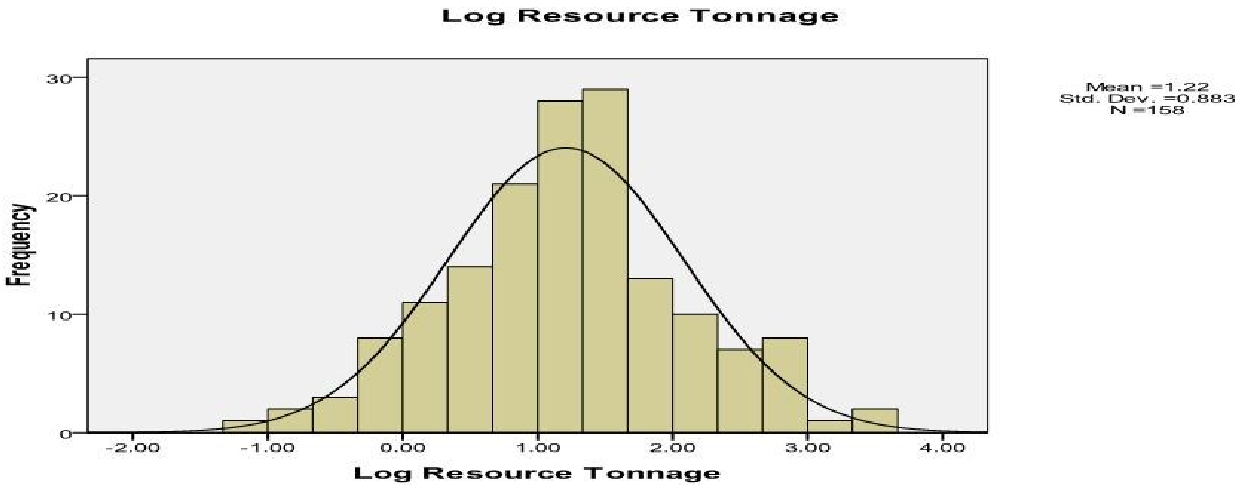

Fig.1 a. Log resource-tonnage

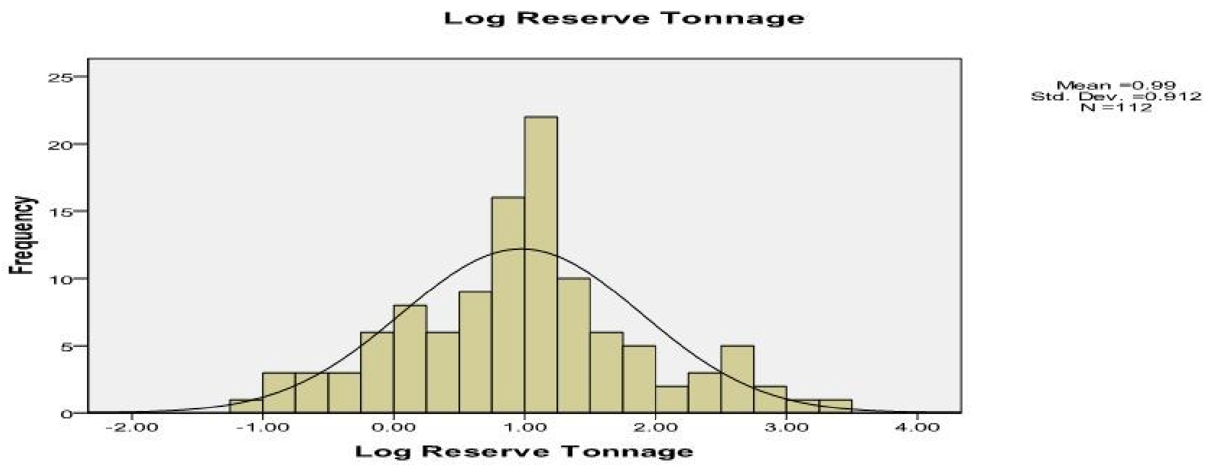

Fig. 1 b. Log reserve-tonnage

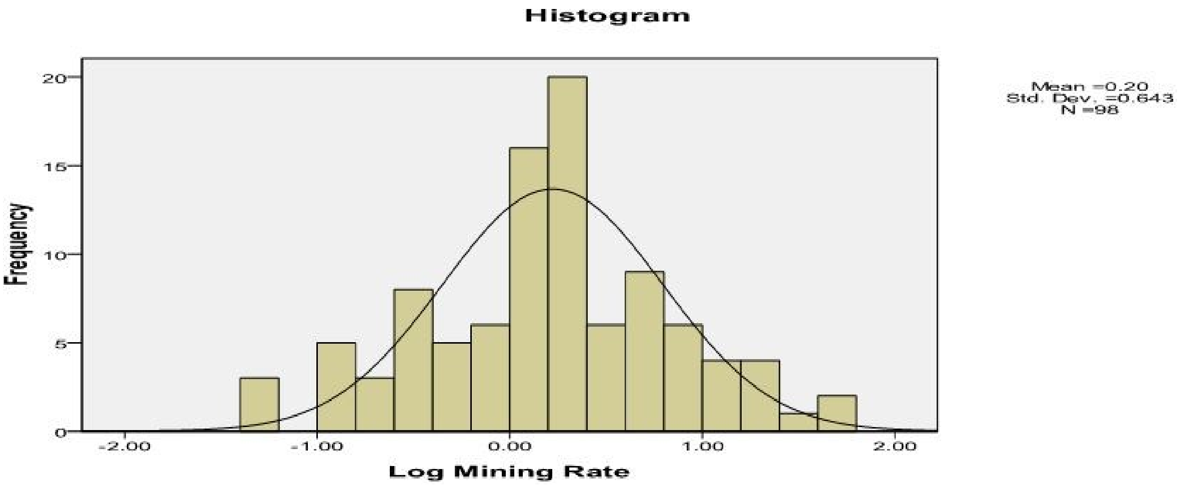

Fig 1 c. Log mining rate

Regression Analysis

Two multiple linear regression analyses using stepwise method of enter in the SPSS software were carried out in order to establish two regression models. The first one is the model of mining rate vs. resource variation and the second one is the model of mining rate vs. reserve variation. Stepwise method of enter was selected since it enables examine the contribution of each independent parameter entered in the analysis. In the first analysis, log mining rate was entered as dependent variable while log resource-tonnage and log resource-grade were entered as independent variables. Similarly, in the second analysis, log mining rate was entered as dependent variable while log reserve-tonnage and log reserve-grade were entered as independent variables.

C. Generated Regression Models of Mining Rate vs. Resource Variation

The main output of the first regression analysis as obtained in the ANOVA and model summary tables is that F ((1, 95) = 205.625, p = .000) with R^2 of .686. This indicates a very strong relationship between log mining rate and log resource-tonnage. Resource-tonnage accounted for 68.6 per cent of the total variation of the mining rate. The unaccounted 31.4 per cent may be contributed by other determinant parameters. Log resource-grade was removed from the analysis by SPSS Software. Coefficients of the independent variable are summarized in Table 2.

Table 2. Coefficients in the Resource Model

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-.670	.072		-9.342	.000
	Log Resource-tonnage	.636	.044	.828	14.340	.000

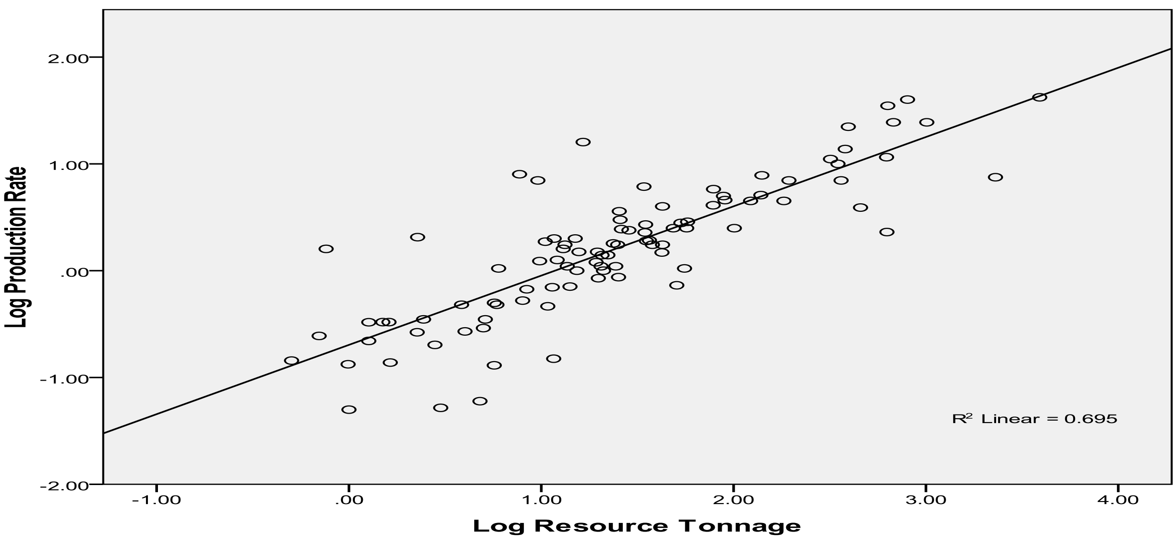

Fig. 2. Scatterplot for log mining rate against log resource tonnage

Based on the coefficients in Table 2, a significant regression model can be estimated in the logarithm and non-logarithm forms as (5) and (6), respectively.

$$Log\ Mining\ Rate = -0.67 + 0.636 \log Resource\ tonnage \quad (5)$$

$$Mining\ Rate\ (Mt) = 0.214\ Resource\ Tonnage^{0.636} \quad (6)$$

Fig. 2 provides further evidence of the linear relation between log mining rate and log resource.

D. Generated Regression Models for Mining Rate vs. Reserve Variation

Similarly, the main output of the second regression analysis as obtained in the ANOVA and model summary tables of the reserve-tonnage is that F (1, 87) = 290.954, *p* = .000) with R^2 of .77. This indicates very strong relationship between mining rate and reserve-tonnage. Reserve-tonnage accounted for 77 per cent of the total variation of the mining rate. The unaccounted 23 per cent may be contributed by other determinant parameters. Log reserve-grade was removed from the analysis by SPSS Software. Coefficients of the independent variable are summarized in Table 3.

Based on the coefficients in Table 3, a significant regression model can be estimated in the logarithm and non-logarithm forms as (7) and (8), respectively.

$$Log\ Mining\ Rate = -0.556 + 0.67 \log Reserve\ tonnage \quad (7)$$

$$Mining\ Rate\ (Mt) = 0.277\ Reserve\ Tonnage^{0.67} \quad (8)$$

Fig. 3 provides further evidence of the linear relation between log mining rate and log reserve-tonnage.

Table 3: Coefficients in the Reserve Model

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-.556	.055		-10.171	.000
	Log Reserve-tonnage	.670	.039	.877	17.057	.000

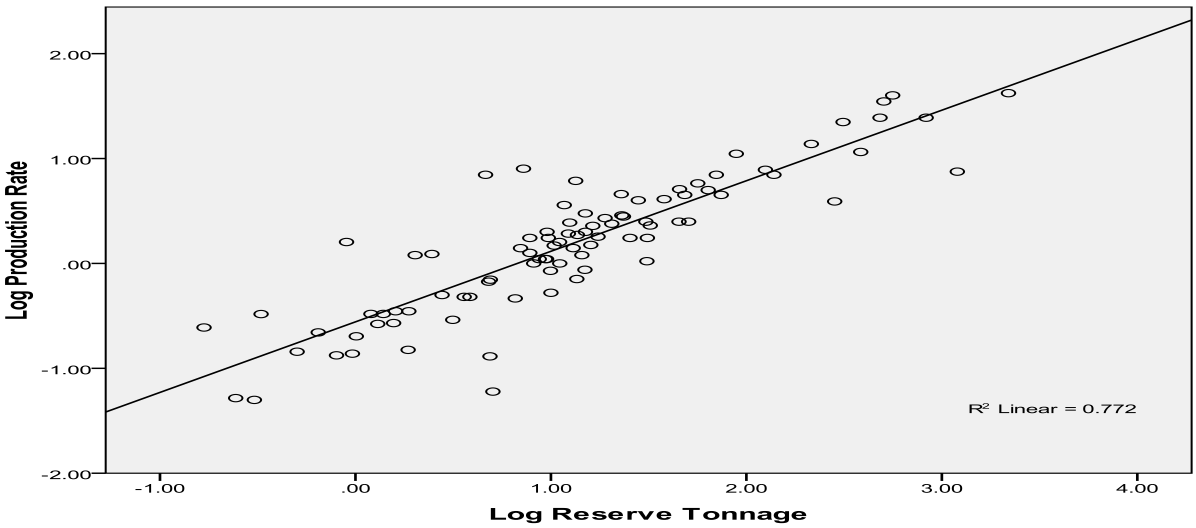

Fig.3. Scatterplot for log mining rate against log reserve tonnage

E. Model Validation

It was decided to validate the two models in two different ways; first - against each other and second - with the values of actual mining rates of known mines.

Comparison of Resource and Reserve Models

Hypothetical resource-tonnage and reserve-tonnage ranging from 0 to 1,800 Mt were selected for validation of the two models. Mining rates predicted by these models were comparable as shown in Fig. 4.

Comparison with Actual Data

The two models were validated based on known or actual data of mining rates, resource-tonnage and reserve-tonnage of gold mines/deposits in the stage of feasibility study whose data were available but were not included in the analysis. Percentage differences between actual and predicted values of mining rates were calculated and the results presented in Table 4. The table indicated that the models could predict mining rates well within the range of errors of ±20 per cent.

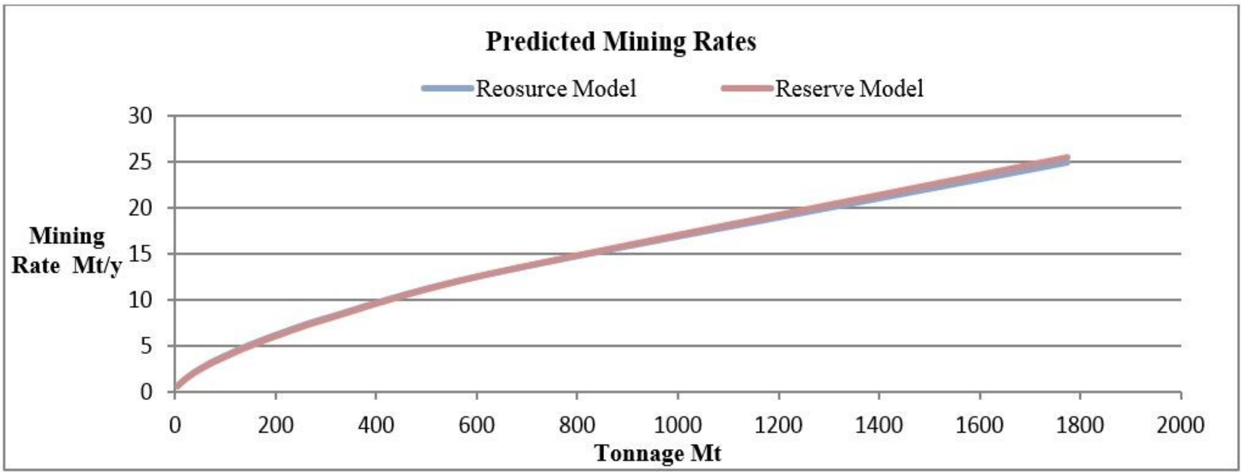

Fig. 4. Mining rates as predicted by resource and reserve models

Table 4. Results of Validation of Resource and Reserve Models

Mine/deposit	Planned Mining Rate	Resource Model		Reserve Model	
		Predicted mining rate	Percentage difference between predicted and plan	Predicted Mining rate	Percentage difference between predicted and plan
Donlin Creek	35	30.95	11.56	27.88	20.32
Lindero	10.8	9.85	8.79	8.60	20.32
Dublin Gulch	9.1	8.05	11.50	8.71	4.24
Esaase	7.5	7.06	5.85	7.19	4.10
Pan	5.5	4.47	18.66	4.72	14.14
Chertovo Koryt	2.2	2.66	-20.90	2.47	-12.51
Tokur	2	2.19	-9.53	1.72	13.64
Chaarat	1.9	2.03	-6.88	1.50	20.57
Quimsacocha	1.1	0.94	14.61	1.20	-9.59
Caspiche	21.7	21.93	-1.06	20.04	7.61
Bissa	1.575	1.48	5.79	1.78	-13.49

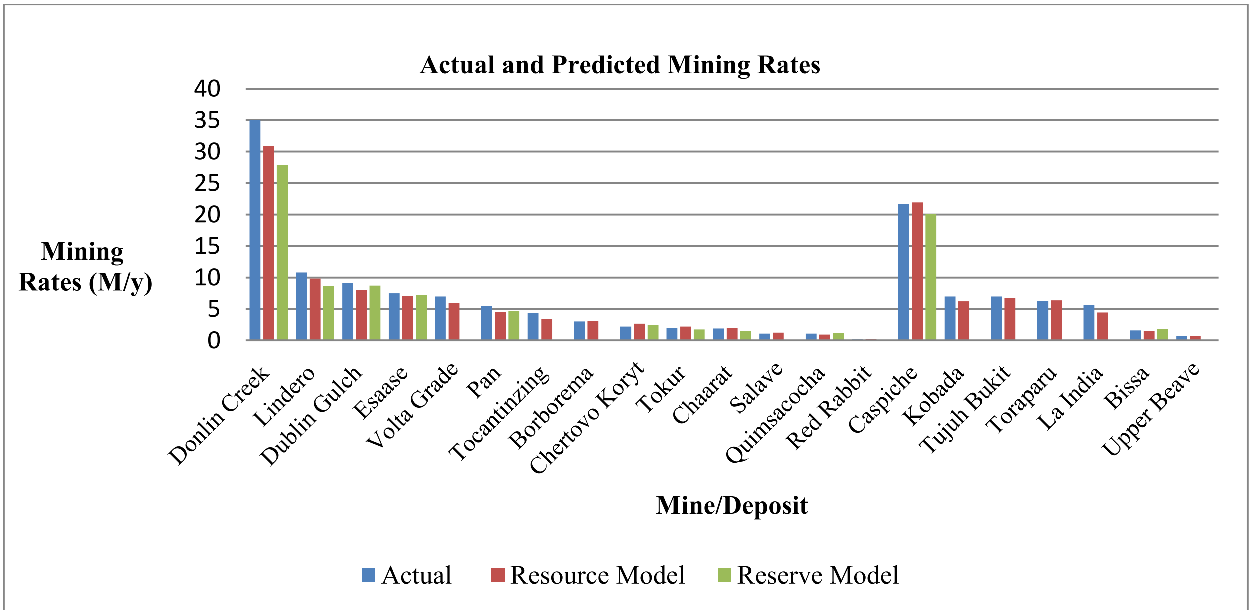

Fig. 5. Plan and predicted mining rates of various gold mines

Fig. 5 provides comparative results of actual and plan mining rates of various gold mines in the stage of feasibility study as predicted by resource and reserve models.

IV. Conclusions

The regression model summary presented in Table 2 and 3 allow concluding the following:

- Mining rate in the gold mining can be estimated based on the resource-tonnage and reserve-tonnage;
- Resource-tonnage in the resource model accounted for 68.6 per cent of the determinant variables of mining rate. The 31.4 per cent unaccounted is contributed by other variables;
- Similarly, reserve-tonnage in the reserve model accounted for 77 per cent of the determinant variables of mining rate. The 23 per cent unaccounted is contributed by other variables;
- The form of the models developed in this study appeared to be similar to other tonnage-based models in the sense that tonnage is the only determinant of mining rate and is characterized by curve. However, the main difference between the developed models and other tonnage-based models discussed in this paper lies on the values of coefficients a and b. Therefore, the results obtained from the developed models are not necessarily the same as those obtained from other models.

Depending on the availability of data, the regression models could be enhanced by adding more determinant parameters such as mining methods, technology used and presence of by-product to account for the unaccounted parameters in both resource and reserve models. Nevertheless, the regression models generated in this study could be used to predict mining rate of gold mine of similar characteristics used in this study in the stage of pre-feasibility, feasibility and construction stage in any country worldwide.

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References

[1] Z. Li, A theoretucal approach to determination of mine life and design capacity, *International Journal of Surface Mining,* Volume 3, 1989, pp 49-50

[2] C. S. Ji, and Y. D. Zhang, A criterion to optimize mine production rate: computer application in the mineral industry eds X. Wang, and Jiang, Swets & Zeldinger, Lisse, 2001.

[3] J. Liu, Determination of optimum mine's capacity with calculated least cost, *Coal Mine Design*, 1992, pp 47-49.

[4] H. K.Taylor, Rates of working mines; a simple rule of thumb: transactions of the institution of mining and metallurgy, 1977, Vol. 95, Section A, pp 2023-204.

[5] T. Dominski, B. Kolankowski, A. Marck, M. Pasternak, and S. Shamba (2017, November) Estimation of the potential production rate design, *Queens's University* Available: www.minewiki.engineering.queensu.ca.

[6] P. McCarthy, Production rate optimization: avoiding the tempetation of tonnage, strategic versus tactical aprroaches in mining, 2011, ed Potvin, Y Australian Centre for Geomechanics, Perth

[7] W. Hustrulid, and M. Kuchta, *Open pit mine: planning and design*, Volume 1, London, Taylors and Francis plc, 2006

[8] H. M. Wells "Optimization of mining engineering design in mineral valuation", *Mining Engineering*, Aspermont UK Team, Volume 30, No. 12, 1978

[9] D. S. Nilsson, "Advantages of high production levels in underground mining", *Mining Engineering*, Aspermont UK Team, Volume 34, No. 8, 1982,

[10] SAIMM (2000), *South Africa Code for Reporting of Mineral Resource and Reserve, the SAMREC Code*, Available: http://www.saimm.co.za,

[11] AusIMM (2005), *Australisia Code for the Reporting of Exploration Results, Mineral Resources and Ore Reserves: The JORC Code*, Available: http://www.jorc.org

[12] USGS (1980), *Principles of a resource/reserve classification for minerals*, USGS Circular 631

[13] CIM (2004), *Definition standards on mineral resources and mineral reseves,* Available: http://www.cim.org

[14] A. J. Sinclair, and G. H. Blackwell, *Applied mineral inventory estimation*, Cambridge, Cambridge University Press 2002

[15] B. Cavender, Determination of the optimum lifetime of a mining project using discounted cash flow and option pricing techniques, *Mining Engineering*, Volume. 44. 1992, pp 1262-1268

[16] Y. Lizotte, and J. Elbrond, Choice of mine-mill capacities and production schedules using Open ended dyanmic programming, *CIM Bulletin*, Volume 75, No. 839, 1982, pp 154-163,

[17] L. D. Smith, A critical examination of the methods and factors affecting the selection of an optimum production rate, *CIM Magazine*, Bulletin Volume. 90 No. 1007, February, 1997

[18] S. A. Sabour, "Mine size optimization using marginal analysis." *Resource Policy 28*, No 3, 2002, pp 145-151

[19] C. R. Tatman Production-rate selection for steeply dipping tabular deposits, *Mining Engineering*, Volume. 53, No 10, October:, 2001, pp 62-64

[20] H. K. Taylor, Rates of working mines: a simple rule of thumb: *Transactions of the Institution of Mining and Metallurgy*, Volume. 95, Section A, 1986, pp 2023-204

[21] D. A. Singer, D. W. Menzie, and K. R Long (1998). A simplified economic flter for open-pit gold-silver mining in the United States: *U.S Geological Survey Open-File Report* 98-207, 10 p., Available: http://geopubs.wr.usgs.gov/ope-file/of98-207/OFR98-207.pdf

[22] D. A Singer, D. W. Menzie, and K. R. Long (2000). A simplified economic flter for underground mining of massive sulfide depsoits. Available:http://geopubs.wr.usgs.gov/ope-file/of01-208/of01-218.pdf

[24] K. R. Long, and D. A Singer (2001), A simplified economic filter for open-pit mining and heap-leach. Available: http://geopubs.wr.usgs.gov/ope-file/of01-218/of1-218.pdf

[25] K. R. Long, (2009), A test and re-estimation of Taylor's empirical capacity-reseve relationship. Available: http://link.springer.com/article/10.1007%2Fs11053-009-90088-y.

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