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An Approach for Engineering Tuning of PID-Controller with Dynamic Object from Second Order

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***Abstract* – An approach is proposed for engineering adjustment of the PID-controller with a dynamic second order object. There is a proposal to solve the problem by solving the characteristic equation. As a result of the third row dynamic system analysis, the adjustment parameters of the PID-controller are calculated. The transitional processes of the closed system (object-controller) are dealt with by assignment and disturbance. For the transitional process by assignment, overshoot $\sigma = 4{,}0\ \%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0,6% inaccuracy is observed in theory. Therefore, the proposed approach for engineering adjustment for a PID-controller with a second-order dynamic object is suitable for use in third-order dynamic systems analysis.**

***Keywords* – PID-Controller, Tuning, Dynamic System, Second Order Object, Transfer Function.**

I. Introduction

Third order dynamic systems can be obtained in the following cases [1, 2, 3]:

- Object two aperiodic links operating with I-controller;
- Object two aperiodic links working with PI-controller;
- Object two aperiodic links, working with PID-controller with perfect differentiation;
- Object three aperiodic links operating with a P-controller;
- Object three aperiodic links, working with a PD-controller with perfect differentiation;
- Object oscillating link operating with PI-controller;
- Object oscillating link operating with PID-controller with ideal differentiation and
- Object oscillating link operating with a real-time differentiation first-order PD-controller.

The operation of the so-defined objects with stated linear controllers results in dynamic systems of third order. In the study of these systems - an analysis of the dynamic characteristics and determination of the desired adjustment of the controller, two approaches are used. The first approach addresses the universal methodologies and guidelines developed for more sophisticated systems. A great deal of these methodologies are also applicable to lower order systems. The second approach explores the differential equations of the second or third order, which is considered to be easy as the equations are relatively well studied.

As a disadvantage of the first approach, it can be noted that it does not always give accurate results. In some cases, it can not be used. Its advantage is that it is easier and more suitable for engineering work.

As a disadvantage of the second approach (above all for third order systems) it can be noted the relative complexity of the research in engineering work. An important advantage here is higher accuracy.

The theoretical investigation of third order systems is generally not complicated. For conducting quick and accurate engineering calculations related to tuning the controllers, there are not always suitable nomograms and formulas from the first approach.

II. Problems with the Adjustment of Controllers in third Order Systems

Third and higher order dynamic systems are often used in industrial automation systems for a variety of production processes, but due to their complexity, few authors have attempted to do theoretical research on them [6,7]. The complexity is that the roots of the characteristic equation of the closed ACS (automatic control system) is three, and it is not clear how the third real root influences the stability of the system, and hence the indicators of quality of the transitional processes.

III. Possible Options for Solution of the Assigned Task

In analyzing third order dynamic systems, the determination of dependencies between quality indicators and system parameters is considerably more complicated. One of the possible options for solving the task is through the use of Prof. Vishnegradski's diagram [1]. The diagram he suggests allows to judge not only sustainability but also some key quality indicators. In the study of dynamic systems of third order, he concluded that the nature of the transitional process can be determined without solving the characteristic equation of the system. For this purpose, it is sufficient for hyperbola built according to its parameters - X and Y to be supplemented with three auxiliary curves [1]. He has given an original word formulation of his criterion, which states: To be a dynamic third-order system sustainable, it is both necessary and sufficient to fulfill the following two conditions: 1. All the coefficients of the characteristic equation must be positive; 2. The average output minus the output of the final coefficients of the characteristic equation of the system must be positive. Failure to comply with these conditions will make the third order dynamic system unstable or at the limit of resistance.

Other possible options for solving this task are by using Ziegler & Nicols first method, Koppelovich's nomograms and nomograms given in [2]. These are methods for determining the parameters for adjusting the controllers by known data for the transitional characteristic of the control object [3, 5].

The purpose of this paper is to offer an engineering adjustment for a proportional-integral-differential PID-controller with a dynamic second order object by solving the characteristic equation of the closed system.

IV. Proposal for Solving the Problem by Solving the Characteristic Equation

Figure 1 shows the structural diagram of a ACS comprising a second order object (two aperiodic units) and a PID-controller.

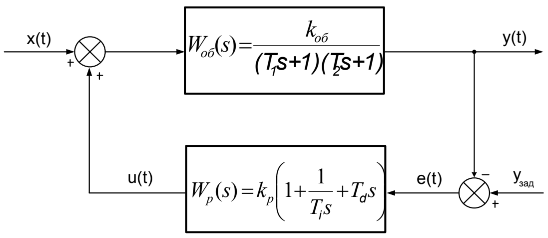


Fig. 1. A system with a second order object and a PID-controller

The transmission function of the closed system (fig.1) regarding the assignment is the type

$$W_{зад}(s)=\frac{Y(s)}{Y_{зад}(s)}=\frac{W_{об}(s).W_p(s)}{1+W_{об}(s)\cdot W_p(s)}=$$

$$=\frac{\frac{k_{об}}{(T_1s+1)(T_2s+1)}\cdot k_p\left(\frac{T_iT_ds^2+T_is+1}{T_is}\right)}{1+\frac{k_{об}}{(T_1s+1)(T_2s+1)}\cdot k_p\left(\frac{T_iT_ds^2+T_is+1}{T_is}\right)}=$$

$$=\frac{T_iT_ds^2+T_is+1}{\frac{T_1T_2T_i}{k_{об}k_p}s^3+\frac{\left(T_1+T_2+k_{об}k_pT_d\right)T_i}{k_{об}k_p}s^2+\frac{\left(1+k_{об}k_p\right)T_i}{k_{об}k_p}s+1}. \quad (1)$$

The transmission function of the closed system (fig.1) regarding the disturbance is the type

$$W_x(s)=\frac{Y(s)}{X(s)}=\frac{W_{об}(s)}{1+W_{об}(s)\cdot W_p(s)}=$$

$$=\frac{\frac{k_{об}}{(T_1s+1)(T_2s+1)}}{1+\frac{k_{об}}{(T_1s+1)(T_2s+1)}\cdot k_p\left(\frac{T_iT_ds^2+T_is+1}{T_is}\right)}=$$

$$=\frac{T_i}{k_p}\cdot\frac{s}{\frac{T_1T_2T_i}{k_{об}k_p}s^3+\frac{\left(T_1+T_2+k_{об}k_pT_d\right)T_i}{k_{об}k_p}s^2+\frac{(1+k_{об}k_p)T_i}{k_{об}k_p}s+1}. \quad (2)$$

We propose that the analysis of the third-order dynamic system be carried out with a successively connected oscillating and aperiodic link, i.

$$W(s)=\frac{1}{T_o^2s^2+2\xi T_0s+1}\cdot\frac{1}{Ts+1}. \quad (3)$$

Assuming that the time constant of the aperiodic link (first order low pass filter) is equal to the time constant of the oscillating link, i. $T=T_0$ is obtained

$$W(s)=\frac{1}{T_o^2s^2+2\xi T_0s+1}\cdot\frac{1}{T_0s+1}. \quad (4)$$

For the polynomial in the denominator of expression (4) the characteristic equation is obtained

$$(T_o^2s^2+2\xi T_0s+1)(T_0s+1)=T_o^3s^3+(2\xi+1)T_0^2s^2+(2\xi+1)T_0s+1 \quad (5)$$

If we equal the corresponding coefficients in front of s^3, s^2 etc. from the characteristic equation (5) to the coefficients of s^3, s^2 etc. of the polynomial in the denominator of expression (1), the transfer function of the closed system regarding the assignment will have the final appearance

$$W_{зад}(s)=k_{зад}\cdot\frac{T_iT_ds^2+T_is+1}{T_o^3s^3+(2\xi+1)T_0^2s^2+(2\xi+1)T_0s+1}, \quad (6)$$

where $k_{зад}=1$ is called a coefficent of the system assignment.

The transmission function of the closed disturbance system will have the final appearance

$$W_x(s)=k_x\cdot\frac{T_0s}{T_o^3s^3+(2\xi+1)T_0^2s^2+(2\xi+1)T_0s+1}, \quad (7)$$

where $k_x=\frac{T_i}{k_p}\cdot\frac{1}{T_0}=\frac{T_i}{k_p}\cdot\sqrt[3]{\frac{k_{об}k_p}{T_1T_2T_i}}=\sqrt[3]{\frac{T_i^2k_{об}}{k_p^2T_1T_2}}$ is called the system disturbance factor.

By comparing the coefficients in front of the corresponding degrees of s in the polynomials of expressions (1) and (6), dependencies between the parameters of the transition process and the parameters of the system can be determined. Equivalent time constant is

$$T_o=\sqrt[3]{\frac{T_1T_2T_i}{k_{об}k_p}}. \quad (8)$$

Similarly, the attenuation coefficient ξ is determined. For it two expressions of s^2 and s of (6) are obtained, ie.

The first expression that can be determined ξ is

$$(2\xi+1)T_0^2=\frac{\left(T_1+T_2+k_{об}k_pT_d\right)T_i}{k_{об}k_p}. \quad (9)$$

If we only express ξ we obtained

$$\xi = \frac{1}{2}\left[\frac{(T_1 + T_2 + k_{o\delta}k_pT_d)T_i}{T_0^2 k_{o\delta}k_p} - 1\right]. \qquad (10)$$

The second expression from which can be determined ξ is

$$(2\xi + 1)T_0 = \frac{(1 + k_{o\delta}k_p)T_i}{k_{o\delta}k_p}. \qquad (11)$$

If we express only ξ it is obtained

$$\xi = \frac{1}{2}\left[\frac{(1 + k_{o\delta}k_p)T_i}{T_0 k_{o\delta}k_p} - 1\right]. \qquad (12)$$

If the expressions (9) and (11) are divided into one another, it is obtained

$$T_o = \frac{T_1 + T_2 + k_{o\delta}k_pT_d}{1 + k_{o\delta}k_p}. \qquad (13)$$

If the expressions (10) and (12) are equal to one another, i.

$$\frac{1}{2}\left[\frac{(T_1 + T_2 + k_{o\delta}k_pT_d)T_i}{T_0^2 k_{o\delta}k_p} - 1\right] = \frac{1}{2}\left[\frac{(1 + k_{o\delta}k_p)T_i}{T_0 k_{o\delta}k_p} - 1\right] \qquad (14)$$

and then simplified, an expression of the type (13) is obtained. This confirms that the expressions (8) and (13) are equal, i

$$T_o = \sqrt[3]{\frac{T_1T_2T_i}{k_{o\delta}k_p}} = \frac{T_1 + T_2 + k_{o\delta}k_pT_d}{1 + k_{o\delta}k_p}. \qquad (15)$$

If an expression (15) is solved regarding the time constant of integration T_i, it is obtained

$$T_i = \frac{(T_1 + T_2 + k_{o\delta}k_pT_d)^3 k_{o\delta}k_p}{T_1T_2(1 + k_{o\delta}k_p)^3}. \qquad (16)$$

The time constant of differentiation T_d can be determined by an expression (10), ie.

$$T_d = \frac{(2\xi + 1)T_0^2 k_{o\delta}k_p - (T_1 + T_2)T_i}{k_{o\delta}k_pT_i} \qquad (17)$$

The proportionality coefficient of the controller k_p can be determined by an expression (11), ie.

$$k_p = \frac{T_i}{[(2\xi + 1)T_0 - T_i]k_{o\delta}} \qquad (18)$$

Example: Transitional process of object is given with two aperiodic links. The following algorithm performs the following:

1. Take the transitional process of the object that is smooth and normalizing.
2. Since the object model is of second order – fig. 2 (two consecutively connected aperiodic links with equal time constants) - the transitional characteristic is monotone with a transient delay, it is chosen to approximate the method of Ormans [4]. After the approximation, it is determined: $k_{o\delta} = 1$, $T_1 = T_2 = 19.5$ sec.

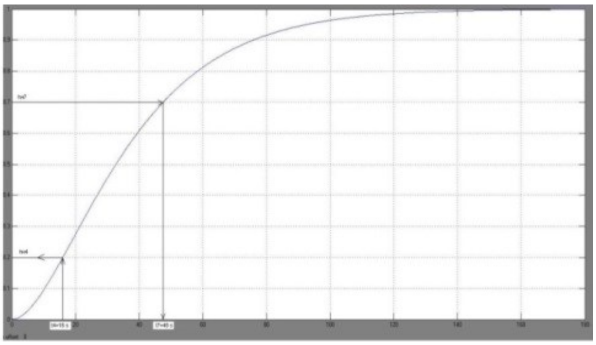


Fig. 2. Transitional process of control object

3. By the expressions (16), (17) and (18), the adjustment parameters of the PID-controller are calculated using the iteration procedure.

First, calculate the value of k_p, at a set value of $T_i \approx 3\tau_a$, where τ_a is determined after a Cupfmuler approximation

$$k_p = \frac{T_i}{[(2\xi + 1)T_0 - T_i]k_{o\delta}} = 1.027.$$

Then calculate the value of T_d again at the same set value of T_i

$$T_d = \frac{(2\xi + 1)T_0^2 k_{o\delta}k_p - (T_1 + T_2)T_i}{k_{o\delta}k_pT_i} = 4.24 \text{ sec}.$$

Finally T_i is calculated, if its value is close to the one above, the calculation procedure is terminated.

$$T_i = \frac{(T_1 + T_2 + k_{o\delta}k_pT_d)^3 k_{o\delta}k_p}{T_1T_2(1 + k_{o\delta}k_p)^3} = 25.793 \text{ sec}$$

If the value of T_i differs greatly from the set above, the calculation procedure starts from the beginning by selecting a value T_i, of so to minimize the difference between the set value and the value obtained.

4. By the expression (12) the damping factor ξ is calculated and approximately what is the value of the overshoot σ from [3]

$$\xi = \frac{1}{2}\left[\frac{(1 + k_{o\delta}k_p)T_i}{T_0 k_{o\delta}k_p} - 1\right] = 0,7$$

$$\sigma^2 = \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}.2\pi\right) = 0{,}0021$$ or only

$\sigma = 4{,}6\ \%$.

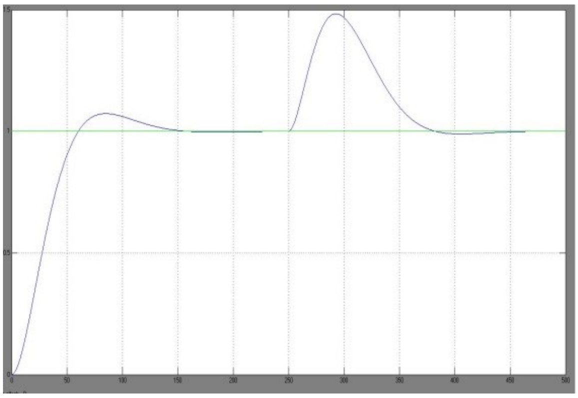


Fig. 3. Transitional processes by assignment and by disturbance

5. Determine the maximum dynamic deviation y_1 in the expression given in [3]

$$y_1 = \exp\left(-\frac{\zeta}{\sqrt{1-\zeta^2}}\right) = 0{,}37.$$

6. If any of the above two parameters does not meet the prerequisites for quality, adjust the controller.

The transitional processes of the closed system (fig.1) by assignment and by disturbance are shown in fig. 3. For the transitional process by assignment, overshoot $\sigma = 4{,}0\ \%$ occurs. If a comparison of the overshoot of the transitional process by assignment (obtained in simulation) is made, 0, 6 % inaccuracy is observed in theory. Therefore, the proposed sub-process for engineering adjustment of a PID-controller with a dynamic object with two aperiodic links is suitable for use in the analysis of third-order dynamic systems.

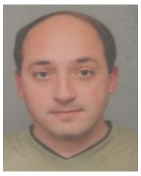
V. Conclusions

An approach is proposed for engineering adjustment of the PID-controller with a dynamic second order object (two aperiodic links with equal time constants). There is a proposal to solve the problem by solving the characteristic equation. As a result of the analysis of the third order dynamic system, the adjustment parameters of the PID-controller are calculated.

References

[1] N. Naplatanov, Theory of Automatic Control - Volume 1, Linear Systems, "Technika", Sofia, 1976, pp. 226-229.
[2] H. Hinov, S. Tsonkov et al. Automation of production - part 2. Automation of technological processes, "Technika", Sofia, 1978, pp. 170-173.
[3] I. Dragotinov, I. Ganchev, Automation of technological processes, UFT, Plovdiv, 2003, pp. 182-192.
[4] J. Badev, Identification of Systems Execise Guide, UFT, Plovdiv, 2013, pp. 44-49.
[5] G. Terziyski, I. Dragotinov, Automation identification of PI and PID controllers, Journal of the Technical University - Sofia, Plovdiv branch, Volume 21, book 1, 2015, pp. 59-64.
[6] H. Patel, S. Chaphekar, Developments in PID Controllers: Literature Survey, International Journal of Engineering Innovation & Research, Volume 1, Issue 5, 2012, pp. 425-430.
[7] J. Rana, R. Prasad, R. Agarwal, Designing of a Controller by using Model Order Reduction Techniques, International Journal of Engineering Innovation & Research, Volume 5, Issue 3, 2016, pp. 220-223.

Authors' Profiles



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