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Why we Live in a Penrose Fractal Pointless Noncommutative Multiverse :- A Simple Proof Using the Bijection Formula of E-Infinity Cantorian Spacetime

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Abstract – **This short paper gives in a nutshell a simple proof as to why we are living in a Penrose "pointless" fractal multiverse. The analysis is based on A. Connes' noncommutative geometry and the corresponding E-infinity formulation using its bijection formula.**

Keywords – **Noncommutative Pointless Geometry, E-infinity, Penrose Multiverse, Menger-Urysohn Theory, Bijection Formula, Golden Mean, Empty Set.**

I. Introduction

The archetypical example of A. Connes noncommutati--ve geometry [1]-[10] which at the same time the simplest and clearest is understandably the famous Penrose fractal tiling multiverse [7]-[9]. This generic and fundamental example is fully described by the von Neumann-Connes dimensional function as explained lucidly in Connes seminal work on noncommutative geometry [1], [2] as well as in various other important variations given later on by a number of authors [2], [5] [10] - [14].

The present short paper will argue that more than just a mathematical example, Penrose multiverse is a highly probable candidate for the true topological-geometrical and physical nature of reality, i.e. the cosmos we inhabit [7], [10], [13]. To this end we present a possible simple proof based on a reformulation of von Neumann-Connes' dimensional function using the so called bijection formula of E-infinity Cantorian spacetime theory in a fractal version of Kaluza-Klein five dimensional manifold [15]-[20]. We will start by introducing the basic notions of the Menger-Urysohn dimensional theory [21]-[23] and subsequently move directly to the reformulation of the noncommutative dimensional function using the E-infinity bijection [10]-[13]. Finally we show how this reformulati--on leads to results supported by mainstream cosmological measurements and observations on the large scale structure of the universe [24], [25] as well as various other well known facts on the ordinary scale of the classical world [26], [27].

II. The Menger-Urysohn Deductive Dimensional Theory

Fundamental to our analysis as well as to the von-Neumann-Connes dimensional function of noncommutati--ve geometry [1]-[11] is the understanding of the Menger-Urysohn deductive dimensional theory. The simplest and most intuitive way to explain this theory is to proceed using the elementary example of a three dimensional cube as follows [1]-[13]:

The cube is evidently three dimensional and its surface is two dimensional. Consequently we have the following deductive equation namely that a 3D cube gives a 2D surface. Consequently 2D surface leads to 1D lines and 1D lines have zero D end points. In simple terms we may write

$$D - 1 = n \qquad (1)$$

and then make an imaginative extrapolation to ask the following: what is the surface of a zero dimensional, i.e. point? Proceeding inductively it must be

$$\begin{aligned} D - 1 &= 0 - 1 \\ &= -1 \end{aligned} \qquad (2)$$

which is termed in topology the dimension of the neighborhood of an abstract point and is commonly referred to as "empty set". Two things come out of these simple but ingenious thoughts, which were introduced to mathematics independently by the Austrian-American mathematician Karl Menger and the distinguished Russian mathematician P. Urysohn who tragically died at a very tender age [18]-[21], namely the zero set and the empty set. Thus zero is clearly not empty. Leading mathematicia--ns working in the area at the time were then able to reason that the classical Cantor set is an example of a zero set with a topological Menger-Urysohn dimension equal zero. However why stop here since we can make a Cantor set increasingly "thinner" until it gradually disappears altogether [18]-[24]. This would correspond in our deductive equation to asking about the dimension of the neighbourhood of the neighbourhood of a point which would be $-1-1=-2$ and so on ad infinitum until we reach $n=-\infty$. In turn $-\infty$ would mean that our Cantor set vanishing completely. This vanishing corresponds to nothingness where there exists neither a question nor of course an answer. That way the notion of the degree of emptiness of an empty set was born [28]-[31] and used extensively in the last decades by the present author in making high energy physics and cosmology mathematically consistent [12]-[14]. Our main conclusions are thus the following:

(a) A classical Cantor set is topologically a zero set [28], [31].

(b) A thin Cantor set will start with the neighbourhood of the Cantor points which is the empty set with a topological dimension minus one [31]-[44].
(c) When the Cantor set becomes thinner and thinner, then at infinitely thin Cantor set will have a topological dimension minus infinity and the Cantor set vanishes completely [1]-[10].

That way we may stress emphatically that zero is neither nothing nor empty and similarly empty is not nothing and all the three notions should not be confused and must be held apart clearly in logic and mathematics as it is in transfinite set theory and that this must have a considerable bearing on reality, i.e. physics and cosmology [18]-[20].

On the other hand topological dimensions are too general and too specific and have nothing to say about some fine structural details of a manifold to capture the difference between smoothness and non-differentiability [12], [19], [32] let alone disjointment. This situation leads ultimately to the recognition by some of the finest applied and pure mathematics of this and the last century to teach us that we need two and not only one dimensions to capture all the quintessential differences and similarities between two distinct manifolds, namely a topological dimension and a Hausdorff fractal dimension corresponding to the topological dimension. We discuss this interesting and novel situation further in connection with the von Neumann-Connes dimensional function as well as its bijection formula equivalence of E-infinity theory [1], [10], [31], [32].

III. The Dimensions Function of Noncommutative Penrose Universe and the Bijection Formula of Cantorian Spacetime

It may be that the famous von- Neumann-Connes' dimensional function [1] of the generic example of a noncommutative manifold, namely the Penrose pointless fractal multiverse [41] is the first mainstream mathematical model which explicitly uses the golden mean and the Fibonacci properties in its fundamental equation. There have been a few theories which do similar things but they came later such as the four dimensional fusion algebra which links the topological quantum field theory with sub factors as well as the more recent golden anyons theory [14]. However all these theories seem to have come into existence after A. Connes work as well as the development of E-infinity theory which started independent of the work of Connes but then joined it and benefited considerably from Connes' great mathematical insights [1]-[14]. For these reasons we will concentrate on the work of A. Connes and the said dimensional function as well as its more lucid incarnation as a bijection formula [6]-[10] [18]-[20]. It is instructive to reverse the historical order and start with the rather more general and lucid bijection formula and then we will find that it is much easier to reconnect to the dimensional function of Penrose multiverse [7], [10]. In its most general form, the bijection formula states that the Hausdorff dimension of a relevant space $d_c^{(n)}$ as a function of the corresponding topological dimension n of the same space is given by [13], [34]

$$d_c^{(n)} = \left(1/d_c^{(o)}\right)^{n-1} \tag{3}$$

where $d_c^{(o)}$ is the backbone or zero set Hausdorff dimension of the space or fractal manifold in question. Setting $d_c^{(o)} = \phi$ where $\phi = \left(\sqrt{5}-1\right)/2$ is the golden mean is equivalent to identifying our backbone zero set with the Hausdorff dimension of a randomly constructed counterpart of the classical deterministic Cantor set which is given by $d_c^{(o)} = \ell n2 \;\; \ell n3 \quad 0.6309297536$ which is not that far off the random Cantor set of Mauldin and Williams with $d_c^{(o)} = \phi = 0.618033989$. Now let us proceed from the above to determining the Hausdorff dimension for the zero set and the empty set. For the zero set, i.e. $n=0$ the result is trivial because [20], [24], [32], [34]

$$\begin{aligned} d_c^{(o)} &= \left(1/\phi\right)^{n-1} \\ &= \left(1/\phi\right)^{-1} \\ &= \phi \end{aligned} \tag{4}$$

exactly as should be. For the empty set on the other hand, i.e. for $n=-1$ one finds [30]-[34]

$$\begin{aligned} d_c^{(-1)} &= \left(1/\phi\right)^{-1-1} \\ &= \left(1/\phi\right)^{-2} \\ &= \phi^2 \end{aligned} \tag{5}$$

It is not difficult to obtain the same result using the dimensional function although the relation between $n=0$, $n=-1$ and ϕ, ϕ^2 is not as clear. Let us write down the dimensional function first which reads [1]-[5]

$$D(a,b) = a + b(\phi),\ a,\ b \in Z \tag{6}$$

Taking a = 0 and b = 1 we find that

$$\begin{aligned} D(0,1) &= \phi \\ &= d_c^{(o)} \end{aligned} \tag{7}$$

which is the Hausdorff dimension of the zero set space. On the other hand, taking a = 1 and b = -1 we find that [1]-[10]

$$\begin{aligned} D(1,-1) &= \phi^2 \\ &= d_c^{(-1)} \end{aligned} \tag{8}$$

which is the Hausdorff dimension of the empty set. It is easily reasoned that the way of finding any Hausdorff dimension is to proceed in a Fibonacci manner [10], [27] starting from any two successive dimensions. The drawback in this case is that the unique correspondence between the topological dimension and its Hausdorff fractal dimensional partner is not as clear as in the case of using the bijection formula as the reader can ascertain by himself via going through these elementary computations. There are a few more remarkable things about the union and intersection of the zero set and the empty set and the resulting measures. First the union of the Hausdorff dimension leads to

$$\phi + \phi^2 = 1 \tag{9}$$

as far as the Hausdorff dimensions are concerned, which means the union leads to the unit interval. Consequently ϕ^2 must correspond to a fat Cantor set with positive measure because the zero set ϕ is by construction a thin Cantor set measure zero [11], [13], [19], [28]. By contrast the intersection of both sets leads to an emptier set

$$(\phi)(\phi^2) = \phi^3 \tag{10}$$

and while ϕ^3 is a normed topological probability of finding a Cantor "point" in the ϕ^3 set, the un-normed inverse probability $1/\phi^3$ leads to a fractal version of Einstein's spacetime D = 4, namely the by now famous [31]-[34]

$$D = d_c^{(4)} = 1/\phi^3 = 4 + \phi^3 \tag{11}$$

with the Russian doll-like continued fraction [13], [34]

$$\begin{aligned} D &= d_c^{(4)} \\ &= (1/\phi)^{4-1} \\ &= 4 + \phi^3 \\ &= 4 + \cfrac{1}{4 + \cfrac{1}{4 \ldots\ldots}} \end{aligned} \tag{12}$$

We have shown elsewhere [15], [17], [35], [36] every point in the fractal-like Einstein spacetime corresponding to equation 12 is also four dimensional and how adding a spin ½ degree of freedom, i.e. an additional Kaluza-Klein extra dimension to the fractal Einstein spacetime leads to a fractal version of the K-K theory with the remarkable dimension

$$\begin{aligned} D_F &= (K-K) = 1 + (4 + \phi^3) \\ &= 5 + \phi^3 \end{aligned} \tag{13}$$

as a combined fractal manifold [35,36].

IV. Some "Physical" Interpretations and the Main two Components of the Maximal Cosmic Energy Density of the Universe

Let us start from the maximal cosmic energy density arising from a world formula interpretation of the famous formula of special relativity $E = mc^2$ which we traced back in previous publications to the pioneering work of many scientists, in particular the remarkable $E = kmc^2$, $1/2 \le k \le 1$ of the great Russian physicist Nikolay Emov [37]. Rewriting it with a view on $D = 5 + \phi^3$ of the fractal Kaluza-Klein spacetime one finds [36], [37].

$$E = \left(\frac{5+\phi^3}{5+\phi^3} \right) mc^2 \tag{14}$$

and on separating the "fractal" irrational zero set part ϕ^3 from the integer one 5 and finding

$$\begin{aligned} E &= E(O) + E(D) \\ &= \left(\frac{\phi^3}{5+\phi^3} \right) + \left(\frac{5}{5+\phi^3} \right) \end{aligned} \tag{15}$$

From the preceding equation it becomes evident that

$$E(O) = \left[(\phi^3)/(5+\phi^3) \right] mc^2 \tag{16}$$

is the accurately measured ordinary energy density of our universe while

$$E(D) = \left[(5)/(5+\phi^3) \right] mc^2 \tag{17}$$

is the dark energy sector density which comprises the sum of pure dark energy density and the dark matter energy density. The result is in astounding agreement with accurate cosmic measurements and observations as well as with numerous previous analysis using a variety of methods and theories [35]-[37]. This alone shows that with a probability bordering on certainty, the Penrose pointless fractal tiling multiverse [41] is the topological-geometrical reality of the cosmos we are living in [32]-[37]. In retrospect we also see that E(O) being linked to the zero set fractal part ϕ^3 makes a great deal of sense because zero is looked upon naively as a point and a point in the mental picture we naively have for a particle. In other words the quantum particle is represented in our theory by a zero set quasi point without the problems associated with the infinity arising from the very nature of a point without any extensions which leads then to nasty singularities. However a Cantor point is something else all together uniting what naively could never be united for being a point which upon magnification, reveals itself as an entire Cantor set with uncountable infinitely many points and so on ad infinitum [13], [19], [34]. In other words one will not encounter the absurd concept of a classical point which has no extension and whenever we write the word point we mean a Cantor point which is hardly a point in any classical sense.

The empty set on the other hand is modelling the quantum wave as explained in various earlier publications [6]-[20] [40]-[41] and will not be discussed here any further as it is sufficient to direct the interested reader to the relevant literature [31], [32], [40], [41].

Our final word, and the point which we cannot stress strongly enough for attentive reader, is that E-infinity is a pointless spacetime theory in the sense of J. von Neumann and A. Connes and as such it is well equipped to be free of gauge anomalies and consequently free of 90 percent of the source of problems facing fundamental physics and cosmology [41]. This seemingly theoretical point is never the less of immense engineering practical consequence for designing energy reactors and fuelless space crafts [24]-[44].

V. Conclusion

Penrose fractal tiling multiverse is by construction golden mean proportioned and necessarily so as to leave no gaps and have no overlapping. It is that way on all scales since it is a fractal obeying fractal scaling as explained by many authors, including Sir Roger Penrose himself. It was the leading French pure mathematician and theoretical physicist A. Connes who first showed that Penrose fractal multiverse is a generic example for his noncommutative geometry proposal and by extending some aspect of von Neumann so called continuous and pointless geometry [8] [34]-[41], A. Connes was able to devise a golden mean based dimensional function, which describes this fractal manifold completely. The present author followed this conclusion partially independently and showed that Penrose fractal space is also a generic example for his E-infinity Cantorian spacetime manifold. That way E-infinity theory and noncommutative geometry fused together to a unit complimented by four dimensional fusion algebra as well as the golden mean Fibonacci theory which united fractional quantum Hall effect with Frank Wilczek anyons and via the present author the theory of ordinary and dark energy density sectors. In the course of doing all that it became clear that all the four golden mean based theories are basically equivalent. This would still not mean that Penrose fractal tiling is the reality of the cosmos we are living in, however taking the extra ordinary agreement of the various theoretical results with the accurate cosmic measurements and observations, as well as the golden mean based masses of the fundamental elementary particles of high energy physics, it seems plausible that the golden mean number system [13], [32], [34] is the language used by nature to express its blueprint master plan and it becomes almost a compelling conclusion that the Penrose fractal tiling is the optimal model which combines reality and the golden mean number system. This is not visible only on the quantum energy scale as well as on the large scale of the cosmos, but in fact it is observed in many things which we encounter on our own human scale be it in phyllotaxis or biology as well as visual art as well as classical music. The golden mean number system is really and truly universal and so is Sir Roger Penrose's fractal multiverse. With this we rest our case.

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AUTHOR'S PROFILE

Professor **M.S. El Naschie** was born in Cairo, Egypt on 10th October 1943. He received his elementary education in Egypt. He then moved to Germany where he received his college education and then his undergraduate education at the Technical University of Hannover where he earned his (Dipl-Ing) diploma, equivalent to a Master's degree in Engineering plus being a professional chartered engineer. After that he moved to the UK where he enlisted as a post graduate student in the stability research group of the late Lord Henry Chilver and obtained his Ph.D. degree in structural mechanics under the supervision of Professor J.M.T. Thompson, FRS. After his promotions up to the rank of full professor, he held various positions in the UK, Saudi Arabia and USA and was a visiting professor, senior scholar or adjunct professor in Surrey University, UK, Cornell, USA, Cambridge University, UK and Cairo University, Egypt. In 2012 he ran for the Presidency of Egypt but withdrew at the final stage and returned to academia and his beloved scientific research. He is presently a Distinguished Professor at the Dept. of Physics, Faculty of Science of the University of Alexandria, Egypt.

Professor El Naschie is well known for his research in structural stability in engineering as well as for his work on high energy physics and more recently for his work is cosmology and elucidation of the secret of dark energy and dark matter as well as for proposing a dark energy Casimir nanoreactor. He is the creator of E-infinity theory, which is a physical theory based on random Cantor sets and can be applied to micro, macro and mesoscopic systems.

Professor El Naschie is the single or joint author of about one thousand publications in engineering, physics, mathematics, cosmology and political science. His current h-index is 77 and his i-10 index is 762 and total citations are 32781 according to Google Scholar Citation.