One-Dimensional Radial Fin by Frobenius Method
Versus Two-Dimensional Straight Radial Fin

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Abstract – Fins or extended surfaces are extensively used in engineering applications to increase the efficiency of heat transfer of surfaces. Recent applications in compact heat exchangers increase interest in easy and applicable models for fin systems. A generalized one-dimensional radial fin model has been developed, where the modified power series expansion, the "Frobenius" method, is applied to a specific geometry. The comparison between two models, one-dimensional and two-dimensional, was presented to determine the thermal characteristics in a simple fin system. The one-dimensional model is suitable for compact fin systems, where the ratio is relatively low (K ≤ 6) and the Biot number is not very high (Bi < 0.1). The results obtained are promising and motivating, leading to the conclusion that the implementation of the generalized model should be effective.

Keywords – Extended Surfaces, Compact Heat Exchangers, Modified Power Series, Frobenius.

I. INTRODUCTION

Fins or the extended surfaces are extensively used in engineering applications to increase the heat transfer efficiency of surfaces. Once the temperature distribution through the fin is known, the heat transfer rate and the efficiency can be readily determined. A large variety of fins geometries are used in heat transfer application, and the longitudinal fin of rectangular profile and the one-dimensional radial fin are the most common used [01], [23], [25], [10], [28], [17], [06], [09], [26], [14], [21].

Because the practical importance of the extend surfaces, widely works in this subject is yet developed, and the applications in compact heat exchanger increase the interest in easy and applicable models for fins systems [13], [12], [24], [27], [18].

This work is directly connected to the solution of extended surfaces, as a special topic related to heat conduction theory, which is extension of the classical heat conduction mathematical formulation [07], [22], [03], ([19]; [20]), [11], [05]), [04], [15].

Was developed a generalized one-dimensional radial fin (Figure 01), where the expansion in modified power series, the Frobenius Method, is applied for a particular geometry (Figure 02). The two-dimensional straight radial fin, described for [08], was used as a reference for comparison.

II. THEORETICAL ANALYSIS

A. Frobenius Method

Consider steady-state, one-dimensional heat conduction through a radial fin, with constant conductivity, k and subjected an ambient temperature $T_\infty$. 
\[
\dot{q}_r - \dot{q}_{r+dr} = 2h_2(\alpha r + L_M)dr[T(r) - T_\infty] \quad \text{(01)}
\]

\[
\dot{q}_{r+dr} = \dot{q}_r - \frac{d\dot{q}_r}{dr}dr \quad \text{(02)}
\]

\[
\dot{q}(r) = -kA(r)\frac{dT}{dr} = -k\alpha r L_M \frac{dT}{dr} \quad \text{(03)}
\]

\[
\frac{d}{dr}\left[k\alpha r L_M \frac{dT(r)}{dr}\right] = 2h_2(\alpha r + L_M)[T(r) - T_\infty] \quad \text{(04)}
\]

\[
\frac{d}{dr}\left[r \frac{dT(r)}{dr}\right] = \frac{2h_2(\alpha r + L_M)}{k\alpha L_M}[T(r) - T_\infty] \quad \text{(05)}
\]

By definition:

\[
\theta(r) = \frac{T(r) - T_\infty}{T(r_i) - T_\infty} \quad \text{(06)}
\]

Then, we have:

\[
r^2 \frac{d^2\theta(r)}{dr^2} + r \frac{d\theta(r)}{dr} = \frac{2h_2(\alpha r^2 + L_M r)}{k\alpha L_M} \theta(R) \quad \text{(07)}
\]

\[
R = \frac{r - r_i}{r_o - r_i} \quad \text{with} \quad L_o = r_o - r_i \quad \text{(08)}
\]

\[
\left[R + \frac{r_i}{L_o}\right]^2 \frac{d^2\theta(R)}{dR^2} + \left[R + \frac{r_i}{L_o}\right] \frac{d\theta(R)}{dR} = \frac{2h_2[\alpha(L_o r + r_i)^2 + L_M(L_o R + r_i)]}{k\alpha L_M} \theta(R) \quad \text{(09)}
\]

\[
\mathbb{P}(R) \frac{d^2\theta(R)}{dR^2} + \mathbb{Q}(R) \frac{d\theta(R)}{dR} - \mathbb{W}(R)\theta(R) = 0 \quad \text{(10)}
\]

where

\[
\mathbb{P}(R) = P_1 R^2 + P_2 R + P_3; \quad \mathbb{Q}(R) = Q_1 R + Q_2; \quad \mathbb{W}(R) = W_1 R^2 + W_2 R + W_3 \quad \text{(11)}
\]

and

\[
P_1 = 1.0; \quad P_2 = 2K_1; \quad P_3 = K_1^2; \quad Q_1 = 1.0; \quad Q_2 = K_1; \quad W_1 = \frac{2\beta r^2}{\kappa_2}; \quad W_2 = B_{1a} \left[\frac{4K_2K_1}{\kappa_2}\right] + \frac{2K_1}{\alpha}; \quad W_3 = B_{1a} \left[\frac{2K_2K_1}{\kappa_2} + \frac{2K_1}{\alpha}\right] \quad \text{(12)}
\]

with dimensionless groups defined as:
\[ \begin{align*}
K &= \frac{L_\alpha}{w/2}; K_1 = \frac{r_i}{L_\alpha}; K_2 = \frac{L_M}{w/2}; w = \alpha r_i; L_M = w; B_{11} = \frac{h_2 w}{Z k}; B_{12} = \frac{h_2 w}{Z k} \\
\end{align*} \]

The proposal, in this work, for one validation of the general model (Figure 01), is realized a comparison with a “Two-Dimensional Straight Radial Fin”, presented by COTTA and MIKHAILOV (1997). In this case, the following simplifications were needed:

\[ B_{11} \to \infty; r_i = 0 \to K_1 = 0 \text{ and } \alpha \to \infty \]

Then,

\[ P_2 = 0; P_3 = 0; Q_2 = 0; W_2 = 0 \text{ and } W_3 = 0 \]

and

\[ R^2 \frac{d^2 \theta(R)}{dR^2} + R \frac{d\theta(R)}{dR} - W'_2 R^2 \theta(R) = 0 \]

For convenience, was defined

\[ K_2 = 2; \beta^2 = W_1 \text{ and } R' = \beta R \]

In this case,

\[ R'^2 \frac{d^2 \theta(R)}{dR'^2} + R' \frac{d\theta(R)}{dR'} - R'^2 \theta(R) = 0 \]

or

\[ \frac{1}{R'} \frac{d}{dR'} \left[ R' \frac{d\theta}{dR'} \right] - \theta(R') = 0 \]

In this work the equation 18 is more convenient, because the interest is in obtaining a particular solution of equation 10, by the expansion in modified series of power, called "Frobenius method" in the specialized literature. The Equation 18 has a singular regular point in \( R'=0 \), and By Georg Frobenius (1849-1917) [04] (1986, pag.243), [15] (1969, pag.190), [03] (1966, pag.231), [11] (1962, pag.143), [21] (1955, pag.46-59), [07] (1947, pag.374-376):

\[ \theta(R') = \sum_{n=0}^{\infty} a_n R'^{n+s} \]

\[ \theta'(R') = \frac{d\theta(R')}{dR'} = \sum_{n=1}^{\infty} a_{n-1} (n + s - 1) R'^{n+s} \]

\[ \theta''(R') = \frac{d^2 \theta(R')}{dR'^2} = \sum_{n=2}^{\infty} a_{n-2} (n + s - 2)(n + s - 3) R'^{n+s} \]

Then

\[ \mathcal{P}(R) \sum_{n=2}^{\infty} a_{n-2} (n + s - 2)(n + s - 3) R'^{n+s} + \mathcal{Q}(R) \sum_{n=1}^{\infty} a_{n-1} (n + s - 1) R'^{n+s} - \mathcal{W}(R) \sum_{n=0}^{\infty} a_n R'^{n+s} = 0 \]

or
\[ R^2 \sum_{n=0}^{\infty} a_n(n + s)(n + s - 1)R^{n+s-2} + R' \sum_{n=0}^{\infty} a_n(n + s)R^{n+s-1} - R^2 \sum_{n=0}^{\infty} a_nR^{n+s} = 0 \]

By algebraic manipulation, the following indicial equation was obtained:

\[ a_0[(s^2 - s) + s]R^s = 0 \quad \text{with} \quad a_0 \neq 0 \quad \text{and} \quad s = 0 \]

The roots of the indicial equation are equal zero and the recurrence rule is given by

\[ a_n = \frac{a_{n-2}}{n^2} \]

or

\[ a_2 = \frac{a_0}{2^2}; \quad a_4 = \frac{a_0}{2^44^2}; \quad a_6 = \frac{a_0}{2^44^26^2} \ldots \]

For the situation in analysis, two equal roots, there are two linearly independent solutions, which constitute a fundamental system of solution [15]. The first is:

\[ \theta_1(R) = 1 + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}(\beta R)^{2m} ; \quad a_{2m} = \frac{1}{2^{2m}(m!)^2} \]

The second linearly independent solution contains a logarithmic term and has a form:

\[ \theta_2(R) = [\ln(\beta R)]\theta_1(R) + \sum_{m=1,2,3,\ldots}^{\infty} A_m(\beta R)^m \]

By [07], and [04] the more convenient expression is

\[ \theta_2(R) = -\left[ \ln\left(\frac{\beta R}{2}\right) + \gamma \right] \theta_1(R) + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m} \]

where

\[ H_m = \frac{1}{m} + \frac{1}{m-1} + \ldots + \frac{1}{2} + 1 \quad \text{and} \quad \gamma \equiv 0.5772 \]

\( \gamma \) is known as the Euler-Mascheroni [04] (1986, pag.247) constant.

Then

\[ \theta(R) = a_0\theta_1(R) + a_1\theta_2(R) \]

\[ \theta(R) = a_0[1 + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}(\beta R)^{2m}] + a_1[-\left[ \ln\left(\frac{\beta R}{2}\right) + \gamma \right] \theta_1(R) + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m}] \]

or

\[ \theta(R) = a_0[1 + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}(\beta R)^{2m}] - a_1\left\{ \left[ \ln\left(\frac{\beta R}{2}\right) + \gamma \right] \left[ 1 + \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}(\beta R)^{2m} \right] - \sum_{m=1,2,3,\ldots}^{\infty} a_{2m}H_m(\beta R)^{2m} \right\} \]

The first boundary condition is defined by [08]:
$$\theta(0) = 1 \quad \Rightarrow \quad a_0 = 1 + a_1 \left[ \ln \left( \frac{\beta R_b}{2} \right) + \gamma \right]$$

Finally,

$$\theta(R) = \theta_1(R) + a_1 \left[ \ln \left( \frac{\beta R_b}{2} \right) + \gamma \right] \theta_1(R) + \theta_2(R)$$

$$\theta'(R) = \theta_1'(R) + a_1 \left[ \ln \left( \frac{\beta R_b}{2} \right) + \gamma \right] \theta_1'(R) + \theta_2'(R)$$

where (Figure 02)

$$R_b = R \rightarrow 0$$

For the second boundary conditions:

$$\theta'(1) = -B_{i2}K\theta(1)$$

Then,

$$-\left[ \theta_1(1) + B_{i2}K\theta_1'(1) \right] = a_1 \left[ \ln \left( \frac{\beta R_b}{2} \right) + \gamma \right] \left[ \theta_1(1) + B_{i2}K\theta_1'(1) \right] + \left[ \theta_2(1) + B_{i2}K\theta_2'(1) \right]$$

In this case,

$$a_1 = \frac{-\left[ \theta_1(1) + B_{i2}K\theta_1'(1) \right]}{\left[ \ln \left( \frac{\beta R_b}{2} \right) + \gamma \right] \left[ \theta_1(1) + B_{i2}K\theta_1'(1) \right] + \left[ \theta_2(1) + B_{i2}K\theta_2'(1) \right]}$$

The total exchange heat transfer is given by

$$\dot{q} = -k A_b (T_b - T_\infty) \theta'(0)$$

The dimensionless exchange heat transfer is written in the form, by definition

$$Q_b = \frac{\dot{q}}{h_2A_b(T_b - T_\infty)} \quad \Rightarrow \quad Q_b = \frac{-1}{B_{i2}K} \left( \frac{d\theta}{dR} \right)_{R=0}$$

$$A_b \text{ and } T_b \text{ are the base area and the base temperature respectively}$$

Efficiency is given by

$$\eta = \frac{-1}{B_{i2}K(1 + K)} \left( \frac{d\theta}{dR} \right)_{R=0}$$

**A. Straight Radial Fin**

The formulation, in dimensionless form, is written as:

$$\frac{1}{R} \frac{\partial}{\partial R} \left[ R \frac{\partial \theta(R,Y)}{\partial R} \right] + K' \frac{\partial^2 \theta(R,Y)}{\partial Y^2} = 0$$

with boundary conditions

$$\theta(R_b,Y) = 1; \quad \frac{\partial \theta(1,Y)}{\partial R} + B_{i2}K\theta(1,Y) = 0, \quad 0 \leq Y \leq 1$$
\[
\frac{\partial \theta(R,0)}{\partial Y} = 0; \quad \frac{\partial \theta(R,1)}{\partial Y} + B_{12}K\theta(R,1) = 0, \quad R_b \leq R \leq 1
\]

Fig. 2. Geometry System for Straight Radial Fin Analysis by [08].

**Dimensionless groups are defined as:**

\[
K = \frac{r_e}{w/2}; \quad R_b = \frac{r_b}{r_e}; \quad B_{12} = \frac{h_e(w^2)}{k}
\]

The exact solution of the Two-Dimensional Straight Radial Fin is obtainable by separation of variables, and the dimensionless average temperature at each circumferential section is given by [08]:

\[
\theta_{av}(R) = 2 \sum_{n=1}^{\infty} \frac{\sin^2 \lambda_n}{\lambda_n + \sin \lambda_n \cos \lambda_n} F(\lambda_n, R)
\]

where,

\[
F(\lambda_n, R) = \frac{\{I_0(\lambda_nKR)[B_{12}I_0(\lambda_nK) + \lambda_nI_1(\lambda_nK)] - I_0(\lambda_nKR)[B_{12}I_0(\lambda_nK) - \lambda_nI_1(\lambda_nK)]\}}{\{I_0(\lambda_nKR_b)[B_{12}I_0(\lambda_nK) + \lambda_nI_1(\lambda_nK)] - I_0(\lambda_nKR_b)[B_{12}I_0(\lambda_nK) - \lambda_nI_1(\lambda_nK)]\}}
\]

\(I_v, K_v\) are modified Bessel functions and the \(\lambda_n\)'s are obtained from the solution of the transcendental equation:

\[
\lambda_n \tan \lambda_n = B_{12}
\]

For small \(R\) [19] (1989, pag.493):

\[
I_n(R) \equiv \frac{1.0}{2^n n!} R^n
\]

\(K_n(R) \equiv -I_n R \quad \text{for} \ n = 0 \quad \text{and} \quad K_n(R) \equiv \frac{2^{n-1}(n-1)!}{R^n} \quad \text{for} \ n \neq 0
\]

For \(R \geq 10\):

\[
I_0(R) \equiv \frac{0.3989 e^{-R}}{R^2} \left\{1 + \frac{1}{8R} + \frac{9}{128R^2} + \frac{75}{1024R^3}\right\}
\]

\[
I_1(R) \equiv \frac{0.3989 e^{-R}}{R^2} \left\{1 + \frac{3}{8R} - \frac{15}{128R^2} + \frac{105}{1024R^3}\right\}
\]

\[
K_n(R) \equiv \frac{0.12533 e^{-R}}{R^2} \left\{1 - \frac{1}{8R} + \frac{9}{128R^2} - \frac{75}{1024R^3}\right\}
\]
\[ h_1(R) \equiv \frac{1.2533e^{-R}}{R^2}\left(1 + \frac{3}{8R} - \frac{15}{128R^2} + \frac{105}{1024R^3}\right) \]

For large \( R \):

\[ h_n(R) \equiv \frac{e^R}{\sqrt{2\pi R}} \]

\[ h_n(R) \equiv \frac{\pi}{\sqrt{2R}}e^{-R} \]

The total heat exchange through the fin’s base, in dimensionless form, is given by:

\[ Q_b = \sum_{n=1}^{\infty} \frac{\sin^2 \lambda_n}{[\lambda_n + \sin \lambda_n \cos \lambda_n]} G(\lambda_n) \]

where,

\[ G(\lambda_n) = \left\{ h_1(\lambda_n KR_b)B_{12} B_0(\lambda_n K) + \lambda_n B_1(\lambda_n K)\right\} + h_1(\lambda_n KR_b)B_{12} B_0(\lambda_n K) - \lambda_n h_1(\lambda_n K)\]

\[ \eta = \frac{Q_b}{B_{12} K^2} \]

Table 1. Obtained Results for Modified Bessel Functions.

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<th>X</th>
<th>K0(X)</th>
<th>K1(X)</th>
<th>K2(X)</th>
<th>K3(X)</th>
<th>K4(X)</th>
<th>K5(X)</th>
<th>K6(X)</th>
<th>I0(X)</th>
<th>I1(X)</th>
<th>I2(X)</th>
<th>I3(X)</th>
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<td>2.426E + 00</td>
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<td>1.003E + 00</td>
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<td>4.210E - 01</td>
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Bold: M. NECATTI ÖZISIK (1980; 1989)

Table 2. Six First Eigenvalue of the Equation 46

<table>
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<th>( B_{12} )</th>
<th>( \lambda_1 )</th>
<th>( \lambda_2 )</th>
<th>( \lambda_3 )</th>
<th>( \lambda_4 )</th>
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<td>12.5903</td>
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</table>
In Figure 04, below, the temperature is measured as a function of the radial position for aspect ratio K = 2. The comparison between the models for this aspect ratio value demonstrates that the models, one-dimensional and two-dimensional, present equivalent results for relatively low Biot number. The difference, for average temperature, is only noticeable for Biot number near and above 10.

Fig. 3. Implemented Modified Bessel Functions

III. RESULTS AND DISCUSSION

In Figure 04, below, the temperature is measured as a function of the radial position for aspect ratio K = 2. The comparison between the models for this aspect ratio value demonstrates that the models, one-dimensional and two-dimensional, present equivalent results for relatively low Biot number. The difference, for average temperature, is only noticeable for Biot number near and above 10.

Fig. 4. Average Temperature for Aspect Ratio K = 2 versus Biot Number.
Through Figure 05, with a $K = 6$ aspect ratio, it can be observed that the difference between the models, one-dimensional and two-dimensional, already occurs for Biot number values above 1.0. The same occurs for aspect ratio $K = 10$ (Figure 06).

Fig. 5. Average Temperature for Aspect Ratio $K = 6$ versus Biot Number.

Fig. 6. Average Temperature for Aspect Ratio $K = 10$ versus Biot Number.

Fig. 7. Average Temperature for Biot Number $B_{i2} = 20$ versus Aspect Ratio.
Figure 07, above, demonstrates that for high Biot number value, Biot equal to 20, even for low aspect ratio, the difference between models is noticeable and meaningful. The difference between the models, one-dimensional and two-dimensional, becomes more evident through the rate of heat transfer dimensionless, Figure 08.

The one-dimensional model works properly for low aspect ratio value, in a wide range of Biot number. In fact, the results obtained show that the one-dimensional model presented is suitable for compact systems, where the aspect ratio of the fin is low and the Biot number is not very high.

In Figure 09 there are values for efficiency according to the heat transfer coefficient, where the length of the base and the conductivity of the fin were obtained from an electric motor finned, with K close to 6. It is observed that there is a maximum efficiency for the exchange of heat in all cases. For K = 6 The maximum efficiency corresponds to an approximate value of 80 W/ (m². K), for the heat transfer coefficient, in Biot number less than 7.3 10-3.
IV. CONCLUSIONS

The comparison between two models, one-dimensional and two-dimensional, was presented to determine the thermal characteristics in a simple radial fin system.

The one-dimensional model is simpler and easier to be deployed than the two-dimensional model, as demonstrated by the formulation of Frobenius Method, implemented to a particular geometric situation, used in electrical motors.

The one-dimensional model is suitable for compact fins systems, where the aspect ratio is relatively low (K ≤ 6) and the number of Biot is not very high (Bi<0.1). The smaller the value of the aspect ratio, the greater the range of Biot number in which the one-dimensional model works properly.

It is a first approximation to the generalized one-dimensional model using the Frobenius method, and it can to be used, for example, in finned compact heat exchangers, because of the low aspect ratio.

The results obtained are promising and motivating, leading to the conclusion that the implementation of the generalized model should be effective.

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