International Journal of Engineering Innovation & Research
Volume 10, Issue 6, ISSN: 2277 – 5668

National Per Capita Income and Expenditure Forecasts Based on the GM(1,1) Model

Zenjing Di[1] and Hui Xu[1*]
[1]Department of Mathematics, Faculty of Science, Yanbian University, Yanji, China.
*Corresponding author email id: 361471648@qq.com

Date of publication (dd/mm/yyyy): 08/12/2021

Abstract **– With the initial implementation of the 14th Five-Year Plan, China's economy has shown a good trend of "stable growth and steady improvement", and the quality of life of the people has been steadily improving. Therefore, a reasonable forecast of the national residents' per capita income and expenditure level can reflect China's economic strength, and help the relevant departments to accurately examine the trend of China's residents' average consumption level and formulate and adjust national economic plans according to the forecast results. This paper uses the initial grey forecasting model as the basis, and improves the model by using the weighted sliding average method, and applies the optimized model to forecast the per capita income and expenditure of the nation's residents, so as to provide a scientific basis for promoting the sustainable development of the national economy.**

Keywords **– GM 1.1 Grey Forecasting, Weighted Sliding Average Method, National per Capita Income and Expenditure.**

I. INTRODUCTION

At present, China has experienced the year of achieving the goal of building a moderately prosperous society in all aspects and the closing year of winning the battle against poverty in all aspects. With the continuous introduction of relevant policies, China's economy is growing steadily and the living standard of residents is gradually improving. To study and analyze the income and expenditure level of China's residents at the current stage, establish a prediction model and make accurate forecasts is a prerequisite for ensuring the sustainable development of China's economy. With the introduction of grey system theory, in recent years, many experts and scholars have made use of the theory to conduct forecasting studies on the level of residents' income and expenditure. 2003, Artemisia Jianhua [1] published "The current situation of residents' Income and Expenditure in Xi'an and Modeling Analysis", which predicted the trend of Xi'an residents' per capita income and expenditure through grey model and linear regression model, providing a basis for decision-making in the new round of consumption. 2006, Hu Yu et al [2] quantified the relationship between rural residents' consumption structure and income level in "Grey correlation analysis and trend prediction of farmers' income and consumption structure in Hunan", and used a grey prediction model to forecast farmers' income level and various expenditures from 2006 to 2010, which pointed out the direction for the formulation of relevant economic policies. 2018, Zhu Xiaoyan et al. in "A study of urban residents' income and expenditure in Henan Province based on grey-regression combination prediction model the study of urban residents' income and expenditure in Henan Province"[3] combined the grey GM (1,1) model with the regression model, and used the simulated values of the regression model as the original data series of the grey GM(1,1) model for forecasting, which expanded the scope of application of the forecasting model and provided an idea to improve the accuracy of the original GM(1,1) model.

The increase in disposable income of the nation's residents has led to changes in people's consumption attitudes, and their sense of access, happiness and security has been enhanced, which to a certain extent has given a boost to the social economy. Therefore, it is important to establish a forecasting model and grasp the income and expenditure situation of China's residents in order to promote social stability and facilitate the stable and rapid development of China's economy.

II. Overview of the Grey GM(1,1) Model

In 1982, Professor Deng Julong of the Huazhong University of Technology proposed a new theory that extended the ideas and methods of cybernetics to socio-economic systems - the grey systems theory [4] - on the basis of repeatedly examining the problems of systems containing known and unknown information, and published the first international paper on grey systems "The control problems of grey systems" [5]. Since the establishment of grey systems theory, it has been successfully applied not only in the fields of engineering control, economic management and ecosystems, but also in water resources, meteorology, industry and biological control, etc. Considerable results have been achieved.

Fundamentals of the GM (1,1) Model[6]

1. *Cumulative Generation*

The original data sequence is known $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, a single accumulation of $x^{(0)}$ produces a sequence of

$$x^{(1)}=(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)) =(x^{(0)}(1),x^{(0)}(1)+x^{(0)}(2),\cdots,x^{(0)}(1)+\cdots+x^{(0)}(n)) \tag{1}$$

Where, $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i),k=1,2,\cdots,n$.....

2. *Building the Model*

Construct a sequence of background values from $x^{(1)}$:

$$z^{(1)}=(z^{(1)}(2),z^{(1)}(3),\cdots,z^{(1)}(n)) \tag{2}$$

Where, $z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1),k=2,3,\cdots,n.$

Establishing the grey differential equation ,

$$x^{(0)}(k)+az^{(1)}(k)=b,k=2,3,\cdots,n \tag{3}$$

The corresponding whitening differential equation is

$$\frac{dx^{(1)}}{dt}+ax^{(1)}(t)=b \tag{4}$$

3. *Establishing a Prediction Formula*

Set $u=[a,b]^T$, $Y=[x^{(0)}(2),x^{(0)}(3),\cdots,x^{(0)}(n)]^T$, $B=\begin{bmatrix}-z^{(1)}(2) & 1\\ -z^{(1)}(3) & 1\\ \vdots & \vdots\\ -z^{(1)}(n) & 1\end{bmatrix}$.

Then, by least squares, the estimate of u that minimizes $J(u)=(Y-Bu)^T(Y-Bu)$ is found to be,

$$\hat{u}=[\hat{a},\hat{b}]^T=(B^TB)^{-1}B^TY \tag{5}$$

Solving the equation gives $\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}},k=0,1,\cdots,n-1,\cdots$

A. *Analysis of the GM(1,1) model*

When using the GM (1,1) grey model for forecasting, it requires less raw data, does not need to consider the distribution of the data and the pattern of change, and is simple to operate and easy to test. As a result, GM(1,1) models have been used in many fields. Although the GM(1,1) model has many advantages, the dispersion of the data becomes larger when the original data becomes larger, and the accuracy of the prediction results will also deteriorate [7-8].

III. GM(1,1) Model Improvement

The sliding average method is a simple smoothing forecasting technique, which is based on the simple average method, by sequentially adding and subtracting old and new data period by period to calculate the moving average, strengthening the role of recent data and weakening the influence of distant data, thus effectively eliminating the chance variation factor in statistical data and reducing the error generated by randomness. In this paper, on the basis of reference [8] , we continue to use the weighted sliding average method to optimize the model more accurately, test the original data series, and then use the processed series to build an improved GM(1,1) model to obtain more accurate forecasting results.

A. *Introduction to the Sliding Average Method [9-10]*

1. *Simple Sliding Average Method*

The simple sliding average method treats all time series points as having the same degree of influence on the growth trend of the data, i.e. each element is equally weighted. The formula for the simple sliding average method is

$$Y_k=\frac{1}{n}(X_k+X_{k-1}+\cdots+X_{k-(n-1)}),k=n,n+1,\cdots,N \tag{6}$$

Where, $Y_k(k=n,n+1,\cdots,N)$ is the smoothed time series; $X_k(k=n,n+1,\cdots,N)$ is the original time series; n is the sliding window size; N is the length of the time series.

2. *Weighted Sliding Average Method*

The weighted sliding average method assigns unequal weights to the effect of values at different points in time within the same sliding window on the smoothing result. The weighted sliding average method is calculated using the formula:

$$Y_k=\sum_{i=-q}^{p}w_iX_{k+i},k=q+1,q+2,\cdots,N-p \tag{7}$$

Where, w_i is a weighting factor , and $\sum_{i=-q}^{p}w_i=1$； p,q is any positive integer less than n and $p+q+1=n$。

A. *Testing and Processing of Data[11]*

Assuming reference data of $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, the original sequence can be modified as follows: $y^{(0)}(k)=\sum_{i=-q}^{p} w_i x_{k+i}^{(0)}, k=q+1,q+2,\cdots,N-p$

Where, w_i is a weighting factor , and $\sum_{i=-q}^{p} w_i=1$; p,q is any positive integer less than n and $p+q+1=n$.

The modified sequence level ratios are

$$\lambda(k)=\frac{y^{(0)}(k-1)}{y^{(0)}(k)}, k=2,3,\cdots,n \tag{8}$$

where the sequence $x^{(0)}$ can be grey predicted as data for the model GM(1,1) if all the level ratios $\lambda(k)$ fall within the admissible coverage $\Theta=(e^{-\frac{2}{n+1}},e^{\frac{2}{n+2}})$.

Otherwise, some transformation of the sequence $x^{(0)}$ is also required so that it falls within the admissible cover.

For this case, an appropriate constant c can be taken and a translation transformation made $z^{(0)}(k)=y^{(0)}(k)+c, k=1,2,\cdots,n.$

Such that the sequence $z^{(0)}=(z^{(0)}(1),z^{(0)}(2),\cdots,z^{(0)}(n))$ has a step ratio $\lambda_y(k)=\frac{z^{(0)}(k-1)}{z^{(0)}(k)}\in\Theta, k=2,3,\cdots,n$.

B. *Building the Model*

A GM(1,1) model based on the whitening differential equation yields the predicted values

$$\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}k}+\frac{\hat{b}}{\hat{a}}, k=0,1,\cdots,n-1,\cdots \tag{9}$$

And $\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k), k=1,2,\cdots,n-1,\cdots$

C. *Test Predicted Values*

1. *Residual Test*

Let the residuals be $\varepsilon(k)$ and calculate $\varepsilon(k)=\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}, k=1,2,\cdots,n$

Where, $\hat{x}^{(0)}(1)=x^{(0)}(1)$.

If $\varepsilon(k)<0.2$, the general requirements are considered to be met; If $\varepsilon(k)<0.1$, the higher requirements are considered to be met.

2. *Grade Ratio Deviation Value Test*

The grade ratio $\lambda(k)$ is first calculated from the reference data $x^{(0)}(k-1), x^{(0)}(k)$ and then the corresponding grade deviation $\rho(k)=1-(\frac{1-0.5a}{1+0.5a})\lambda(k)$ is found using the development factor a.

If $\rho(k) < 0.2$, the general requirements are considered to be met; If $\rho(k) < 0.1$, the higher requirements are considered to be met.

IV. IMPROVED MODEL SUPERIORITY TEST

By looking up the yearbook, statistics on the national per capita income and expenditure from 2013 to 2019 are shown in Table I.

Table I. National per capita income and expenditure, 2013-2019.

Year	National per Capita Disposable Income Projection/ RMB 10,000	National per Capita Consumption Expenditure Forecast / RMB 10,000
2013	18310.80	13220.40
2014	20167.10	14491.40
2015	21966.20	15712.40
2016	23821.00	17110.70
2017	25973.80	18322.10
2018	28228.00	19853.10
2019	30732.80	21558.90

A. *Initial Grey Prediction Model*

The initial forecasting model was used to forecast national per capita income and expenditure from 2013 to 2019 and to compare it with actual values.

Table II. National per capita disposable income.

Year	Actual value / RMB million	Forecast Value / Million	Residuals	Relative Error	Grade Ratio Deviation
2013	1.83108	1.83108	0	0.0000%	
2014	2.01671	2.015309	0.001401	0.0695%	0.012290
2015	2.19662	2.192228	0.004392	0.2000%	0.001256
2016	2.38210	2.384678	-0.00258	0.1082%	-0.003138
2017	2.59738	2.594023	0.003357	0.1293%	0.002323
2018	2.82280	2.821746	0.001054	0.0374%	-0.000970
2019	3.07328	3.06946	0.00382	0.1243%	0.000820

Table III. National per capita consumption expenditure.

Year	Actual Value / RMB Million	Forecast Value / Million	Residuals	Relative Error	Grade Ratio Deviation
2013	1.32204	1.32204	0	0.0000%	
2014	1.44914	1.451538	-0.0024	0.1655%	0.013003
2015	1.57124	1.570336	0.000904	0.0576%	0.002186

Year	Actual Value / RMB Million	Forecast Value / Million	Residuals	Relative Error	Grade Ratio Deviation
2016	1.71107	1.698856	0.012214	0.7138%	0.006526
2017	1.83221	1.837895	-0.00568	0.3103%	-0.010356
2018	1.98531	1.988313	-0.003	0.1513%	0.001545
2019	2.15589	2.151042	0.004848	0.2249%	0.003715

B. *Improved Grey Prediction Model*

The improved grey forecasting model was used to forecast national per capita income and expenditure from 2013 to 2019 and to compare it with actual values.

Table IV. National per capita disposable income.

Year	Actual Value / RMB Million	Forecast Value / Million	Residuals	Relative Error	Grade Ratio Deviation
2013	1.83108	1.831266	-0.00019	0.0102%	
2014	2.01671	2.01699	-0.00028	0.0139%	0.011619
2015	2.19662	2.194078	0.002542	0.1157%	0.002319
2016	2.38210	2.386715	-0.00461	0.1937%	-0.001152
2017	2.59738	2.596264	0.001116	0.0430%	0.000597
2018	2.82280	2.824212	-0.00141	0.0500%	0.000014
2019	3.07328	3.072173	0.001107	0.0360%	-0.001045

Table V. National per capital consumption expenditure.

Year	Actual Value / RMB Million	Forecast Value / Million	Residuals	Relative Error	Grade Ratio Deviation
2013	1.32204	1.322167	-0.00013	0.0096%	
2014	1.44914	1.452227	-0.00309	0.2130%	0.012106
2015	1.57124	1.571272	-0.00003	0.0020%	0.004999
2016	1.71107	1.700076	0.01099	0.6425%	0.001982
2017	1.83221	1.839439	-0.00723	0.3945%	-0.004766
2018	1.98531	1.990225	-0.00492	0.2476%	-0.000299
2019	2.15589	2.153373	0.00252	0.1168%	0.001760

C. *Analysis*

By comparing the forecasting results of the initial grey forecasting model and the improved grey forecasting model, it can be found that the mean relative error of the initial grey forecasting model in forecasting the national per capita disposable income is 0.0907%, while the mean relative error of the improved model is 0.0661%; the mean relative error of the initial grey forecasting model in forecasting the national per capita consumption expenditure is 0.2331%, while The mean value of the relative error of the improved model was 0.2323%. Overall, it seems that the improved model has more accurate prediction results and is more feasible.

V. National Per Capita Income and Expenditure Projections

Combined with the above analysis, the improved GM(1,1) forecasting model has higher forecasting accuracy compared with the initial grey forecasting model. The improved GM(1,1) forecasting model was used to forecast the national per capita income and expenditure for the next six years, and the results are shown in Table VI.

Table VI. National per capita income and expenditure projections, 2020-2023.

Year	National Per Capita Disposable Income Projection / RMB 10,000	National Per Capita Consumption Expenditure Forecast / RMB 10,000
2020	3.341904	2.329894
2021	3.635318	2.520886
2022	3.954493	2.727534
2023.	4.301690	2.951122
2024	4.679371	3.193038
2025	5.090212	3.454785

VI. Conclusions

In this paper, based on the initial grey forecasting model, the initial model is optimized and improved by applying the weighted sliding average method to reduce the impact of random errors in the statistical process on the forecasts. A comparative analysis of the two models is also carried out. By comparing the actual and forecast values of the national per capita income and expenditure from 2013 to 2019, it is concluded that the improved model has the advantages of being simple to calculate and easy to test initially, and also has a higher forecasting accuracy. Finally, the model is applied to the forecasting of the national per capita income and expenditure, providing a direction for exploring the changes in the structure of the population's income and expenditure and for promoting supply-side structural reform.

References

[1] Artemisia. Current situation of residents' income and expenditure in Xi'an and modeling analysis [J]. Statistics and Information Forum, 2003(03):43-45.

[2] Hu Yu, Wang Xiong. Grey correlation analysis and trend prediction of farmers' income and consumption structure in Hunan [J]. Consumer Economics, 2006(06): 30-32+7.

[3] Zhu Xiaoyan, Li Bingjun, Yang Weiming. Research on urban residents' income and expenditure in Henan province based on combined grey-regression forecasting model[J]. Henan Science,2018,36(07):1149-1156.

[4] Grey system theory[J]. Journal of Party and Government Cadres,1992(04):46.

[5] DENG JL.Control problem of grey system[J].System and Control Letter,1982,1(5):288-294.

[6] Shoukui Si. Mathematical modeling algorithms and procedures [M]. Naval Aviation Engineering Institute, 554.

[7] Zhang Hai, Jiang Shi-Fang, Shi Kai-Quan. Theoretical flaws and improvement of grey prediction formula[J]. Systems Engineering Theory and Practice,2002(08):140-142.

[8] Hao J,Liu JM. Application of GM(1,1) model improvement technique to groundwater dynamic prediction in Xianyang city[J]. Soil and water conservation research,2011,18(03):252-254+259.

[9] LI Zheng, FANG Hongcai, KE Xizheng, LI Fang, WANG Yanning. Application of sliding average method to MEMS gyro signal trend term extraction[J]. Journal of Electronic Measurement and Instrumentation, 2019, 33(07): 43-49.

[10] Pei Yixuan,Guo Min. Basic principle and application of sliding average method [J]. Journal of Artillery Firing and Control, 2001(01): 21-23.

[11] Shoukui Si. Mathematical modeling algorithms and procedures [M]. Naval Aviation Engineering Institute, 554-555.

[12] 6 People's life 6-1 National per capita income and expenditure of residents. Liu Aihua, Ye Shicai, eds, China Statistical Yearbook, China Statistical Press, 2020, 170, Annals.

AUTHOR'S PROFILE

First Author
Zhenjing Di, Study in the department of mathematics of Yanbian University (Yanji, China).The thesis is a project of Jilin Province The paper is the result of the project "Algorithmic Model of Ordinary Differential Equations" (202010184009) of the Innovation and Entrepreneurship Training Program for University Students.

Second Author
Hui Xu, His major is mathematics. He is now an associate professor in the Department of Mathematics, School of Science, Yanbian University, and he is a master's tutor. His main research direction is mathematical education technology, mathematical modeling and intelligent algorithm.